DYNAMIC ANALYSIS OF SPECIAL CIVIL ENGINEERING STRUCTURES

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A THESIS

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CERTIFICATE

This is to certify that the thesis entitled "**DYNAMIC ANALYSIS OF SPECIAL CIVIL ENGINEERING STRUCTURES** is a bonafide record of work done by **Ms. GEETHA. G.S.** under my supervision and guidance. The thesis is submitted to the **University of Calicut** in partial fulfilment of the requirements for the award of the degree of **Master of Technology** in **Civil Engineering (Structural Engineering).**

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NOMENCLATURE

The following notations are used in this thesis. In some places, same notations are used in different context. In such cases, the meaning of the case is explain in those places.

a	Top width of the section considered
b	Bottom width
D	Mean diameter at the section of the Cooling Tower
E	Modulus of elasticity of the material
f_s	Form factor
G	Shear Modulus
g	Accelaration due to gravity
H	Height of the dam
h	Height of Station i from the bottom
$\bar{h}$	Height of the C.G. of the dam above the base
I	Moment of inertia
$[k]$	Stiffness matrix
$[m]$	Mass matrix
m_i	Mass associated with i^{th} degree of freedom
n_b	Number of bays
p	Natural frequency of vibration
r_o	Offset distance
S_a/g	Average accelaration coefficient

SDOF	Single Degree of Freedom system
T_1	I^{st} mode period in sec.
T_2	II^{nd} mode period in sec.
T_3	III^{rd} mode period in sec.
TDOF	Two Degree of Freedom system
V_i	Absolute value of maximum shear at the i^{th} storey
W	Total weight of the Dam
x	Displacement Vector
$\ddot{x}$	Accelaration Vector
y	Station distance measured from the bottom
α_h	Design horizontal seismic coefficient
β	Ratio of top to bottom width
δ_{ii}	Principal flexibility incluence coefficient
δ_f	Deflection due to flexure
δ_s	Deflection due to shear
θ_f	Rotation due to flexure
	Density of material

CHAPTER I

INTRODUCTION

The essential background for study in the field of earthquake engineering is, of course, the earthquake itself. The detailed study of earthquakes and earthquake mechanisms lies in the province of seismology, but in this studies the earthquake engineer must take a different point of view than the seismologist. Seismologists have foccussed their attention primarily on the global or long range effects of earthquakes and therefore are concerned with very small amplitude ground motions which induce no significant structural responses. Engineers, on the other hand, are concerned mainly with the local effects of large earthquakes, where the ground motions are intense enough to cause structural damage. These so-called strong motion earthquakes are too violent to be recorded by the typical seismographs used by seismologists and have necessitated the development of special types of strong motion seismographs.

Earthquakes systematically bring out the mistakes made in the design and construction - even the most minute mistakes; it is this aspect of earthquake engineering that makes it challenging and fascinating and gives it an educational value far beyond its immediate objectives.

A good portion of the loads on the structures can be considered as static loads requiring static analysis only. Although almost all the loads except the dead loads are transient in nature they are considered as static. The increasing availability of high speed digital computers has initiated the trend that the earthquake motion, blast effects etc are represented as dynamic loads. During an earthquake the structure is subjected to rapid ground displacements and experiences internal forces which include inertia forces, damping forces and elastic forces. The magnitude of these forces are functions of mass of the building, dynamic properties of the building such as its mode shape, periods of vibration and its damping characteristics. The dynamic analysis involves the idealization of the structure so that a mathematical model can be formulated finally, the determination of the response of the mathematical model for suitable ground motion.

In this thesis approximate methods are put forward for finding the natural period of gravity dam, earthdam, cooling tower and vierendeel girder. For performing the modal analysis recommended by IS 1893-1984 modal periods and amplitude of vibration are necessary. So in this thesis simplified equations are proposed to find the eigen pairs of gravity dam, earthdam and cooling tower.

CHAPTER II

DYNAMIC ANALYSIS

2.1 General

For large or complex structures static methods of seismic analysis are not accurate enough and many authorities demand dynamic analysis for certain type and size of structure. Various methods of differing complexity have been developed for the dynamic seismic analysis of structures. They all have in common the solutions of the equations of motion as well as the usual statical relationships of equilibrium and stiffness. For any structure with more than three degrees of freedom such analysis are carried out by matrix methods on computers.

2.2 Different methods of dynamic analysis

The three main techniques currently used for dynamic analysis [6] are

i) Direct integration of the equations of by step-by-step procedure

ii) Normal mode analysis

iii) Response spectrum techniques

Direct integration provides the most powerful and informative analysis for any given earthquake motion. A time-

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ABSTRACT

Natural period is the basic parameter necessary for evaluating the base shear and base moment due to dynamic effects like blast loadings, earthquake motion etc.

In this thesis two approximate mathematical models are put forward for finding the natural period of gravity dams. For cooling tower an approximate method is propounded for finding the natural period. For Vierendeel Girder approximate method of analysis is done and an equation is given for finding the natural period.

Code suggests that seismic coefficient method and response spectrum method are meant only for preliminary design of dams. For final design dynamic analysis is desirable. But it doesn't indicate the type of dynamic analysis. Here modal analysis is performed.

For gravity dam, earthdam and cooling tower a method is suggested for assessing the periods and amplitudes of vibration. This is necessary for performing modal analysis. These approximate methods are important as a means of checking computer results.

dependent forcing function is applied and the corresponding response history of the structure during the earthquake is computed. That is the moment and force diagrams at each of a series of prescribed intervals throughout the applied motion can be found.Three dimensional nonlinear analysis have been devised which can take the three orthoganal accelerogram components from a given earthquake and apply them simultaneously to the structure. In principle, this is the most complete dynamic analysis technique so far devised, and is unfortunately correspondingly expensive to carry out

Linear behaviour is seldom analysed by direct integration, unless mode coupling is involved, as normal mode techniques are easier, cheaper and nearly as accurate.

Normal mode analysis is a more limited technique than direct integration, as its depends on artifically seperating the normal modes of vibration and combining the forces and displacements associated with a chosen number of them by superposition. As with direct integration techniques, actual earthquake accelerograms can be applied to the structure and a stress-history determined, but because of the use of superposition the technique is limited to linear material behaviour. Although modal analysis can provide any desired order of accuracy for linear behaviour. By ncorporating all the modal responses, some approximation is ormally made by using only the first few modes in order to ave computation time.

The most serious shortcoming of linear analysis is that ney do not accurately indicate all the members requiring aximum ductility. For important structures in zones of high eismic risk, non-linear dynamic analysis is required.

The response specturm technique is really a simplified pecial case of modal analysis. The modes of vibration are etermined in period and shape in the usual way and the aximum response magnitudes corresponding to each mode are ound by reference to a response spectrum. But the imitations of this method is that this can be used for reliminary design only.

In this thesis work Modal analysis is performed. In his analysis the mathematical formulation is such that it educes the multidegree of freedom into as many single degree f freedom as the original degrees of freedom, each with an ndependent second order differential equation being similar o the equation of single degree of freedom systems.

.3 Theory of Modal Analysis [12]

For a multi degree of freedom undamped system, the quation of motion can be given as

$[m] \{\ddot{x}\} + [k] \{x\} = \{G\}$ as a coupled equation (2.1)

where {G} is a function of time.

Let the transformation be

$$\{x\} = [a] \{y\} \tag{2.2}$$

$$\{\ddot{x}\} = [a] \{\ddot{Y}\} \tag{2.3}$$

where {y} - principal co-ordinates

[a] - orthogonal eigen vector matrix

Substituting (2.2) and (2.3) in (2.1) we get

$$[m] [a] \{\ddot{y}\} + [k] [a] \{y\} = \{G\} \tag{2.4}$$

Premultiplying (2.4) with $[a]^T$, there

$$[a]^T [m] [a] \{\ddot{y}\} + [a]^T [k] [a] \{y\} = [a]^T \{G\} \tag{2.5}$$

$[a]^T [m] [a] = [M]$ is a diagonal mass matrix

$[a]^T [k] [a] = [k]$ is a diagonal stiffness matrix

$[a]^T \{G\} = \{R\}$

Equation (2.5) becomes

$$[M] \{\ddot{y}\} + [K] \{y\} = \{R\} \tag{2.6}$$

is an uncoupled equation, containing only one variable.

2.4 Code Procedure for Modal Analysis

The modal analysis described in [7] and [9] is a semi automated procedure in which the first step is to obtain the eigen pairs. With the knowledge of eigen pairs, modal analysis can be performed manually.

Steps

1. Determination of eigen pair for the first three modes.
2. Determination of Horizontal seismic coefficient by

$$\alpha_h^{(r)} = \beta \, I \, F_o \, Sa/g^{(r)} \tag{2.7}$$

β - a coefficient depending upon the soil-foundation system (see Table 2.1)

I - A factor dependant upon the importance of the structure (See Table 2.2)

F_o - seismic zone factor for average acceleration spectra (see Table 2.3)

Sa/g - Average acceleration coefficient as read from Fig.2.1 for appropriate natural period and damping of the structure.

3. Determination of Mode participation factor for each mode

$$C^{(r)} = \frac{\sum_{i=1}^{n} W_i \, \phi_i^{(r)}}{\sum_{i=1}^{n} W_i \, [\phi_i^{(r)}]^2}$$

where W_i - weight of floor i

$\phi_i^{(r)}$ - Mode shape coefficient at floor i in r^{th} mode vibration obtained from free vibration analysis

$C^{(r)}$ - Mode participation factor

4. Computation of lateral force and shears for all the three modes.

The lateral load $Q_i^{(r)} = k\ W_i\ \phi_i^{(r)}\ C_r\ \alpha_h^{(r)}$ (2.9)

where k - performance factor depending upon the types of buildings

5. The combination of shears for the three modes.

After getting shear forces in each mode, the total shear force obtained by the superposition of first three modes as follows,

$$V_i = (1 - \gamma) \sum_{r=1}^{3} V_i^{(r)} + \sum_{r=1}^{3} (V_i^{r})^2 \qquad (2.10)$$

where $V_i^{(r)}$ - absolute value of maximum shear at the i^{th} storey in the r^{th} mode; the value of shall be as given in table 2.4.

