



B.TECH DEGREE EXAMINATIONS: MAY 2018

(Regulation 2015)

Fourth Semester

INFORMATION TECHNOLOGY

U15MAT403 : Discrete Mathematics

COURSE OUTCOMES

- CO1:** Have a better understanding of sets and application of set theory.
CO2: Acquire the knowledge of relations, equivalence relations and their properties.
CO3: Understand different kinds of functions.
CO4: Understand logical arguments and constructs simple mathematical proofs.
CO5: Know various graphs and learn different algorithms.
CO6: Acquire the knowledge of partially ordered sets, lattices, Boolean algebra and able to apply in circuits.

Time: Three Hours

Maximum Marks: 100

Answer all the Questions:-

PART A (10 x 1 = 10 Marks)

1. Match List I with List II and select the answer from the codes given below.

CO4 [K₁]

List I	List II
A. $\neg p \vee q$	i. $\neg(p \rightarrow q)$
B. $p \vee q$	ii. $p \rightarrow q$
C. $p \wedge q$	iii. $\neg(p \rightarrow \neg q)$
D. $p \wedge \neg q$	iv. $\neg p \rightarrow q$

- | | A | B | C | D |
|----|-----|----|-----|----|
| a) | ii | i | iii | iv |
| b) | iii | iv | ii | i |
| c) | ii | iv | iii | i |
| d) | iii | i | ii | iv |

2. A function $f : A \rightarrow B$ has an inverse if and only if f is

CO3 [K₁]

- | | |
|--------------------|-------------------------|
| a) 1-1 function | b) Bijective |
| c) Onto - function | d) Neither 1-1 nor onto |

3. If $A = \{0,2,3\}$, $B = \{2,3\}$ and $C = \{1,5,9\}$ then

CO1 [K₂]

- | | | | |
|------------------|------------------|----------------------|-------------------------|
| 1. $B \subset A$ | 2. $\{3\} \in A$ | 3. $\{3\} \subset A$ | 4. $B \cap A \subset C$ |
|------------------|------------------|----------------------|-------------------------|

Which of the above are true?

- | | |
|--------|--------|
| a) 1,3 | b) 1,4 |
| c) 1,2 | d) 2,3 |

4. Let A and B be two non-empty finite sets containing m and n elements. Then the total number of subsets of $A \times B$ is CO1 [K₁]
- a) 2^{mn} b) 2^{m+n}
 c) 2^m d) 2^n
5. Examine the two statements carefully and select the answer using the codes given below: CO2 [K₄]
- Assertion (A): The relation R on the set of all integers defined by “**a R b if a is a multiple of b**” is a equivalence relation
- Reason (R): If the relation is reflexive, symmetric and transitive then it is equivalence relation.
- a) Both A and R are individually true and R is the correct explanation of A b) Both A and R are individually true but R is not the correct explanation of A
 c) A is true but R is false d) A is false but R is true
6. For every $n > 0$ there is a unique Boolean algebra of ----- elements CO6 [K₁]
- a) 2^n b) $2n$
 c) $n+2$ d) $2/n$
7. The steps involved in finding inverse of a function CO3 [K₂]
1. Swap the variables x and y.
 2. Replace y by $f^{-1}(x)$.
 3. Solve for y.
 4. Replace the function f(x) by y in the equation describing the function.
- a) 2-3-4-1 b) 1-3-2-4
 c) 3-4-2-1 d) 4-1-3-2
8. Consider the statements. CO4 [K₂]
1. If it is raining then the roads are wet.
 2. It is raining
- Then the conclusion is
- a) The roads are not wet b) The roads are wet
 c) It is not raining d) It is raining and the roads are not wet
9. Examine the two statements carefully and select the answer using the codes given below: CO5 [K₄]
- Assertion (A) : The number of regions in a complete graph with 4 vertices is 4
- Reason (R) : For any connected planar graph, the number of regions $r = e - v + 2$ where e is number of edges and v is number of vertices.
- a) Both A and R are individually true and R is the correct explanation of A b) Both A and R are individually true but R is not the correct explanation of A
 c) A is true but R is false d) A is false but R is true
10. In a partially ordered set, if every pair of elements has supremum and infimum then it is called a CO6 [K₁]
- a) Boolean algebra b) Poset
 c) Lattice d) Chain

PART B (10 x 2 = 20 Marks)

(Answer not more than 40 words)

