

B.E DEGREE EXAMINATIONS: APRIL/MAY 2012

Fourth Semester

ELECTRONICS AND COMMUNICATION ENGINEERING

MAT107: Random Process and Vector Spaces

Time: Three Hours

Maximum Marks: 100

Answer All Questions:-

PARTA (10 x 1 = 10 Marks)

1. Probability of an impossible event is
a) 0 b) 1 c) -1 d) ∞
2. If A and B are independent and $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{4}$ then $P(A \cap B)$ is
a) $\frac{7}{12}$ b) $\frac{1}{12}$ c) $\frac{1}{9}$ d) $\frac{1}{16}$.
3. Standard deviation of Binomial distribution is
a) npq b) np c) \sqrt{np} d) \sqrt{npq} .
4. If a normal distribution has mean 25 then its mode is
a) 15 b) 50 c) 25 d) 20
5. Which of the following correlation coefficient shows the lowest degree of association?
a) -0.95 b) 0.75 c) 0.38 d) 0.1
6. When the two regression lines coincide then r is
a) 0 b) -1 c) 1 d) 0.5
7. A random process that is not stationary in any sense is called
a) Wide sense stationary b) Evolutionary Process
c) Ergodic Process d) Weakly stationary process
8. For a Poisson process mean and variance are
a) Equal b) Not equal c) Mean greater than variance d) Variance greater than mean
9. If all α_i 's are zero then the vectors are
a) dependent b) linearly dependent c) independent d) linearly independent
10. In the usual notation, $\|x\|$ is given by
a) \sqrt{x} b) $x \cdot x$ c) $\sqrt{x \cdot x}$ d) x^2

PART B (10 x 2 = 20 Marks)

11. A bag contains 3 Red and 4 White balls. Two draws are made without replacement. What is the probability that both the balls are Red.?

12. Find the value of k for the following density function $f(x) = k e^{-|x|}$, $-\infty < x < \infty$.
13. If x is uniformly distributed random variable with mean 1 and variance $\frac{4}{3}$, find $P(x < 0)$.
14. Define the exponential distribution.
15. Let x and y have joint distribution function $f(x, y) = 2, 0 < x < y < 1$. Find the marginal density function of x .
16. The following data were available $\bar{x} = 970, \bar{y} = 18, \sigma_x = 38, \sigma_y = 2, r = 0.6$. Find the line of regression and obtain the value of x when $y = 20$.
17. State any two properties of Poisson Process.
18. Find the mean of a stationary random process whose auto correlation is $18 + \frac{2}{6+\tau^2}$.
19. Define an inner product space.
20. Define an ortho normal set.

PART C (5 x 14 = 70 Marks)

21. a) A bag contains 5 balls and it is not known how many of them are white. Two balls are drawn at random from the bag and they are noted to be white. What is the chance that all the balls in the bag are white?

(OR)

- b) A continuous RV has pdf $f(x) = \begin{cases} a + bx, & 0 \leq x \leq 1 \\ 0, & \text{otherwise} \end{cases}$. If the mean of the distribution is $\frac{1}{2}$ then find a and b . Also find $\text{var}(x)$.

22. a) In a large consignment of electric bulbs 10% are defective. A random sample of 20 is taken for inspection. Find the probability that
 - (i) All are good bulbs
 - (ii) At most 3 defective bulbs
 - (iii) Exactly there are 3 defective bulbs.

(OR)

- b) In a certain factory turning razor blades there is a small chance of $\frac{1}{500}$ for any blade to be defective. The blades are in packets of 10. Use Poisson distribution to calculate the approximate number of packets containing
 - (i) No defective
 - (ii) One defective

(iii) Two defective blades respectively in a consignment of 10,000 packets.

23. a) If the joint pdf x and y is given by $f(x, y) = \begin{cases} \frac{1}{8} (6 - x - y), & 0 < x < 2, 2 < y < 4 \\ 0, & \end{cases}$

Find $P(x < 1 \cap y < 3)$ and $P\left(x < \frac{1}{y} / < 3\right)$.

(OR)

b) The two lines of regression are $8x - 10y + 66 = 0$

$$40x - 18y - 214 = 0$$

The variance of x is 9, Find

(i) The mean values of x and y

(ii) Correlation coefficient between x and y .

24. a) If customers arrive at a counter in accordance with a Poisson Process with a mean rate of 3 per minute, find the probability that the interval between 2 consecutive arrivals is

(i) More than 1 minute

(ii) Between 1 minute and 2 minutes

(iii) 4 Minutes or less

(OR)

b) Three Girls G1, G2, G3, are throwing a ball to each other. G1 always throws the ball to G2 and G2 always throws the ball to G3, but G3 is just as likely to throws the ball to G2 as to G1. Prove that the process is Markovian. Find the transition matrix.

25. a) Verify whether the following vectors are linearly

(i) dependent or independent? If they are dependent give their linear combination

$$v_1 = (1, 2, 0), \quad v_2 = (0, 3, 1), \quad v_3 = (-1, 0, 1)$$

(ii) Prove that the union of two subspaces need not be a subspace

(OR)

b) (i) State and prove the Schwarz inequality.

(ii) By using Gram-Schmidt process, construct the orthonormal basis for $V_3^{\mathbb{R}}$ with standard inner product for the basis $(1, 0, 1), (1, 3, 1), (3, 2, 1)$.