Table 2.1 VALUES OF β FOR DIFFERENT SOIL FOUNDATION SYSTEMS

Type of soil mainly constituting the foundations	VALUES OF FOR					
	Piles Passing through any soil, but resting on soil Type I	Piles not covered under Col 3	Raft Foundations	Combined or isolated RCC Footing with tie beams	Isolated RCC footings without tie beams or unreinforced strip foundations	Well Foundations
(2)	(3)	(4)	(5)	(6)	(7)	(8)
Type 1 Rock or hard soils	1.0	-	1.0	1.0	1.0	1.0
Type II Medium soils	1.0	1.0	1.0	1.0	1.2	1.2
Type III Soft soils	1.0	1.2	1.0	1.2	1.5	1.5

NOTE : The values of β for dams shall be taken as 1.0

Table 2.2 VALUES OF IMPORTANCE FACTOR, I

Sl. No.	STRUCTURES	Value of Importance Factor, I (See Note)
(1)	(2)	(3)
i)	Dams (all types)	3.0
ii)	Containers of inflammable or poisonous gases or liquids	2.0
iii)	Important service and community structures, such as hospitals; water towers and tanks; schools; important bridges; important power houses; monumental structures ; emergency buildings like telephone exchange and fire bridge; large assembly structures like cinemas, assembly halls and subway stations	1.5
iv)	All others	1.0

NOTE : The values of importance factor, I given in this table are for guidance. A designer may choose suitable values depending on the importance based on economy, strategy and other considerations.

Table 2.3 VALUES OF BASIC SEISMIC COEFFICIENTS AND SEISMIC ZONE FACTORS IN DIFFERENT ZONES

Sl. No.	Zone No.	MEETHOD	
		Seismic Coefficient Method	Response Spectrum Method
		Basic horizontal seismic coefficient	Seismic zone factor for average acceleration spectra to be used with Fig. 2.1, F_o
(1)	(2)	(3)	(4)
i)	V	0.08	0.40
ii)	IV	0.05	0.25
iii)	III	0.04	0.20
iv)	II	0.02	0.10
v)	I	0.01	0.05

NOTE : For under ground structures and foundations at 30 m depth or below, the basic seismic coefficient may be taken as 0.5 α_o ; for structures placed between ground level and 30 m depth, the basic seismic coefficient may be linearly interpolated between α_o and 0.5 α_o.

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Table 2.4 DIFFERENT VALUES OF γ

Height, H m	γ
Upto 20	0.4
40	0.6
60	0.8
90	1.0

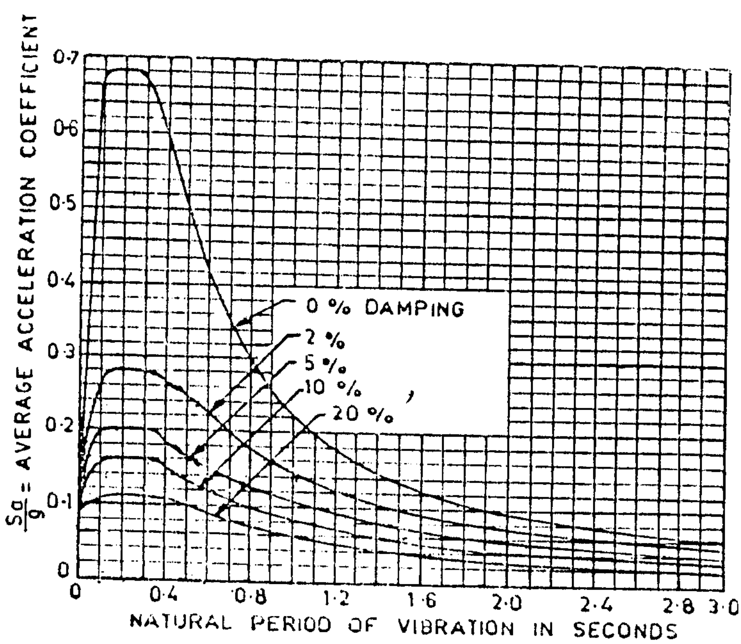


Fig. 2.1 Average accelaration spectra

CHAPTER 3

GRAVITY DAM

3.1 General

Gravity dams usually form an important element of multipurpose projects like hydroelectric, irrigation and flood control. In India, many river valley projects are located in the seismically active zones and thus dams located in these regions will be subjected to dynamic forces caused by earthquakes. It is, therefore, essential that these dams should be designed taking into account anticipated earthquake forces so that they can safely withstand future shocks without seismic damage since the failure of a dam is far more disastrous to a community than that of other structures.

At the failure of an earthquake, besides the normal forces, namely, self weight, water pressure and silt or earth pressure, inertia and hydrodynamic forces act on a dam. The inertia force is the product of mass and acceleration and their force acts in the direction opposite to that of the ground motion. The horizontal inertia force acts from u/s to D/S as well as from D/S to u/s. Similarly the vertical inertia force acts from downwards as well as from upwards to downward direction. An acceleration downwards decreases the eight. The dam is designed from the worst combination ie or the horizontal and vertical inertia forces.

The dynamic behaviour of dam-reservior system is sually evaluated treating the dam and reservior as two ncoupled system, namely, dynamic responce of the dam gnoring the effect of reservior water and hydrodynamic ressure on the dam to represent the effect of reservoir vater. It is invariably assumed that the interaction effects etween the dam and reservoir are small so that the solutions of the uncoupled system can be combined to obtain a complete solution for the response of the dam.

.2 Literature Review

) As early as 1933, Westergaard Propounded an empirical formula for finding the natural period of concrete dams[5]. Westergaard finds the time of vibration for a concrete gravity dam of triangular section, reservior empty, with a modulus of elasticity of 2000000lb/inch^2

$$t_s = \frac{h^2}{2000\ l} \tag{3.1}$$

Where t_s - time of vibration in seconds

h - height of dam in feet

l - base length in feet

Taking the variation of modulus of elasticity and converting the equation into S.I unit.

$$t_s = \frac{0.1926\ H^2}{l\sqrt{E_c}} \qquad (3.2)$$

where

E_c - Modulus of elasticity in Mpa

H - height of dam in metres

l - baselength in metres

Westergaard's equation is very potent for finding the period with amazing simplicity for use in preliminary design of dams subjected to earthquake effects.

ii) For concrete dams 1S: 1893-1894 gives the formula for finding the fundamental period of vibration.

$$T = 5.55\ \frac{H^2}{B}\sqrt{\frac{W_m}{gE_s}} \qquad (3.3)$$

Where

T - fundamental period in seconds

H - height of dam in metres

B - basewidth of dam in metres

W_m - unit weight of the material of the dam kgf/m^3

g - accelaration due to gravity m/sec^2

E_s - Modulus of elasticity of the material kg/m^2

Seismic coefficient method for dams upto 100 m height nd response spectrum method for dams greater than 100 m height re meant only for preliminary design of dams. For final esign dynamic analysis is desirable.

ii) Manicka Selvam, Palaniraj and Shaji in the year 1987 roposed an equation to find the fundamantal period of concrete ams using finite element method[14].

$$T = \frac{0.67\, H\, \alpha^{0.2}}{\sqrt{E_c}} \qquad (3.3)$$

here

T - fundamental periods in seconds

H - height of the dam in metres

E_c - Young's Modulus of concrete in N/mm^2

α - Ratio of top width to bottom width of dam

iv) Manicka Selvam, Kathiroli and Geethakumari in the year 1992 proposed an equation for finding the natural period of vibration of concrete dams[13]. For finding the natural period of the system, the fundamental frequency must be known. A closed form solution was put forward for finding the fundamental frequency.

$$\text{ie} \quad \frac{1}{P^2} = \frac{0.3 \rho H^2 b^2}{G\,(b-c)^2} \qquad (3.4)$$

Where

ρ - density of concrete

H - Total hight of the dam in metres

b - bottom width is metres

c - Top width in metres

But this formula is useful only for preliminary calculations only. For getting a satisfactory solution for the period, exact dynamic analysis is a must.

3.3 Natural Period

In the design of concrete and masonry dams IS: 1893-1984 suggests the earthquake forces shall be considered besides the hydrodynamic forces. For determining the same, dynamic analysis is necessary for preliminary design seismic coefficient method or response spectrum method in recommended depending upon the height for finding the base shear and base moment induced in the dam during earthquake. To determine the same, natural period of the dam must be known.

For this purpose, the following formulas put forward in the code

$$T = 5.55 \frac{H^2}{B} \sqrt{\frac{W_m}{gE_s}} \tag{3.5}$$

T - fundamental period in second

H - height of the dam in metres

B - Base width of the dam in metres

W_m - Unit weight of the material of the dam kgf/m^3

g - Accelaration due to gravity in m/sec^2

E_s - Modulus of electricity of the material kg/m^2

The provenence of the formula is not known, however the equation is simple. Therefore it would be desirable to substantiate the validity of the same through mathematical model. With this objective in view in this thesis a realistic method is propounded for assessing the period.

