Register Number:……………..

KUMARAGURU
college of technology
character is life

B.E DEGREE EXAMINATIONS: MAY 2018

(Regulation 2015)

Fourth Semester

COMPUTER SCIENCE AND ENGINEERING

U15MAT404 : Partial Differential Equations and Transforms

COURSE OUTCOMES

CO1: Frame partial differential equations and to get solution.

CO2: Know how to find the Fourier Series and half range Fourier Series of a function given explicitly or to find Fourier Series of numerical data using harmonic analysis.

CO3: Solve one dimensional wave equation, one dimensional heat equation and two dimensional heat equation in steady state using Fourier Series (Cartesian co-ordinates only).

CO4: Apply the Fourier transform, Fourier sine and cosine transform to certain functions and use Parseval's identity to evaluate integrals.

CO5: Evaluate Z – transform for certain functions.

CO6: Estimate Inverse Z – transform of certain functions and to solve difference equations using them.

Time: Three Hours **Maximum Marks: 100**

Answer all the Questions:-

PART A (10 x 1 = 10 Marks)

1. Match the following CO1 [K_2]

List I (PDE)	**List II (Types of the equation)**
A. $5p^2 - q^2 = 2pq$	i. $F(z, p, q) = 0$
B. $p^2 y(1+x^2) = qx^2$	ii. $px + qy + f(p,q) = z$
C. $p^3 + q^3 = 27z$	iii. $F(p,q) = 0$
D. $z = px + qy + p^2$	iv. $f(x, p) = \varphi(y, q)$

	A	B	C	D
a)	ii	iv	i	iii
b)	ii	iv	iii	i
c)	iii	iv	ii	i
d)	iii	iv	i	ii

2. The particular integral of $(D^2 - D'^2)z = x^2$ is CO1 [K_3]

a) $\frac{x^4}{12}$ b) $\frac{x^4}{4}$

c) $\frac{x^3}{3}4x$ d) $\frac{x^3}{12}$

3. Which of the following statements are correct. CO2 [K_3]

1. If a periodic function f(x) is even, its Fourier expansion contains only sine terms.
2. If $f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$ then f(0) is $\frac{-\pi}{2}$.
3. Any function f(x) can be expressed as a Fourier series if it satisfies Dirichlet's condition.

4. If $f(x)=|x|$ is expanded as a Fourier series in $(-\pi,\pi)$ then b_1 is $-\pi$

a) 1,2 b) 2,3

c) 3,4 d) 1,4

4. In steady state, the Cartesian form of the two dimensional heat flow equation is CO3 [K1]

a) $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ b) $\frac{\partial^2 y}{\partial t^2}=\alpha^2\left(\frac{\partial^2 u}{\partial x^2}\right)$

c) $\frac{\partial u}{\partial t}=\alpha^2\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)$ d) $\frac{\partial^2 u}{\partial x^2}=0$

5. Examine the two statements carefully and select the answer using the codes given below. CO3 [K4]

Assertion (A): The one dimensional heat equation $u_t = a^2 u_{xx}$ is of Parabolic type.

Reason (R): The value of B^2-4AC in one dimensional heat equation is zero.

a) Both A and R are Individually true and R is the correct explanation of A b) Both A and R are Individually true but R is not the correct explanation of A

c) A is true but R is false d) A is false but R is true

6. The Fourier sine transform of e^{-2x} is CO4 [K3]

a) $\sqrt{\frac{2}{\pi}}\left[\frac{2}{s^2+2^2}\right]$ b) $\sqrt{\frac{\pi}{2}}\left[\frac{2}{s^2+2^2}\right]$

c) $\sqrt{\frac{\pi}{2}}\left[\frac{s}{s^2+2^2}\right]$ d) $\sqrt{\frac{2}{\pi}}\left[\frac{s}{s^2+2^2}\right]$

7. The steps involved in finding the Fourier series expansion of $f(x)=x^2 \ in -\pi<x<\pi$ and deduction of $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+$............ CO2 [K2]

1. If f(x) is even, hence $b_n=0$
2. Obtain the Fourier co efficient a_0, a_n
3. Substitute the point of discontinuity at $x=\pi$ to get the required deduction.
4. Substitute the coefficients in Fourier series expansion.

a) 2-4-3-1 b) 1-2-4-3

c) 2-1-3-4 d) 1-2-3-4

8. The Z-Transform of the function $(-1)^n$ is CO5 [K3]

a) $\frac{1}{z-1}$ b) $\frac{z}{z+1}$

c) $\frac{z}{z-1}$ d) $\frac{1}{z+1}$

9. Examine the two statements carefully and select the answer using the codes given below. CO4 [K4]

Assertion (A): The function f(x) = $e^{\frac{-x^2}{2}}$ is self reciprocal under Fourier transform.

