Register Number:……..

KUMARAGURU college of technology
character is life

B.E/ B.TECH DEGREE EXAMINATIONS: MAY 2018

(Regulation 2017)

Second Semester

U17MAI2201T : VECTOR CALCULUS AND TRANSFORMS

(Common to CSE/IT)

COURSE OUTCOMES

CO1: Evaluate double and triple integrals in Cartesian coordinates and apply them to calculate area and volume.
CO2: Apply the concepts of vector differentiation
CO3: Apply various integral theorems for solving engineering problems involving cubes and rectangular parallelepipeds.
CO4: Represent periodic functions as infinite trigonometric Fourier series
CO5: Know about difference equations and apply inverse Z-transform to solve them

Time: Three Hours **Maximum Marks: 100**

Answer all the Questions:-

PART A (10 x 1 = 10 Marks)

1. Match the List I with List II CO1 [K4]

List I	List II
A) $\int_0^1 \int_0^1 xydxdy$	i) 1/6
B) $\int_0^1 \int_0^1 dxdy$	ii) 1/2
C) $\int_0^1 \int_0^x dx\,dy$	iii) 1/4
D) $\int_0^1 \int_0^1 x^2 ydxdy$	iv) 1

	A	B	C	D
a)	ii	i	iii	iv
b)	iii	iv	ii	i
c)	ii	iv	iii	i
d)	iii	i	ii	iv

2. CO1 [K_2]

Change the order of integration $\int_0^a \int_{a-y}^{\sqrt{a^2-y^2}} ydxdy$

a) $\int_0^a \int_{a-x}^{\sqrt{a^2-x^2}} dxdy$

b) $\int_{a-x}^{\sqrt{a^2-x^2}} \int_{-a}^{a} ydxdy$

c) $\int_0^{a}\int_{a-x}^{\sqrt{a^2-x^2}} x\,dx\,dy$

d) $\int_0^{a}\int_{a-x}^{\sqrt{a^2-x^2}} y\,dy\,dx$

3. Which of the following statements are true? CO2 [K_3]

(i) $\text{Div}((\vec{A}+\vec{B}) = div(\vec{A}) + div(\vec{B})$

(ii) If ϕ is a constant then grad ϕ is non-zero.

(iii) Divergence of $\vec{r}$ is zero.

(iv) Curl of an irrotational vector is zero.

a) 1,3 b) 1,4

c) 1,2 d) 2,3

4. In a conservative field $\vec{F}$, if C is any simple closed curve then $\int_C \vec{F}.d\vec{r} =$ CO3 [K_1]

a) ∞ b) 1

c) 0 d) 2

5. CO2 [K_4]

Assertion (A): The vector $\vec{F} = x^2\,\vec{i} + y^2\,\vec{j} + z^2\,\vec{k}$ is solenoidal at all points.

Reason (R): A vector $\vec{F}$ is said to be solenoidal vector if $\nabla.\vec{F} = 0$

a) Both A and R are individually true and R is the correct explanation of A

b) Both A and R are individually true but R is not the correct explanation of A

c) A is true but R is false

d) A is false but R is true

6. The RMS value of a function ($\bar{y}$) for f(x) = x^2 in $(0, l)$ is CO4 [K_3]

a) $\frac{l^4}{5}$ b) $\frac{\sqrt{2}l^2}{\sqrt{5}}$

c) $\frac{l^4}{3}$ d) $\frac{\sqrt{2}l^2}{\sqrt{3}}$

7. The sequence of steps to compute the harmonic of the Fourier series for given f(x) is CO4 [K_4]

1. Find the unknown co-efficients from the table values using numerical calculation.

2. Write the Fourier series for f(x) with unknown coefficient upto the specified harmonics.

3. Find the Fourier coefficients for the specified harmonics using the numerical values.

4. Write the Fourier series for given f(x) with its coefficient.

a) 2-1-3-4 b) 1-3-2-4

c) 3-4-2-1 d) 4-1-3-2

8. Greatest directional derivative of ϕ is CO2 [K_1]

a) $\nabla\phi$ b) $|\nabla\phi|$

c) *grad* ϕ d) *Curl* ϕ

9. CO5 [K_4]

Assertion (A): If $\{f(n)\}$ is a casual sequence then z transform $\sum_{n=0}^{\infty} f(n)z^{-n}$ is called one sided or unilateral z transform.