For finding the natural period, frequency must be known. Frequency analysis of a dam can be performed in two ways. One avenue is to use the formulation of partial differential equation, treating the dam as a continuous system. Since the profile of the dam is irregular, theortical solution of the partial differential equation is an impossible task. The other alternative is to analyse the dam as a discrete system using the finite element technique. The disadvantage is apart from time factor, proficiency in several disciplines is required which is a serious hindrance and is beyond the reach of many designers. Here a simple theory is suggested using the elementary principiles of strength of materials and structural dynamics for assessing the natural period in a rational manner.

3.4 Proposed SDOF Model

In general, a complex system with mass continuously distributed such as the dam may be approximated as a SDOF system and its vibration characteristics can be studied. Conversion of the dam into a SDOF system is rather a crude way of performing dynamic analysis. Fig 3.1 shows a typical profite of the dam, it is the simplified model of a more complex outline.

It is assumed that the structure is founded on rck which will not settle or slide due to loss of strength during vibration. In otherwords, a perfect fixing at the bottom is assumed which leads to the illation that the mass close to the bottom will not oscillate during vibration.

At present little is known about how much mass is actually participating in the motion. In the absence of reliable information on this aspect the entire mass is assumed to concentrate at mid height of the dam. With this premises the dam is converted into a S.D.O.F system.

3.4.1 Computation of spring factor

The lateral dimension is such that the deformation of the dam consists of both shearing and bending. The variation f accelarating due to gravity (g) and modulus of elasticity E) are linear. We can add the deformation due shear and lexure to get the combined effect.

.4.1.1 Shearing Deflection Component

A cantilever of height h with rectangular c/s is shown n Fig. 3.2, with a unit load acting at the top where the direct flexibility influences coefficient is needed. The width of the beam perpendicular to the paper is considered as unity.

Shearing deformation $d\delta$ of the small element ABCD with height dx is given by[20]

$$d\delta = \frac{1.2\ dx}{GA} \qquad (3.6)$$

where G = shearing modulus

A = b x 1 c/s area

b = width of the cantilever

Integrating between limits 0 and h Eqn. (3.6) beams

$$\delta = \frac{1.2\ h}{Gb} \qquad (3.7)$$

In Fig. 3.3 a tapering cantilever with unit load acting at the top. An element ABCD with height dx is considered at a distance x from top. The element ABCD is so small, that it can be considered as a rectangle so that AB = CD.

Similar to Eqn. (3.6), the small contribution of the element ABCD to shearing deflection is

$$d\delta' = \frac{1.2\ dx}{GA} \qquad (3.8)$$

where A = AB x 1

$$A = [a + (\frac{b-a}{h})^{x}]\times 1 \qquad (3.9)$$

Substituting Eqn. (3.9) in eqn. (3.8) and integrating between the limits 0 and h leads to

$$\delta' = \frac{1.2\ h}{G\ (b-a)} \ln b/a \qquad (3.10)$$

Which is valid for $0 < a < b$. Using eqn. (3.10) the shearing deflection contribution due to unit load is found.

3.4.1.2. Bending Reflecting Component

In Fig. 3.4 the tapering cantilever is subjected to unit load at the top,

$$\text{depth at section } x - x = [a + (\frac{b-a}{h})x] \qquad (3.11)$$

$$\text{Moment of Inertia} = I = \frac{1}{12} b' \left[a + \left(\frac{b-a}{h}\right)x\right]^3 \qquad (3.12)$$

Where b' is unity

The differential eqn. of flexure beam

$$EI \frac{d^2y}{dx^2} = -M \qquad (3.13)$$

Upon substitution for I from eqn. (3.12) and observing

M = 1 x x, eqn. (3.13) becomes

$$E \frac{d^2y}{dx^2} = \frac{12\,x}{\left[a + \frac{(b-a)}{h} x\right]^3} \qquad (3.14)$$

The kinematic B.Cs are

$$\text{Where} \quad x = 1, \quad \frac{dy}{dx} = 0$$

$$x = 1, \quad y = 0$$

Using the B.Cs and integrating eqn. (3.14) leads to

$$Y = \delta'' = \frac{6h^3}{E\,(b-a)^3} \left\{2 \ln b/a + (a/b - 1) - \frac{(2b-a)\ (b-a)}{b^2}\right\}$$

(3.15)

It is valid when $0 < a < b$. Eqn. (3.15) gives the contribution due to flexure when unit load acts at the top of the beam.

The flexibility influence coefficient $\delta = \delta' + \delta''$ (3.16)

The spring factor $k = 1/\delta$ (3.17)

3.4.2 Mass of the Dam

$$\text{Average width } a = \frac{b+t}{2}$$

$$\text{Mass} = \frac{a \times H \times \text{density}}{g}$$

where

H - ht of dam

density of concrete 24 KN/m^3

g - acceleration due to gravity

The mass of the dam m is known,

The natural frequency p of SDOF is given by

$$p^2 = k/m = 1/m\delta \qquad (3.18)$$

The natural period $T = 2\pi/p$ (3.19)

The procedure described above will be exemplified using a numerical example. See Appendix (1).

.5 MODELLING THE DAM INTO TWO DEGREE OF FREEDOM SYSTEM

A better accuracy for the frequency may be obtained if ie structure is treated as a discrete system with many egrees of freedom. However, modelling the complex structure nto two or more degrees of freedom has a concomitant isadvantage, ie., it becomes more and more difficult to etermine the stiffness influence coefficients as well as the requencies associated with various degrees of freedom. This s especially true in the case of a determinate structure uch as the dam. Further, for more than 3 D.O.F. use of a omputer is inevitable for finding the fundamental frequency f the system dynamic matrix is formulated in terms of tiffness matrix. On the other hand if flexibility influence oefficients are available, the well known Rayleigh and todola methods can be used manually. In the case of n .O.F. system, $\frac{n(n+1)}{2}$ flexibility influence coefficients re required. Two disadvantages are inherent in those two ethods. First, the labour involved is more. Second, the etermination of off-diagonal elements of the flexibility atrix is more cumbersome than the direct flexibility nfluence coefficients.

A method which dispenses with the above disadvantage is he one proposed by Dunkerley. The classic Dunkerley's equation is of the form $1/p^2 = \sum m_i \delta_{ii}$ (3.20)

m_i mass associated with i^{th} degree of freedom

δ_{ii} principal flexibility influence coefficients

p fundamental frequency

3.5.1 APPLICATION OF DUNKERLEY'S EQUATION TO A DAM

To find the fundamental frequency using eqn. (3.20), the dam shown in Fig. 3.5 is discretised into two parts of equal height.

The mass of potion ABCD is assumed to be concentrated at mid height at Station (1) and the mass of the portion of CDEF is lumped at mid height at Station (2).

For these two segments, the flexibility influence coefficients δ_{11} and δ_{22} are found as done in the case of S.D.O.F. system using eqn. (3.10) and (3.15). Then using (3.20) the Dunkerley frequency is found, from which the natural period is determined using eqn. (3.19).

This is examplified using a numerical example. See Appendix 2.

3.5.2 CORRECTION FOR DUNKERLEY FREQUENCY

For a stable system in which the frequencies are real and distinct, the Dunkerley frequency gives lower bound solution. How much the frequency falls short of the true frequency depends upon the number of elements into which the continuous systems is discretized. A study of several Dunkerley's frequencies with 2 D.O.F onwards show that the frequency is less by an amount varying from 15 % to as low as 2 %. In the present case dam is discretized into two parts which is the bare minimum possible. It is likely that the frequency may be lower by about 15%. Therefore, the Dunkerley frequency is increased by a factor 1.14. In other words, Dunkerley's period is divided by 1.14.

3.6 DISCUSSION OF RESULTS

The prediction by the SDOF model and TDOF model is shown along with the code equation in Table 3.1. It is seen that the solution of the SDOF model is consistently and slightly higher than that of the code equation. ON the other hand TDOF model gives fairly accurate results compared with the prediction of code equation.

The only assumption made in SDOF model is that the mass acts at mid height of the dam. The nature of solution lends support to the validity of this assumption. The concepts involved in the two models are exceeding simple and amply attest the fidelity of the code equation. Another distinguishing feature of code equation is its sheer implicity by which the solution can be obtained very quickly.

Natural period of vibration is a logical index used to obtain the base shear and base moment during preliminary design. Owing to the complex profile of the dam exact dynamic analysis is very tedions. The code suggests response specturm method for preliminary design. For use in response spectrum method, a formula is suggested by the code for computing the fundamental period. In this two approximate mathematica models are put forward for reckoning the natural period. It is found that the prediction by the two models vouch for the efficacy of the code equation.

3.7 IMPORTANCE OF DETAILED ANALYSIS

Past as well as current practice in the aseismic design of dams is usually based on Psendostatic methods. In these methods, a design seismic coefficient is adopted keeping in view the past seismic history of the region but without giving regard to the dynamic properties of the structure itself. The codes of practice of various countries specify values of seismic coefficients for the design of dams more or less on an empirical basis. However, the continued use of these empirical values of seismic coefficients appears to given them some semblance of an authoritative design criterion, although, they lack any rational justification. The approach may be reasonable for the purpose of preliminary design and may even suffice for less important dams located in areas which are not visited by severe earthquakes. But, it will generally not be adequate and appropriate for more important dams and in those regions where severe earthquakes are expected to occur. The seismic coefficient method leads to an inefficient design and for an efficient design, a detailed dynamic analysis of the dam due to anticipated earthquake ground motion is essential.

The seismic coefficient method is independent of the dynamic property of the structure. The response spectrum method is scientific as it centers around the vibration characteristic of the dam, via, the fundamental period. However, this method will generally not be sufficient and appropriate for more important dams to be constructed in those regions where severe earthquakes are possible.