11. Find the sets A and B if $A - B = \{1,3,7,11\}$; $B - A = \{2,6,8\}$ and $A \cap B = \{4,9\}$. CO1 [K₂]
12. The relation R on the set $A = \{1,2,3,4,5\}$ is defined by the relation a R b if and only if '3 divides a - b'. List the elements of R and R^{-1} . CO2 [K₂]
13. If R and S be a relation on a set A represented by the matrices CO2 [K₃]
- $$M_R = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} \text{ and } M_S = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}.$$
- Find the matrices that represents $R \cup S$ and $R \cap S$.
14. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ and $g : \mathbb{R} \rightarrow \mathbb{R}$ are functions defined by $f(x) = x^2 + 3x + 1$, $g(x) = 2x - 3$. Find $(f \circ g)(x)$. CO3 [K₃]
15. If A is a subset of U then prove that $\psi_A^-(x) = 1 - \psi_A(x)$ for all $x \in U$. CO3 [K₃]
16. Prove that $(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$ is tautology. CO4 [K₃]
17. Define the two types of quantifiers in predicate calculus CO4 [K₁]
18. Draw the graph that is both Eulerian and Hamiltonian. CO5 [K₂]
19. State Handshaking theorem. CO5 [K₁]
20. Let $A = \{a, b, c\}$ and $\rho(A)$ be its power set. Draw the Hasse diagram of $(\rho(A), \subseteq)$ CO6 [K₃]

Answer any FIVE Questions:-

PART C (5 x 14 = 70 Marks)

(Answer not more than 300 words)

Q.No. 21 is Compulsory

21. Suppose 100 out of 120 Computer Science students study atleast one of the following languages: French, German and Russian. It is given that 65 students study French, 45 students study German, 42 students study Russian, 20 students study French and German, 25 students study French and Russian and 15 students study German and Russian. Find the number of students who study CO1 [K₅]
- (i) All the three languages
 - (ii) Only French and German, but not Russian
 - (iii) Only French and Russian, but not German
 - (iv) Only Russian and German, but not French
 - (v) Only French
 - (vi) Only German
 - (vii) Only Russian
22. (i) Determine whether the relation R on the set of all integers is an equivalence relation, where aRb if and only if $a \equiv b \pmod{7}$ (7) CO2 [K₄]
- (ii) Let $R = \{(b, c), (b, e), (c, e), (d, a), (e, b), (e, c)\}$ be a relation on the set $A = \{a, b, c, d, e\}$. Find the transitive closure using Warshall's algorithm. (7) CO2 [K₃]

23. (i) Show that the function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = \frac{x}{x^2 + 1}$, for all $x \in \mathbb{R}$, is neither one-one nor onto. (7) CO3 [K₄]

(ii) Let $A = \{1,2,3,4\}$, $P_1 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 2 & 4 & 1 \end{pmatrix}$ and $P_2 = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 1 & 3 \end{pmatrix}$. (7) CO3 [K₃]

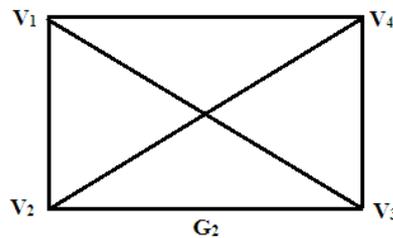
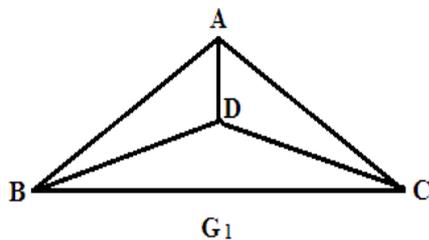
Verify that $(P_1 \circ P_2)^{-1} = P_2^{-1} \circ P_1^{-1}$.

24. (i) Show that the following premises are inconsistent. (7) CO4 [K₄]

$p \rightarrow q$, $p \rightarrow r$, $q \rightarrow \neg r$ and p

(ii) Show that the premises “One student in this class knows how to write programs in JAVA” and “Everyone who knows how to write programs in JAVA can get a high paying job” imply the conclusion “Someone in this class can get a high paying job.” (7) CO4 [K₄]

25. (i) Define isomorphism. Examine whether the following pair of graphs are isomorphic or not. (7) CO5 [K₄]



(ii) Prove that the number of odd vertices in a graph is always even. (7) CO5 [K₄]

26. (i) Show that every chain is distributive lattice. (7) CO6 [K₄]

(ii) In any Boolean algebra show that $ab' + a'b = 0$ if and only if $a = b$. (7) CO6 [K₄]

27. (i) Simplify the Boolean function $f(a,b,c) = [(ab)'c]'[(a'+c)(b'+c)]'$. (7) CO6 [K₄]

(ii) Find the Principal conjunctive and disjunctive normal form of the following. (7) CO4 [K₄]
 $(P \wedge Q) \vee (\neg P \wedge R) \vee (Q \wedge R)$.
