

B.E DEGREE EXAMINATIONS: NOV/DEC 2014

(Regulation 2009)

Fourth Semester

ELECTRONICS AND COMMUNICATION ENGINEERING

MAT107: Random Processes And Vector Spaces

Time: Three Hours

Maximum Marks: 100

Answer all the Questions:-

PART A (10 x 1 = 10 Marks)

1. A typist types 2 letters erroneously for every 100 letters. The probability that the tenth letter typed is the first letter with error is
 - a) 0.0002
 - b) 0.0167
 - c) 0.1484
 - d) 0.0073
2. If X is a random variable with $E(X) = 1$ and $\text{Var}(X) = 4$. The $E(X^2) =$
 - a) 3
 - b) 5
 - c) 4
 - d) 6
3. If $b_{xy} = 0.8$ $b_{yx} = 0.45$, then $r =$ _____
 - a) $r = 0.36$
 - b) $r = 0.6$
 - c) $r = 0.8$
 - d) $r = 0.45$
4. If X and Y are independent random variables with variances 2 and 3, then the variance of $2x + 3y$ is
 - a) 34
 - b) 35
 - c) 36
 - d) 37
5. If both T (index set) and S (state space) are discrete, then the random process is called a _____
 - a) Discrete random process
 - b) Continuous random process
 - c) Discrete random sequence
 - d) Continuous random sequence
6. A non null persistent and a periodic state is _____
 - a) Markov
 - b) Stationary
 - c) reducible
 - d) Ergodic
7. If the input to a time – invariant, stable linear system is a W.S.S process, then the output will also be a -----

b) (i) If $P(X=x) = \frac{x}{15}$, $x=1,2,3,4,5$. Find (i) $P(X=1 \text{ or } X=2)$ (6)

(ii) $p\left(\frac{1}{2} < X < \frac{5}{2} / X > 1\right)$.

(ii) The mileage X (in thousand of miles) which are owners get with a certain kind of tyres is a random variable having a probability density function (8)

$$f(x) = \begin{cases} \frac{1}{20} e^{-\frac{x}{20}}, & x > 0 \\ 0 & x \leq 0 \end{cases}$$

Find the probabilities that one of these tyres will last (i) at most 10,000 miles. (ii) Anywhere from 16,000 to 24,000. (iii) At least 30,000 miles.

22. a) (i) The joint p.d.f of the two dimensional random variable (X, Y) is given by (7)

$$f(x, y) = \begin{cases} \frac{8xy}{9} & 1 \leq x \leq y \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

Find (i) Marginal densities of X and Y.

(ii) The conditional density functions $f(x/y)$ and $f(y/x)$.

(ii) In a partially destroyed laboratory record of an analysis of a correlation data, the following results only are legible. Variance of X = 9, Regression equations are $8x - 10y + 66 = 0$, $40x - 18y = 214$ (7)

What are the mean values of X and Y

The standard deviation of Y

The co-efficient of correlation between X and Y.

(OR)

b) The joint p.d.f of the two dimensional random variable (X, Y) is

$$f(x, y) = \begin{cases} 2-x-y, & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Find the correlation coefficient between X and Y.

23. a) Consider two random processes $X(t) = 3 \cos(\omega t + \theta)$ and

$Y(t) = 2 \cos\left(\omega t + \theta - \frac{\pi}{2}\right)$ where θ is a random variable uniformly distributed in

$(0, 2\pi)$. Prove that $|R_{XY}(\tau)| \leq \sqrt{R_{XX}(0)R_{YY}(0)}$.

(OR)

- b) Let $(X_n : n=1, 2, 3, \dots)$ be a Markov chain with state space $S = \{0, 1, 2\}$ and one-step transition probability matrix.

$$P = \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & 1 & 0 \end{bmatrix}$$

- i) Is the chain ergodic? Explain.
 ii) Find the invariant probabilities.

24. a) If $Y(t) = X(t+a) - X(t-a)$, where $X(t)$ is a WSS process, then show that

i) $R_{YY}(\tau) = 2R_{XX}(\tau) - R_{XX}(\tau - 2a) - R_{XX}(\tau + 2a)$

ii) $S_{YY}(w) = 4 \sin^2 aw S_{XX}(w)$

(OR)

- b) (i) The input to the RC filter is a White noise process with auto correlation function (7)

$$R_{xx}(\tau) = \frac{N_0}{2} \delta(\tau). \text{ If the frequency response } H(\omega) = \frac{1}{1 + j\omega RC}, \text{ find the auto}$$

correlation and the mean square value of the output process.

- (ii) The auto correlation function of the random telegraph signal process is given by (7)

$$R(\tau) = A^2 e^{-2\alpha|\tau|}. \text{ Determine the PSD of the random signal.}$$

25. a) (i) Verify whether the vectors $(1 \quad 4 \quad -2), (-2 \quad 1 \quad 3), (-4 \quad 11 \quad 5)$ are (7)

linearly dependent or independent. If they are linearly dependent write their linear combinations.

- (ii) Applying Gram – Schmidt process, construct orthonormal basis for $V_3(\mathbb{R})$ with (7)

standard inner product for the following basis $(1 \ 0 \ 1), (1 \ 3 \ 1), (3 \ 2 \ 1)$

(OR)

- b) (i) State and prove the Schwarz inequality. (7)

- (ii) Let V be the set of polynomial of degree ≤ 2 together with zero polynomial. V is (7)

a real inner product space with inner product defined by $(f, g) = \int_{-1}^1 f(x)g(x)dx$.

Starting with the basis $(1, x, x^2)$, obtain an orthonormal basis for V .