Reason (R): The function $\{f(n)\}$ is defined as $f(n) = 0$ *for* $n < 0$.

a) Both A and R are Individually true and R is the correct explanation of A

b) Both A and R are Individually true but R is not the correct explanation of A

c) A is true but R is false d) A is false but R is true

10. The expansion of the z-transform of $z[y(n+1)]$ in solving finite difference equation is CO5 [K_1]

a) $z - zy(0)$ b) $zy(z) + y(0)$

c) $zy(z) - zy(0)$ d) $zy(z) + zy(0)$

PART B (10 x 2 = 20 Marks)
(Answer not more than 40 words)

11. Sketch the region of integration for evaluating $\iint_R xydxdy$ where R is the region bounded by the line $x+2y=2$ lying in the first quadrant. CO1 [K_3]

12. Evaluate $\int_0^2 \int_1^3 \int_1^2 xy^2 zdzdydx$ CO1 [K_2]

13. Find grad φ at the point (1,-2,-1) when $\varphi = 3x^2y - y^3z^2$ CO2 [K_3]

14. Prove that $\vec{F} = (y^2 - z^2 + 3yz - 2x)\vec{i} + (3xz + 2xy)\vec{j} + (3xy - 2xz + 2z)\vec{k}$ is both solenoidal and irrotational. CO2 [K_3]

15. Use Green's theorem in a plane to evaluate $\int_c [(2x-y)dx + (x+y)dy]$, where c is the boundary of the circle $x^2 + y^2 = r^2$ in the xoy plane. CO3 [K_3]

16. Can the function $f(x) = \frac{1}{x-2}, (0 \le x \le 3)$ be expanded as a Fourier Series? CO4 [K_4]

17. Expand $f(x) = x, 0 < x < \pi$ in Fourier Sine series CO4 [K_3]

18. Prove that $Z(n) = \frac{z}{(z-1)^2}, |z| > 1$ CO5 [K2]

19. If $Z[f(t)] = F(z)$, then prove that $f(0) = \lim_{z \to \infty} F(z)$ CO5 [K2]

20. Explain Fibonacci Sequence. CO5 [K1]

Answer any FIVE Questions:-
PART C (5 x 14 = 70 Marks)
(Answer not more than 350 words)

21. (a) Change the order of integration in $\int_0^1 \int_y^{2-y} xydxdy$ and then evaluate it. (7) CO1 [K_5]

(b) Express the volume of the sphere $x^2 + y^2 + z^2 = a^2$ as a volume integral and hence evaluate it. (7) CO1 [K_5]

22. (a) Show that $\vec{F} = (y^2 + 2xz^2)\vec{i} + (2xy - z)\vec{j} + (2x^2z - y + 2z)\vec{k}$ is irrotational and hence find it's scalar potential. (7) CO2 [K_4]

(b) Prove that $\nabla^2(r^n) = n(n+1)r^{n-2}$ where $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ and $\vec{r} = |\vec{r}|$ and hence deduce $\nabla^2(\frac{1}{r})$. (7) CO2 [K_4]

23. Verify Gauss Divergence Theorem for $\vec{F} = x^2\,\vec{i} + y^2\,\vec{j} + z^2\,\vec{k}$ where S is the surface of the cuboid formed by the planes $x=0; x=a; y=0; y=b; z=0; z=c$. CO3 [$K_5$]

24. (a) Find the Fourier series expansion of period $2l$ for the f unction $f(x)=(l-x)^2$ in the range $(0, 2l)$. (7) CO4 [K_3]

(b) Find the Fourier series expansion for $f(x)=x^2$ in $(-\pi, \pi)$. Use Parseval's identity to prove $\frac{\pi^4}{90}=1+\frac{1}{2^4}+\frac{1}{3^4}+\frac{1}{4^4}+.....$ (7) CO4 [K_5]

25. (a) Determine the first two harmonics of the Fourier series for the following table values (7) CO4 [K_4]

x	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$
y	1.98	1.30	1.05	1.30	-0.88	-0.25

(b) Using Convolution theorem evaluate inverse Z-transform of $\left[\frac{z^2}{(z-1)(z-3)}\right]$ (7) CO5 [K_3]

26. (a) From $y_n = a2^n + b(-2)^n$, derive a difference equation by eliminating the constants. (4) CO6 [K_3]

(b) Solve the difference equation $y_{n+3} - 3y_{n+1} + 2y_n = 0$ given that $y_0 = 4, y_1 = 0,$ $y_2 = 8$ (10) CO6 [K_5]

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