Knowing this fact, IS 1893 - 1984 recommends that detailed dynamic analysis is desirable for the design of dams. But it does not indicate the type of dynamic analysis to be performed for final design of dams. Using dynamic analysis, calculations can be made of the earthquake induced vibrations of the structure and these will indicate the general nature and amplitude of the deformation that can be expected during the earthquake.

The dynamic analysis centers around the stiffness matrix which can be formulated in a number of ways depending upon the discretion of the analyst.

In the first part of this thesis, a realistic solution is put forward for arriving at the stiffness matrix and mass matrix for the determination of eigen pairs. Finally a hand computation solution is indicated for finding the eigen pairs which is found to be in close agreement with the computer solution.

Here modal analysis is performed for which modal frequencies and modal amplitudes combinedly known as eigen pairs are required. Of the many modes of vibration that are possible, it is found that the first three lower modes contribute predominantly to the response of the system. For the determination of the eigen pairs, two approaches exist.

i) In the first one the dam is treated as a body with mass continuously distributed. This leads to the formulation of a partial differential equation which can be solved numerically.

ii) The second alternative is to consider the dam as a discrete system with mass lumped at selected locations.

This results in the formulation of ordinary partial differential equation of the following form

$$[m]\{\ddot{x}\} + [k]\{x\} = 0 \qquad (3.21)$$

where

[m] - mass matrix, a diagonal matrix

[k] - lateral or translational stiffness matrix

$\{\ddot{x}\}$ - acceleration vector, a column matrix

{x} - displacement vector,, a column matrix.

Eqn. (3.21) refers to equation of motion of free vibration of a system having n translational degrees of freedom.

Once the matrices [m] and [k] are formulated, there are many mathematical eigen value theories for obtaining the eigen pairs using a computer. Here Jacobi's program is used in the extraction of eigen pairs.

3.8 Formulation of [m] and [k] Matrices

Formulation of [m] and [k] matrices are possible is a variety of ways using finite element method and mechanics of mateirals approach.

In general, for using the finite element analysis of a complex structure such as the dam, proficiency in various disciplines is required. Secondly, preparation of a program is an involved one entailing considerable amount of time and effort. Because of this fact, the FEM is beyond the reach of many designers. It is also a fact that at times, the strength of materials solution happens to be more accurate than the FEM answer.

On the otherhand, the mechanics approach is less tedious which can be comprehended and performed easily by a nonspecialist. The [m] and [k] matrices can be easily set up with less effort and time by making judicious assumptions.

IS : 1893 - 1984 does not indicate the type of dynamic analysis to be performed for final design of dams. That is, it does not promulgate explicitly as to how to formulate the stiffness matrix, [k]. In view of these ramifications, the analyst is at liberty to choose any method for arriving at the stiffness matrix.

3.9 EVOLUTION OF THE PROPOSED THEORY FOR OBTAINING THE MASS MATRIX [m] AND STIFFNESS MATRIX [k]

A typical c/s of the dam shown in Fig. 3.6. The profile of the dam is irregular which is not amenable for mathematical treatment. Hence to arrive at a closed form solution, the irregular geometry is simplified as a trapezium as shown in Fig. 3.1.

The tapering section is considerably deep at the bottom and very slender at the top. Because of this fact bending deformation increases gradually, from the bottom towards the top. It is assumed that both shearing and flexural deformations are present in any section. Secondly, it is supposed that the dam is firmly fixed at the bottom enabling the structure to behave as a vertical cantilever. Since the cantilever is a determinate structure, finding the flexibility influence coefficients will be much easier than the stiffness influence coefficients.

3.9.1 Formulation of the Flexibility Matrix $[\delta]$

Since the dam is considered as a linear structure, total deformation at any section may be obtained by superposing the shearing and bending deflection. This means

$$[\delta] = [\delta_f] + [\delta_s] \tag{3.22}$$

where

$[\delta]$ - translational flexibility matrix

$[\delta_f]$ - flexibility matrix due to bending deformation

$[\delta_s]$ - flexibility matrix due to shearing effect

In Fig. 3.7 a until load is applied at the top. Using mechanics theory, the shearing deflection δ_s, the flexural deflection δ_f and the rotation due to flexure θ_f can be found. Since the derivation is very involved, only the results are reported below.

$$\delta_s = \frac{1.2\ h}{G\ (b-a)} \ln [b/a] \qquad (3.23)$$

$$\delta_f = \frac{6\ h^3}{E\ (b-a)^3} \{2\ln [b/a] + [a/b - 1] - \frac{(2b-a)\ (b-a)}{b^2}\} \qquad (3.24)$$

$$\theta_f = \frac{12\ h^2}{E(b-a)^2} \{\frac{(2b-a)}{2b^2} - \frac{1}{2a}\} \qquad (3.25)$$

where

a - width of the dam corresponding to station i

b - bottom width

h - height of station i from bottom

G - shear modulus

E - Young's modulus

The above equations are valid for $0 < a < b$. Eqn.(3.23) enables the formulation of flexibility matrix due to shearing effect. Eqn. (3.24) and Eqn. (3.25) facilitate the formulation of flexibility matrix due to flexural effect. Superposition of these matrices results in the flexibility matrix due to combined effect.

3.9.2 Formulation of Mass Matrix [m]

The mass acts along the various D.O.F. The matrix will be a lumped matrix having non-zero diagonal elements.

Formulation of the flexibility matrix [δ] and the mass matrix [m] are illustrated in Appendix II.

3.10 PROGRAMME OF STUDY

In this study, in all twenty dams of varying heights were investigated as described in the illustrative example. Convergence occured when the dam was discretized into seven segments. For all the dams eigen pairs were found using Jacobi's program. Using the same, modal analysis was performed for 5% damping exactly along similar lines described in [7]. For performing modal analysis first three modes were considered.

3.11 PROPOSED METHOD

The modal analysis described in the code* is a semiautomated procedure in which the first step is to obtain the eigen pairs. Without the use of a computer, it is not possible to reckon the eigen pairs. With the knowledge of eigen pairs, modal analysis can be performed manually very easily. Therefore a simple method is proposed to get the eigen pairs as accurately as possible.

From the detailed study of frequencies and mode shapes of twenty dams, the following expressions for the periods of first three modes and the corresponding eigen vectors are obtained.

3.11.1 Period of the Dam

i) Fundamental period, $T_1 = \frac{158.11}{E_c} \left[\frac{D}{420} + \beta^{3.815} \right]$ (3.26)

where T_1 - first mode period in seconds

E_c - Young's modulus of concrete in MPa

D - Total height of the dam in metres

β - Top to bottom width ratio

ii) Period of second mode, $T_2 = 0.414\ T_1$ (3.27)

iii) Period of third mode, $T_3 = 0.238\ T_1$ (3.28)

Basis of the Empirical Equation No. 3.26

The equation was arrived based on the following assumptions.

$$T_1 = C_1 D + \beta^x \quad (3.29)$$

where C_1 and x are constants.

Using the results obtained from the computer study, the constants C_1 and x were evaluated by trial and error. Then the Eqn. (3.29) was modified as shown in Eqn. (3.26) to account for the variation in the Young's modulus of the material.

3.11.2 Eigen Vectors of the Various Modes

The three mode shapes along with the details are shown in Fig. 3.8.

i) Fundamental Mode :

The expression for the amptitude at any station i is given as

$$a = \left[\frac{x}{H}\right]^{1.6} \qquad (3.30)$$

where a - amplitude of station i

x - height of station i from bottom

H - height of top most station from bottom

ii) Second Mode :

For portion AB shown in Fig. 3.8.

$$a = 0.4 \sin \frac{\pi x}{z} \quad \text{for} \quad 0 < x < z \qquad (3.31)$$

where $z = 0.782\ H$

For portion BC

$$a = \frac{-(y-z)}{(H-z)} \qquad (3.32)$$

where y - station distance measured from bottom

iii) Third Mode :

For portion AB shown in Fig. 3.8.

$$a = 0.4 \; \text{Sin} \; \frac{2 \pi x}{z} \qquad (3.33)$$

where $z = 0.955 \; H$

For portion BC shown in Fig. 3.8.

$$a = \frac{(y-z)}{(H-z)} \qquad (3.34)$$

where y = height of station situated in portion BC, measured from bottom

Using the above expressions, the natural periods and the corresponding modal amplitudes (See Fig. 3.9) may be obtained for the first three modes by suitably discretizing the dam.

Prediction of Eqn. (3.26), Eqn. (3.27) and Eqn. (3.28) are shown in Table 3.2 along with the computer solution for the twenty dams analysed. The eigen vectors obtained from Eqn. (3.30), Eqn. (3.31), Eqn. (3.32), Eqn. (3.33) and Eqn. (3.34) are shown along with computer solution for the 310 m height dam in Table 3.3. The dam was discretized into seven divisions.

3.11.3 Comparison of Computer Solution With the Suggested Hand Computation Procedure

Using the computer results and hand computation solution for the eigen pairs, the dynamic analysis was performed along similar lines described in [7]. The results obtained for shear are compared for the 310 m height dam in Table 3.4.