Reason (R): $F\left[e^{\frac{-x^2}{2}}\right]=e^{\frac{-s^2}{2}}$.

a) Both A and R are Individually true and R is the correct explanation of A b) Both A and R are Individually true but R is not the correct explanation of A

c) A is true but R is false d) A is false but R is true

10. $Z^{-1}\left[\frac{az}{(z-a)^2}\right]$ is — CO6 [K3]

a) a^n b) a^{-n}

c) na^n d) na^{-n}

PART B (10 x 2 = 20 Marks)
(Answer not more than 40 words)

11. Form the partial differential equation by eliminating f from $z = x^2 + 2f\left(\frac{1}{y} + \log x\right)$ — CO1 [K4]
12. Find the complete integral of $p(1+q) = qz$ — CO1 [K3]
13. State the Dirichlet's conditions for a function to be expanded as a Fourier Series. — CO2 [K2]
14. Find the RMS value of the function $f(x) = x - x^2$ in -1 < x < 1 — CO2 [K3]
15. Write down all the possible solutions of two dimensional heat equation in steady state. — CO3 [K2]
16. The ends A and B of a rod of length 10cm long have their temperature kept at $20^0 C$ *and* $70^0 C$. Find the steady state temperature distribution of the rod. — CO3 [K4]
17. If $F\{f(x)\} = F(s)$ then prove that $F\{f(x)\cos ax\} = \frac{1}{2}[F(s-a) + F(s+a)]$ — CO4 [K3]
18. State the Convolution theorem for Fourier Transforms. — CO4 [K2]
19. Find the Z transform of $\frac{a^n}{n!}$ — CO5 [K3]
20. State the initial and final value theorems on Z transforms. — CO5 [K2]

Answer any FIVE Questions:-
PART C (5 x 14 = 70 Marks)
(Answer not more than 300 words)

Q.No. 21 is Compulsory

21. A uniform string is stretched and fastened to two points l cm apart. Motion is started by displacing the string into the form of the curve $y = k\sin^3\left(\frac{\pi x}{l}\right)$ and then releasing it from this position at time t=0. Determine the displacement of the point of the string at a distance x from one end at time t. — CO3 [K4]

22. (i). Solve $\left(D^2 - DD' - 20D'^2\right)z = e^{5x+y} + \sin(4x-y)$ (7) — CO1 [K5]
(ii). Solve $x(y-z)p + y(z-x)q = z(x-y)$ (7) — CO1 [K4]

23. (i). Find the Fourier Series expansion of period 2π for the function y=f(x) which is defined in (0,2π) by means of the table of values given below. Find the series up to the second harmonic. (7) — CO2 [K3]

x	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π
y	1.0	1.4	1.9	1.7	1.5	1.2	1.0

(ii). Prove that in the interval $0 < x < l$, $x = \frac{l}{2} - \frac{4l}{\pi^2}\left(\cos\frac{\pi x}{l} + \frac{1}{3^2}\cos\frac{3\pi x}{l} + \ldots\ldots\ldots\right)$ and deduce that $\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \ldots\ldots\ldots = \frac{\pi^4}{96}$ (7) CO2 $[K_4]$

24. A bar 10cm long with insulated sides has its ends A and B kept at $20^0 C$ and $40^0 C$ respectively, until steady – state conditions prevail, that is until the temperature at any interior point no longer changes with time. The temperature at A is then suddenly raised to $50^0 C$ and at the same instant that at B is lowered to $10^0 C$. Find the subsequent temperature function u(x,t) at any time. CO3 $[K_5]$

25. (i). Show that the Fourier transform of $f(x) = \begin{cases} a^2 - x^2, & |x| < a \\ 0, & |x| > a > 0 \end{cases}$ is $2\sqrt{2/\pi}\left(\frac{\sin as - as\cos as}{s^3}\right)$. Hence deduce that $\int_0^\infty \frac{\sin t - t\cos t}{t^3} = \frac{\pi}{4}$ (7) CO4 $[K_4]$

(ii). Using transform methods evaluate $\int_0^\infty \frac{x^2 dx}{(x^2 + a^2)^2}$ where a>0. (7) CO4 $[K_4]$

26. (i). Find the Z-transform of $\frac{1}{n(n+1)}$, $n \geq 1$ (6) CO5 $[K_3]$

(ii). Solve $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$ *given* $y_0 = y_1 = 0$ (8) CO6 $[K_5]$

27. (i). Using convolution theorem find the inverse Z-transform of $\frac{z^2}{\left(z - \frac{1}{2}\right)\left(z - \frac{1}{4}\right)}$ (7) CO6 $[K_5]$

(ii). Solve $z = px + qy + p^2 - q^2$ (7) CO1 $[K_3]$

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