Register Number:……………..

B.E DEGREE EXAMINATIONS: MAY 2017

(Regulation 2015)

Fourth Semester

COMPUTER SCIENCE AND ENGINEERING

U15MAT404: Partial Differential Equations and Transforms

COURSE OUTCOMES

CO1: Frame partial differential equations and get solution.

CO2: Know how to find the Fourier Series and half range Fourier Series of a function given explicitly or to find Fourier Series of numerical data using harmonic analysis.

CO3: Solve one dimensional wave equation, one dimensional heat equation and two dimensional heat equation in steady state using Fourier Series (Cartesian co-ordinates only).

CO4: Apply the Fourier transform, Fourier sine and cosine transform to certain functions and use Parseval's identity to evaluate integrals.

CO5: Evaluate Z-transform for certain functions.

CO6: Estimate Inverse Z-transform of certain functions and to solve difference equations using them.

Time: Three Hours **Maximum Marks: 100**

Answer all the Questions:-

PART A (10 x 1 = 10 Marks)

1. Match List I of types of partial differential equations with List II of their complete solutions CO1 [K_2]

List I	List II
A. $F(p,q)=0$	i. $f(z,a)=x+ay+c$
B. $F(x,p,q)=0$	ii. $z=ax+by+f(a,b)$
C. $z=px+qy+f(p,q)$	iii. $z=ax+\varphi(a)y+c$
D. $F(z,p,q)=0$	iv. $z=f(x,a)+ay+c$

	A	B	C	D
a)	ii	i	iii	iv
b)	iii	iv	ii	i
c)	i	iv	ii	iii
d)	iii	i	ii	iv

2. The steps involved in solving finite difference equations using Z-transform are CO6 [K_2]

1.The solution is y_n = sum of the residues of $Y(z)z^{n-1}$ at the poles.

2.Take Z-transform on both sides of the given difference equation.

3.Find $Z[y_n]=Y(z)$ using the expansions of Z- transforms on the difference equation.

4. Calculate $Y(z)z^{n-1}$ and hence find the poles.

a) 2-3-4-1 b) 1-3-2-4

c) 2-4-3-1 d) 4-2-3-1

3. Consider the following statements: CO2 [K_2]

1) In the process of finding Fourier series for a function by harmonic analysis,the successive harmonics increase rapidly.

2) $f(x)=\cos x$ is a periodic function with period π.

3) For the Fourier series expansion of a function $f(x)$ in the interval $(c,c+2\pi)$, $f(x)$ has finite number of maxima and minima.

4) When an odd function $f(x)$ is expanded in a Fourier series over the interval $(-\pi,\pi)$,the series will have cosine terms only.
Which of these statements are incorrect?

a) 1,3,4 b) 1,2,3

c) 1,2,4 d) 2,3,4

4. Match List I of Fourier transforms of functions with List II of their formulas. CO4 [K_1]

List I	List II
A. $F_s[xe^{-ax}]$	i. $\sqrt{\frac{2}{\pi}}\left(\frac{a^2-s^2}{(a^2+s^2)^2}\right)$
B. $F_s[f(ax)]$	ii. $\frac{1}{a}F_s\left(\frac{s}{a}\right)$
C. $F_c[xe^{-ax}]$	iii. $\sqrt{\frac{2}{\pi}}\frac{2as}{(a^2+s^2)^2}$
D. $F_c[f(ax)]$	iv. $\frac{1}{a}F_c\left(\frac{s}{a}\right)$

	A	B	C	D
a)	ii	i	iii	iv
b)	iii	iv	ii	i
c)	i	iv	ii	iii
d)	iii	ii	i	iv

5. Examine the two statements carefully and select the answer using the codes given below: CO2 [K_1]

Assertion(A): The root mean square value of the function $y = x^2$ in $(-\pi,\pi)$ is $\frac{\pi^2}{\sqrt{5}}$.

Reason (R): R.M.S value of $f(x)$ in the interval $(-l,l)$ is $\bar{y} = \sqrt{\frac{1}{2l}\int_{-l}^{l}[f(x)]^2\,dx}$

a) A is false but R is true

b) Both A and R are individually true but R is not the correct explanation of A

c) A is true but R is false

d) Both A and R are individually true and R is the correct explanation of A

6. If $F_s\left(\frac{e^{-ax}}{x}\right) = \sqrt{\frac{2}{\pi}}\tan^{-1}\left(\frac{s}{a}\right)$, then $F_s\left(\frac{1}{x}\right)$ is CO4 [K_3]

a) $\frac{\pi}{2}$ b) $\frac{\pi}{4}$

c) $\frac{\sqrt{\pi}}{2}$ d) $\sqrt{\frac{\pi}{2}}$

7. The steps involved in obtaining the solution of a Lagrange's linear equation are CO1 [K_2]
1. The general solution is given by $u = f(v)$ or $v = f(u)$ where the function is arbitrary.
2. Form the subsidiary equations $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$.
3. Obtain two independent solutions $u = a$ and $v = b$ from these equations.
4.Find P, Q, R as functions of x, y, z from the linear partial differential equation $Pp + Qq = R$

a) 2-3-4-1 b) 1-3-2-4
c) 3-4-2-1 d) 4-2-3-1

8. Consider the following statements: CO3 [K_1]
1) In the transverse vibration of a stretched string, the effect of friction is negligible.
2) In one dimensional heat flow,the rate of heat flow across an area is proportional to the area and temperature gradient parallel to the area .
3) In two dimensional heat flow, the flow is along curves lying in parallel planes instead of straight lines.
4) A differential equation may be elliptic in one region, parabolic in another and hyperbolic in some other region.
Which of these statements are correct?
a) 1,2,4 b) 1,3,4
c) 2,3,4 d) 1,2,3