Table 3.1 PROPERTIES OF THE DAM AND PREDICTION OF PERIOD BY SDOF MODEL AND TDOF MODEL

Height of the dam H (m)	Top width t (m)	Bottom width b (m)	Natural period T in sec by SDOF Model	Natural period T in sec by Code Eqn.	Natural period T in sec by TDOF Model
310.0	10.0	220	0.818	0.758	0.753
294.5	9.5	209	0.777	0.721	0.716
279.0	9.0	198	0.736	0.683	0.678
263.5	8.5	187	0.695	0.645	0.640
248.0	8.0	176	0.654	0.607	0.603
232.5	7.5	165	0.614	0.569	0.565
217.0	7.0	154	0.573	0.531	0.527
201.5	6.5	143	0.532	0.493	0.490
186.0	6.0	132	0.491	0.455	0.452
170.5	5.5	121	0.450	0.417	0.414

11	155.0	5.0	110	0.409	0.379	0.377
12	139.5	4.5	99	0.368	0.341	0.339
13	124.0	4.0	88	0.327	0.303	0.301
14	108.5	3.5	77	0.286	0.265	0.264
15	93.0	3.0	66	0.245	0.228	0.226
16	77.5	3.0	55	0.205	0.190	0.189
17	62.0	3.0	44	0.164	0.152	0.152
18	46.5	3.0	33	0.124	0.114	0.116
19	31.0	3.0	22	0.083	0.076	0.079
20	15.5	3.0	11	0.043	0.038	0.041

Table 3.2 PREDICTION OF PERIOD BY PROPOSED METHOD AND COMPARISON WITH COMPUTER SOLUTION OF GRAVITY DAM

Sl. No.	Dam Height H in Metres	Base Width b in Metres	Top Width in Metres	Period T_1		Period T_2		Period T_3	
				Proposed Equation in Sec.	Computer Solution in Sec.	Proposed Equation in Sec.	Computer Solution in Sec.	Proposed Equation in Sec.	Computer Solution In Sec.
1	310.00	220.00	10.00	0.738	0.738	0.306	0.306	0.176	0.176
2	294.50	209.00	9.50	0.701	0.701	0.290	0.290	0.167	0.167
3	279.00	198.00	9.0	0.664	0.664	0.275	0.275	0.158	0.158
4	263.50	187.00	8.5	0.627	0.628	0.260	0.260	0.149	0.149
5	248.00	176.00	8.0	0.590	0.590	0.245	0.245	0.141	0.141
6	232.50	165.00	7.5	0.554	0.553	0.229	0.229	0.132	0.132
7	217.00	154.00	7.0	0.517	0.517	0.214	0.214	0.123	0.123
8	201.50	143.00	6.5	0.480	0.480	0.199	0.199	0.114	0.114
9	186.00	132.00	6.00	0.443	0.443	0.183	0.183	0.105	0.105
10	170.50	121.00	5.50	0.406	0.406	0.168	0.168	0.097	0.097

155.00	110.00	5.00	0.369	0.369	0.153	0.153	0.088	0.088
139.50	99.00	4.50	0.332	0.333	0.138	0.138	0.079	0.079
124.00	88.00	4.00	0.295	0.296	0.122	0.122	0.070	0.070
108.50	77.00	3.50	0.258	0.259	0.107	0.107	0.061	0.061
93.00	66.00	3.00	0.221	0.222	0.092	0.029	0.053	0.053
77.50	55.00	3.00	0.185	0.186	0.076	0.077	0.044	0.044
62.00	44.00	3.00	0.148	0.157	0.061	0.061	0.035	0.035
46.50	33.00	3.00	0.111	0.116	0.046	0.046	0.026	0.026
33.00	22.00	3.00	0.074	0.080	0.031	0.031	0.018	0.017
15.50	11.00	3.00	0.044	0.044	0.018	0.015	0.010	0.008

Table 3.3 PREDICTION OF AMPLITUDE BY PROPOSED METHOD AND COMPARISON WITH COMPUTER SOLUTION FOR GRAVITY DAM

Station Number from the top	Weight in KN	First Mode Shape		Second Mode Shape		Third Mode Shape	
		Computer Solution	Proposed Equation	Computer Solution	Proposed Equation	Computer Solution	Proposed Equation
1	11226.4	1.000	1.000	-1.000	-1.000	1.000	1.000
2	24698.6	0.735	0.765	-0.254	-0.294	-0.338	-0.262
3	38169.8	0.511	0.555	0.187	0.141	-0.518	-0.400
4	51641.5	0.330	0.371	0.362	0.332	-0.103	-0.156
5	65113.2	0.190	0.217	0.347	0.399	0.305	0.229
6	78584.5	0.089	0.096	0.225	0.319	0.384	0.399
7	92056.7	0.022	0.016	0.121	0.071	0.157	0.194

Table 3.4 Comparison of shear values for 310 m height gravity dam

Station No.	By Computer Solution Shear KN	By Proposed equation Shear KN
1	6967	7140
2	14002	14402
3	20991	21734
4	27937	29912
5	34625	36510
6	40746	44247
7	43401	46527

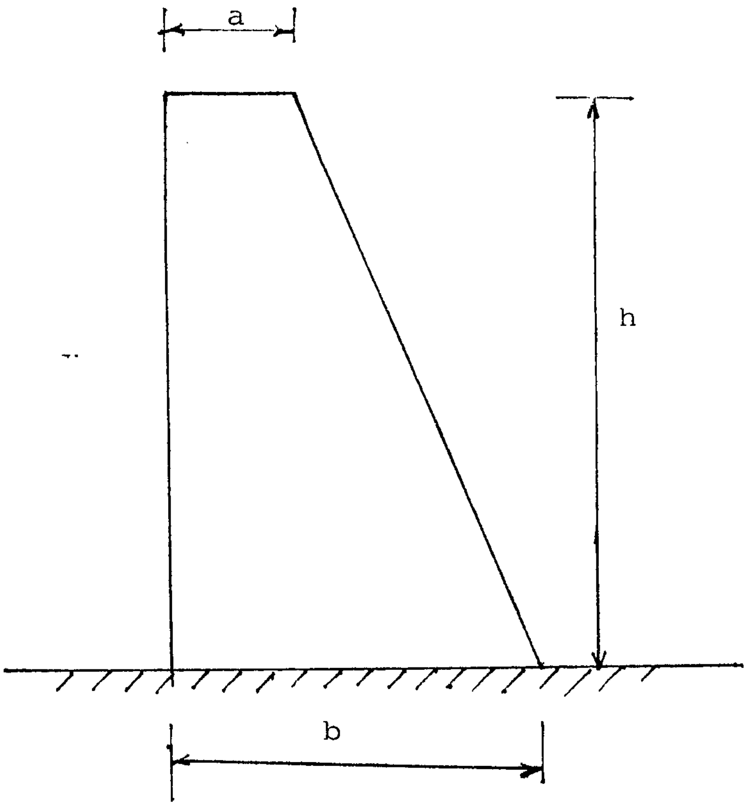


Fig. 3.1 Simplified Profile of a Gravity Dam

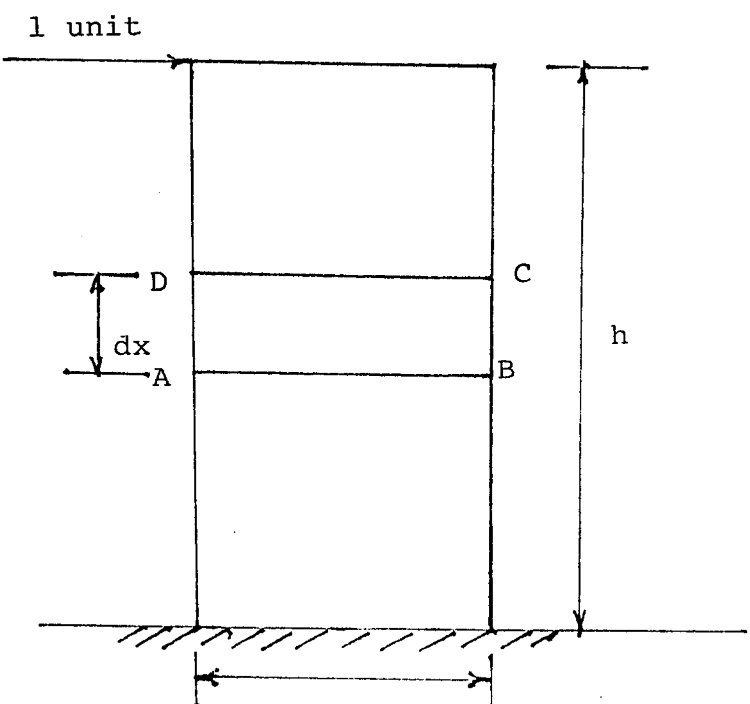
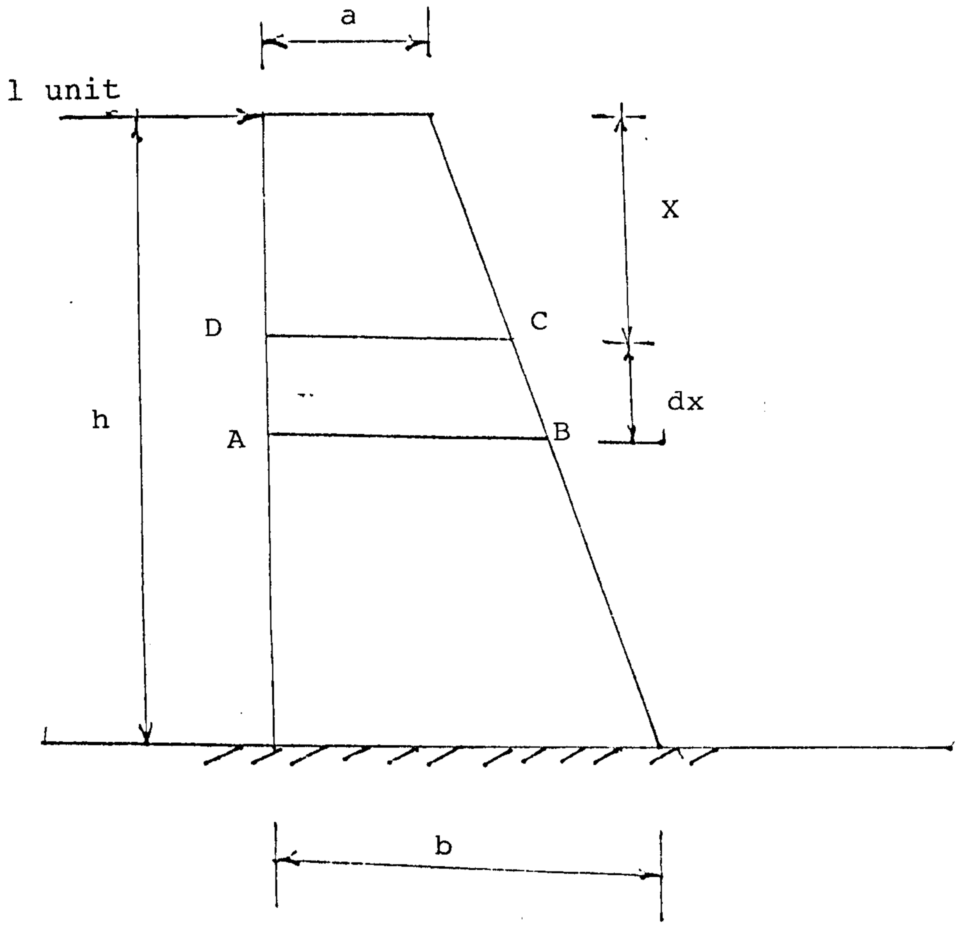
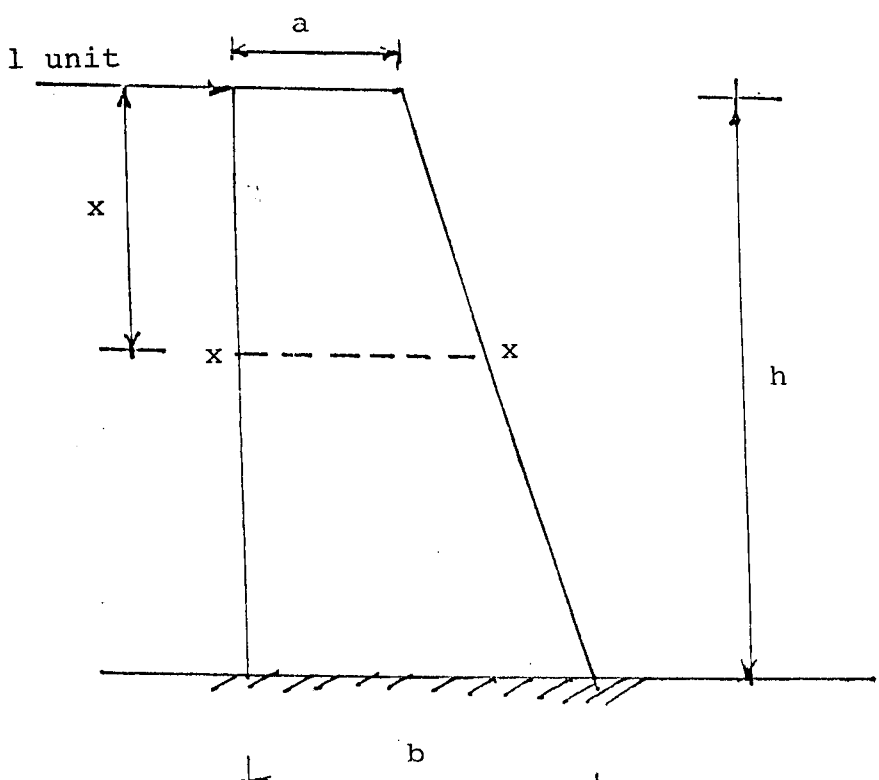