9. Examine the two statements carefully and select the answer using the codes given below: CO3 [K_2]
Assertion(A):The temperature distribution of a metal plate in steady-state is $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$.
Reason (R): In steady state, the temperature u is independent of t so that $\frac{\partial u}{\partial t}=0$.
a) Both A and R are individually true and R is the correct explanation of A
b) Both A and R are individually true but R is not the correct explanation of A
c) A is true but R is false
d) A is false but R is true

10. Z-transform of $n\,a^n$ is CO5 [K_1]
a) $\frac{z}{(z-a)^2}$ b) $\frac{az}{(z+a)^2}$
c) $\frac{az}{(z-a)^2}$ d) $\frac{z}{(a+z)^2}$

PART B (10 x 2 = 20 Marks)
(Answer not more than 40 words)

11. Form the partial differential equation by eliminating the arbitrary functions from $z=f(x^2+y^2)$ CO1 [K_3
12. Find the complete integral of the partial differential equation $p=2qx$ CO1 [K_2]
13. Find the constant term in the Fourier series expansion of $f(x)=x^2+x$ in $(-2,2)$. CO2 [K_3]
14. If the half-range sine series of $x(\pi-x)$ in $0\le x\le\pi$ is $\sum b_n \sin nx$, find the value of $\sum b_n^2$ CO2 [K_2]
15. Classify the partial differential equation: $\frac{\partial^2 u}{\partial x\partial y}=\left(\frac{\partial u}{\partial x}\right)\left(\frac{\partial u}{\partial y}\right)+xy$ CO3 [K_2]
16. A rod of 30 cm long has its ends A and B kept at $20^\circ c$ and $80^\circ c$ respectively until the steady state conditions prevail. Find the steady state temperature in the rod. CO3 [K_3]
17. Find the Fourier sine transform of $\frac{1}{x}$ CO4 [K_3]
18. If $F[f(x)]=F(s)$, then prove that $F[e^{iax}f(x)]=F(s+a)$, CO4 [K_2]
19. Find the value of $Z\{f(n)\}$ when $f(n)=na^n$ CO5 [K_3]
20. State the final value theorem under Z-transform. CO5 [K_2]

Answer any FIVE Questions:-
PART C (5 x 14 = 70 Marks)
(Answer not more than 300 words)

Q.No. 21 is Compulsory

21. (i) Solve: $x(y^2+z)p - y(x^2+z)q = z(x^2-y^2)$ (7) CO1 [K$_3$]

(ii) Find the Fourier transform of $f(x)$ given by $f(x)=\begin{cases}1, |x|<a\\ 0, |x|>a>0\end{cases}$ and hence evaluate $\int_0^\infty \frac{\sin x}{x}dx$. (7) CO4 [K$_4$]

22. (i) Solve: $z = px + qy - 4p^2q^2$ (7) CO1 [K$_3$]

(ii) Solve: $(D^2+2DD'+D'^2)z = \sinh(x+y)+e^{x+2y}$ (7) CO1 [K$_3$]

23. (i) Find the half-range sine series of $f(x)=l-x$ in $(0,l)$. Hence prove that $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\cdots\cdots\cdots\infty=\frac{\pi^2}{6}$. (7) CO2 [K$_4$]

(ii) A function $y=f(x)$ is given by the following table of values. Make a harmonic analysis of the function in $(0,T)$ upto the second harmonic by taking $\theta=\frac{2\pi}{T}x$. (7) CO2 [K$_3$]

x:	0	$T/6$	$T/3$	$T/2$	$2T/3$	$5T/6$	T
y:	0	9.2	14.4	17.8	17.3	11.7	0

24. A rectangular plate is bounded by lines $x=0$, $x=a$, $y=0$ and $y=b$ and the edge temperatures are $u(0,y)=0$, $u(x,b)=0$, $u(a,y)=0$ and $u(x,0)=5\sin\frac{5\pi x}{a}+3\sin\frac{3\pi x}{a}$. Find the steady-state temperature distribution at any point of the plate. CO3 [K$_5$]

25. (i) Find the Fourier cosine transform of $f(x)=\begin{cases}x, 0<x<1\\ 2-x, 1<x<2\\ 0, x>2\end{cases}$ (7) CO4 [K$_3$]

(ii) Using Parseval's identity, evaluate $\int_0^\infty \frac{dx}{(x^2+a^2)^2}$ and $\int_0^\infty \frac{x^2}{(x^2+a^2)^2}dx$ if $a>0$ (7) CO4 [K$_4$]

26. (i) Find the inverse Z-transform of $\frac{z(z+1)}{(z-1)^3}$ using residue method. (7) CO5 [K$_5$]

(ii) Solve the equation $y_{n+2}-5y_{n+1}+6y_n=4^n$ given $y(0)=0, y(1)=1$ (7) CO5 [K$_3$]

27. A string is stretched and fastened to two points l apart. Motion is started by displacing the string into the form $y=3(lx-x^2)$ from which it is released at time $t=0$. Find the displacement of any point on the string at a distance of x from one end at any time t. CO3 [K$_5$]

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