Fig. 3.2 Computation of Shearing Deflection in a



ig. 3.3 Computation of Displacement Contribution Due to Shear



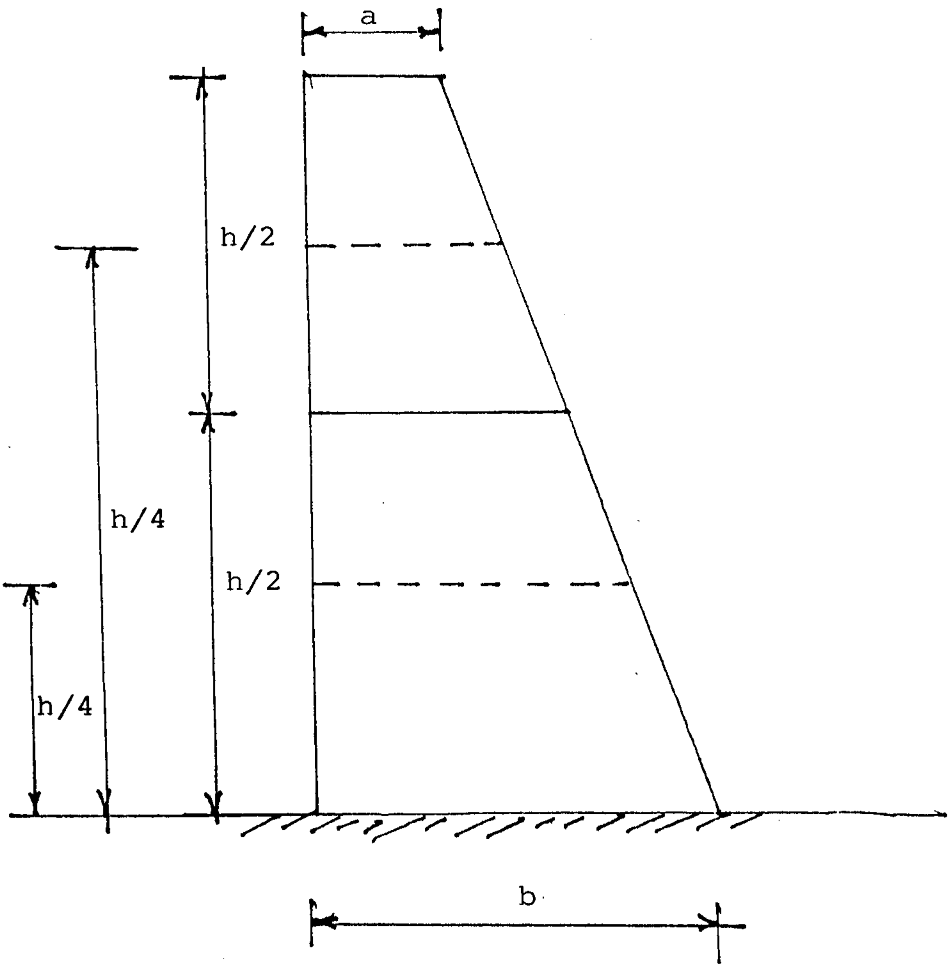


Fig. 3.5 Two Degree of Freedom Model

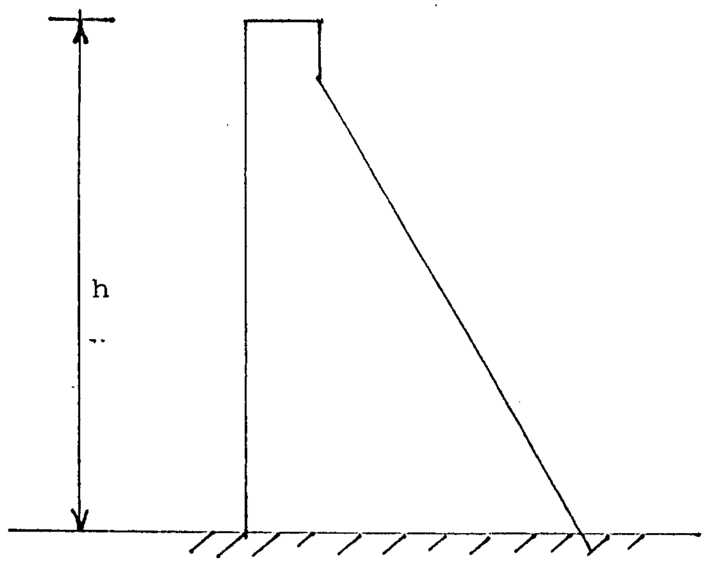


Fig. 3.6 Complex Profile of a Gravity Dam

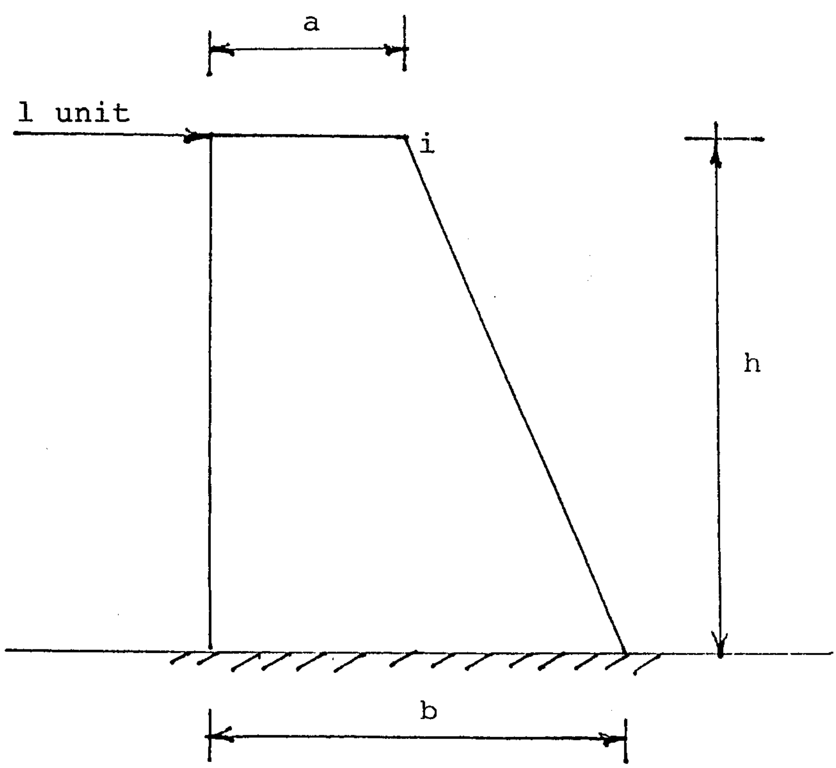


Fig. 3.7 Unit Load Acting at Station i

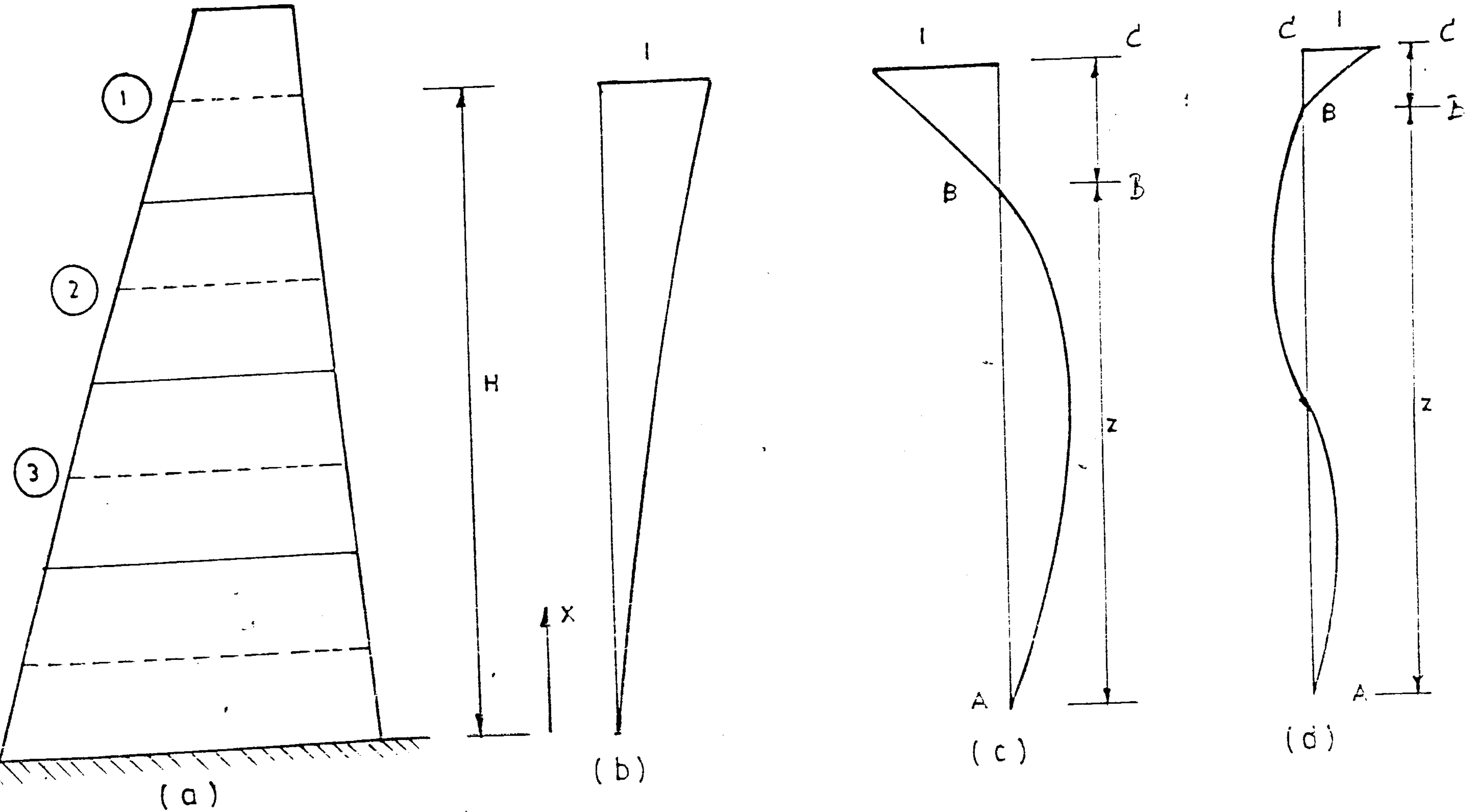


Fig. 3.8 Various Mode Shapes

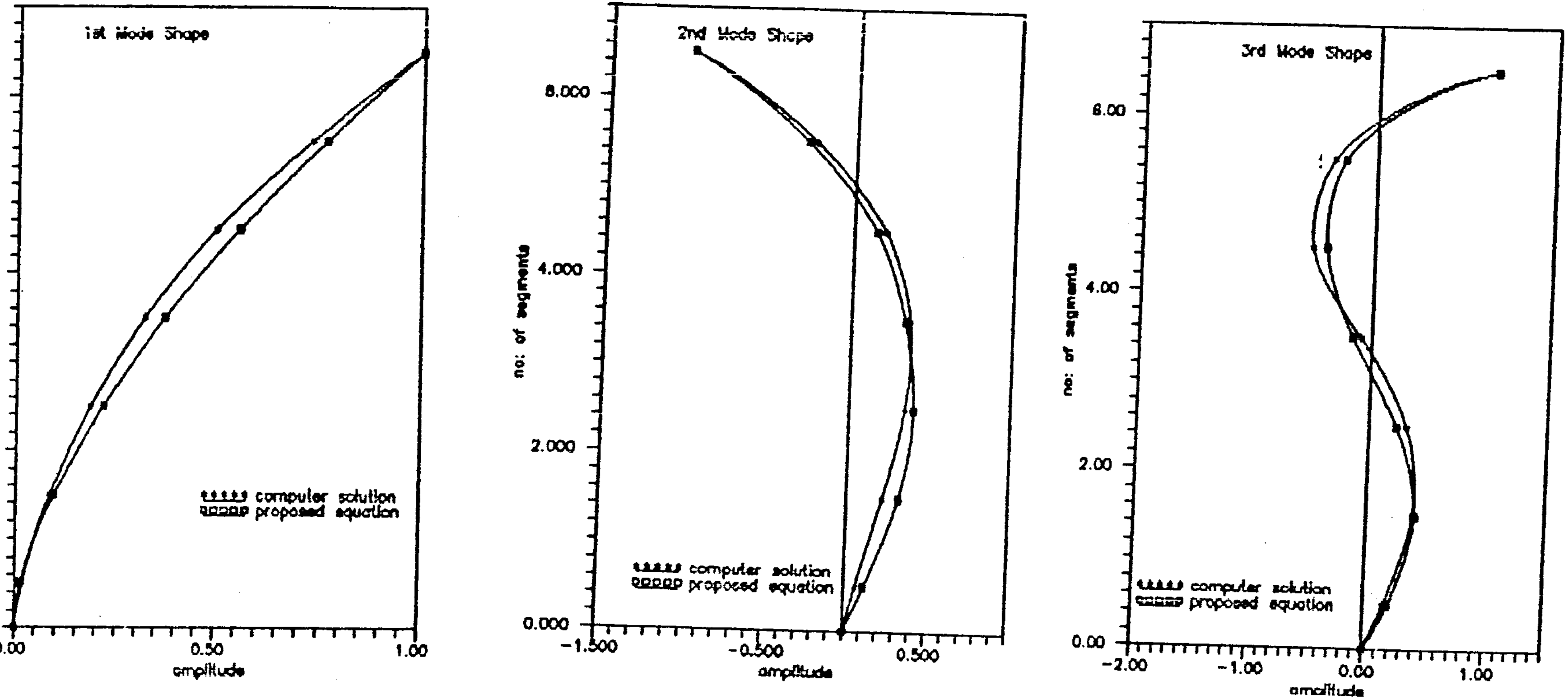


Fig. 3.9 Tree Mode Shapes of Gravity Dam

CHAPTER 4

EARTH DAM

4.1 GENERAL

The ever present need for designing safe structures and the prevention of possible catastrophics has focussed much attention on the performance of earthdams in earthquake motion. Many dams damaged by earthquakes proved their safety against static forces but the damage resulted by superimposition of dynamic forces.

A rigid structure will follow the oscillations of its base as there can be no relative deformation from base to top. For a flexible structure the oscillations to which its base is subjected serve to set the structure vibrating. Earthdam is considered as a flexible structure for determining the dynamic response.

Earthdam vibrates when subjected to ground motion during an earthquake requiring thereby a dynamic analysis of the structure for its design.

Currently the period is computed based on the assumption that the mass density and shear modulus are constant through out the dam. But actually it varies along the height of the dam due to compaction and self weight.

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Here a linear variation of shear modulus and density is assumed and dynamic analysis is carried out.

4.2 PROPOSED METHOD

4.2.1 Formulation of Flexibility Matrix

Following assumptions are made in the formulation of flexibility matrix.

1. There exists a perfect fixity between the foundation and the superstructure of the dam.
2. The earthdam behaves as an elastic system
3. The second order effect caused by the axial shortening due to the self weight is negligible.
4. The vibration mode shape is similar to that of a deep cantilever beam.

For earth dams shear deformation is more predominant than flexural deformation. By neglecting the bending deformation the flexibility influence coefficients can be computed. In the case of earth dam both the profile of the dams and properties of the materials varies along the height (See Fig. 4.1). The very same expression for the flexibility matrix due to shear for the gravity dam can be used to find the flexibility matrix for earthdam.

$$\delta_s = \frac{1.2\ h}{G\ (b-a)} \ln b/a \qquad (4.1)$$

where G - varies along the height of the dam

4.2.2 Computation of Mass Matrix

Only difference in calculating the lumped mass associated with each translational degree of freedom from the previous chapter is due to the variation in the density along the height.

4.3 PROGRAMME OF STUDY

In this study, in all ten dams of varying were investigated. Convergence occured when the dam was discretized into thirteen segments. For all the dams eigen pairs were found using Jacobi's program. Using the same, modal analysis was performed for 10% damping exactly given in[7]. For performing modal analysis first three modes were considered.

4.4 PROPOSED METHOD

The modal analysis described in the code is a semiautomated procedure in which the first step is to obtain the eigen pairs. Without the use of a computer, it is not possible to reckon the eigen pairs. With the knowledge of eigen pairs, modal analysis can be performed manually very easily. Therefore a simple method is proposed to get the eigen pairs as accurately as possible.

4.4.1 Period of the Dam

i) fundamental period, $T_1 = [\frac{D}{57.25} - \beta^{1.605}]$ (4.2)

ii) period of second mode, $T_2 = 0.475\ T_1$ (4.3)

iii) period of third mode, $T_3 = 0.31\ T_1$ (4.4)

4.4.2 Eigen Vectors of the Various Modes

i) Fundamental Mode

The expression for the amplitude at any station i is given as

$$a = \left(\frac{x}{H}\right)^{0.95} \quad (4.5)$$

ii) Second Mode

For portion AB shown in Fig. 3.8.

$$a = 0.5 \ \text{Sin} \frac{\pi x}{z} \quad \text{for } 0 < x < z \quad (4.6)$$

where $z = 0.65\ H$

For portion BC

$$a = \frac{-(y-z)}{(H-z)} \quad (4.7)$$

ii) Third Mode

For portion AB shown in Fig. 3.8.

$$a = 0.2 \text{ Sin } \frac{2 \Pi x}{z} \qquad (4.8)$$

where $z = 0.802 \text{ H}$

For portion BC

$$a = \frac{(y-z)}{(H-z)} \qquad (4.9)$$

Using the above expressions, the natural periods and corresponding modal amplitudes (See Fig. 4.2) may be obtained for the first three modes by suitably discretizing the dam.

4.5 CODE PROCEDURE

1. Determine the fundamental period of the structure from the formula

$$T = 2.9 \, H_t \sqrt{\rho / G} \qquad (4.10)$$

T - fundamental period of the earth dam in sec.

H_t - height of the dam above toe of the slopes

ρ - mass density of shell material

G - Modulus of rigidity of shell material

2. Determine S_a/g for this period T and 10% damping from average acceleration spectrum curves given in Fig. 2.1.

3. Compute the design seismic coefficient α_h using the equation $\alpha_h = \beta I F_o S_a/g$ (4.11)

4. The base shear V_B and base moment M_B may be obtained by the following formula

$$V_B = 0.6 W \alpha_h \quad (4.12)$$

$$M_B = 0.9 W \bar{h} \alpha_h \quad (4.13)$$

where W - total weight of the dam

$\bar{h}$ - height of the centre of gravity of the dam above the base in metres.

4.6 Results and Discussions

The results obtained for the period from computer solution, proposed equation and the code procedure are compared in Table 4.1, 4.2 and 4.3.

The modal analysis is done by taking the first three modes only but the contribution to shear and bending moment due to second and third mode is more. The reason is at low period the value of S_a/g is almost same.

Table 4.1 COMPARISON OF PERIOD BY PROPOSED METHOD AND COMPUTER SOLUTION FOR EARTH DAM

Height (m)	Top Width (m)	Bottom Width (m)	Period (T_1) Sec. Code Equation	Period (T_1) sec.		Period (T_2) sec.		Period (T_3) sec.	
				Computer Solution	Proposed Equation	Computer Solution	Proposed Equation	Computer Solution	Proposed Equation
100	10	410	1.712	1.740	1.744	0.827	0.828	0.539	0.541
90	9	369	1.541	1.565	1.569	0.744	0.745	0.486	0.486
80	8	328	1.370	1.391	1.395	0.662	0.663	0.432	0.432
70	7	287	1.198	1.217	1.220	0.579	0.580	0.379	0.379
60	6	246	1.027	1.043	1.045	0.496	0.496	0.324	0.324
50	5	205	0.856	0.869	0.870	0.414	0.413	0.270	0.270
40	4	164	0.685	0.696	0.696	0.311	0.311	0.216	0.216
30	3	123	0.514	0.522	0.523	0.248	0.248	0.162	0.162
20	2	82	0.342	0.348	0.347	0.165	0.165	0.108	0.108
10	1	41	0.171	0.174	0.171	0.083	0.081	0.054	0.053

Sl. No.	Station Number From The Top	Weight in kN	First Mode Shape		Second Mode Shape		Third Mode Shape	
			Computer Solution	Proposed Equation	Computer Solution	Proposed Equation	Computer Solution	Proposed Equation
1	1	3171.769	1.000	1.000	-1.000	-1.000	1.000	1.000
2	2	7065.162	0.966	0.924	-0.848	-0.771	0.643	0.596
3	3	11011.725	0.907	0.850	-0.612	-0.543	0.177	0.192
4	4	15012.243	0.831	0.771	-0.342	-0.314	-0.209	-0.065
5	5	19066.716	0.741	0.693	-0.084	-0.086	-0.396	-0.163
6	6	23174.163	0.643	0.612	0.125	0.120	-0.374	-0.200
7	7	27335.565	0.542	0.537	0.268	0.294	-0.209	-0.161
8	8	31550.922	0.442	0.458	0.340	0.425	0.065	-0.060
9	9	35819.253	0.346	0.379	0.348	0.493	0.171	0.063
10	10	40141.539	0.255	0.298	0.306	0.488	0.256	0.162
11	11	44516.799	0.172	0.217	0.230	0.411	0.248	0.200
12	12	48946.014	0.096	0.133	0.139	0.274	0.170	0.162
13	13	53429.184	0.030	0.047	0.044	0.096	0.058	0.062

Table 4.3 Comparison of Shear Values for 100 m Height for Earth Dam

tation No.	By Computer Solution Shear KN	By Proposed equation Shear KN
1	9911.37	1047.75
2	2439.39	2636.61
3	3859.82	4043.41
4	4898.08	5084.28
5	5922.51	5907.89
6	7238.70	6662.20
7	8573.50	7611.93
8	9496.66	8939.55
9	10546.40	10840.64
10	11923.71	13416.88
11	13561.72	16359.39
12	14921.11	18841.72
13	15447.25	19869.22

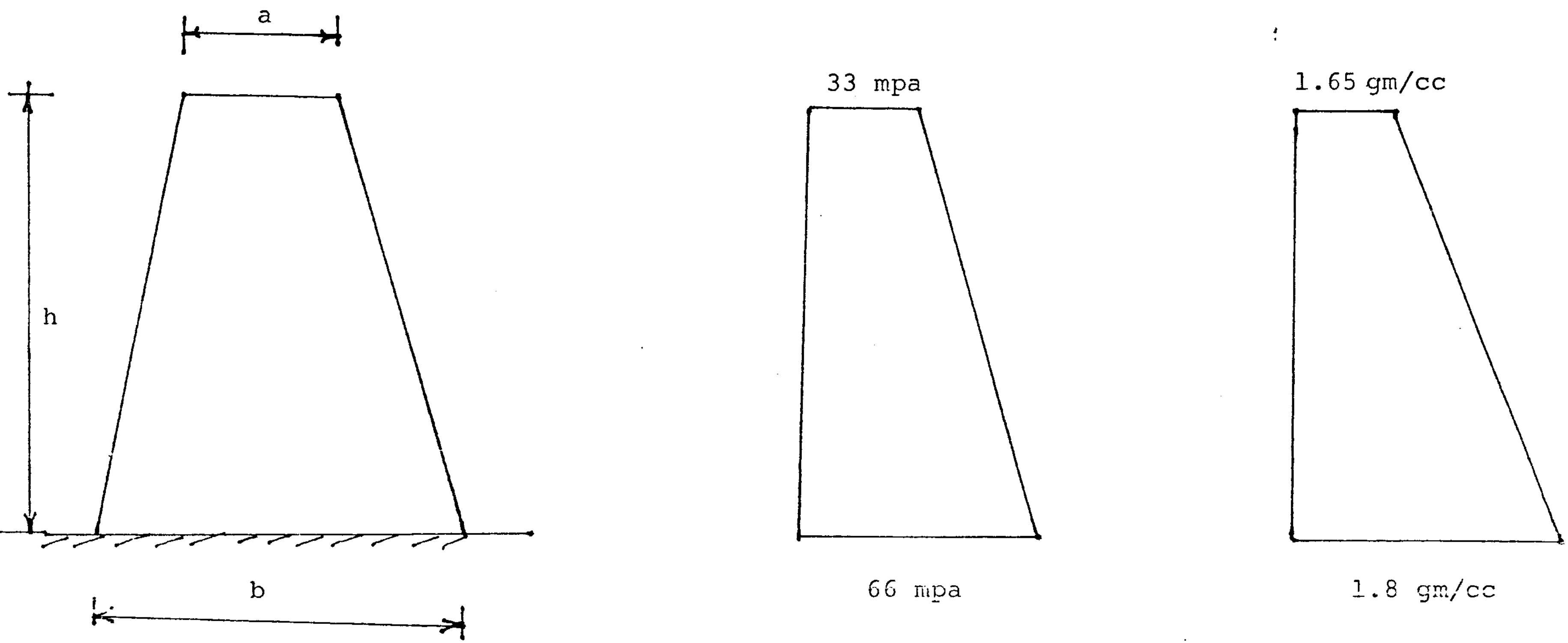


Fig. 4.1 Cross Secion of Earth Dam Variation of Shear Modulus and density

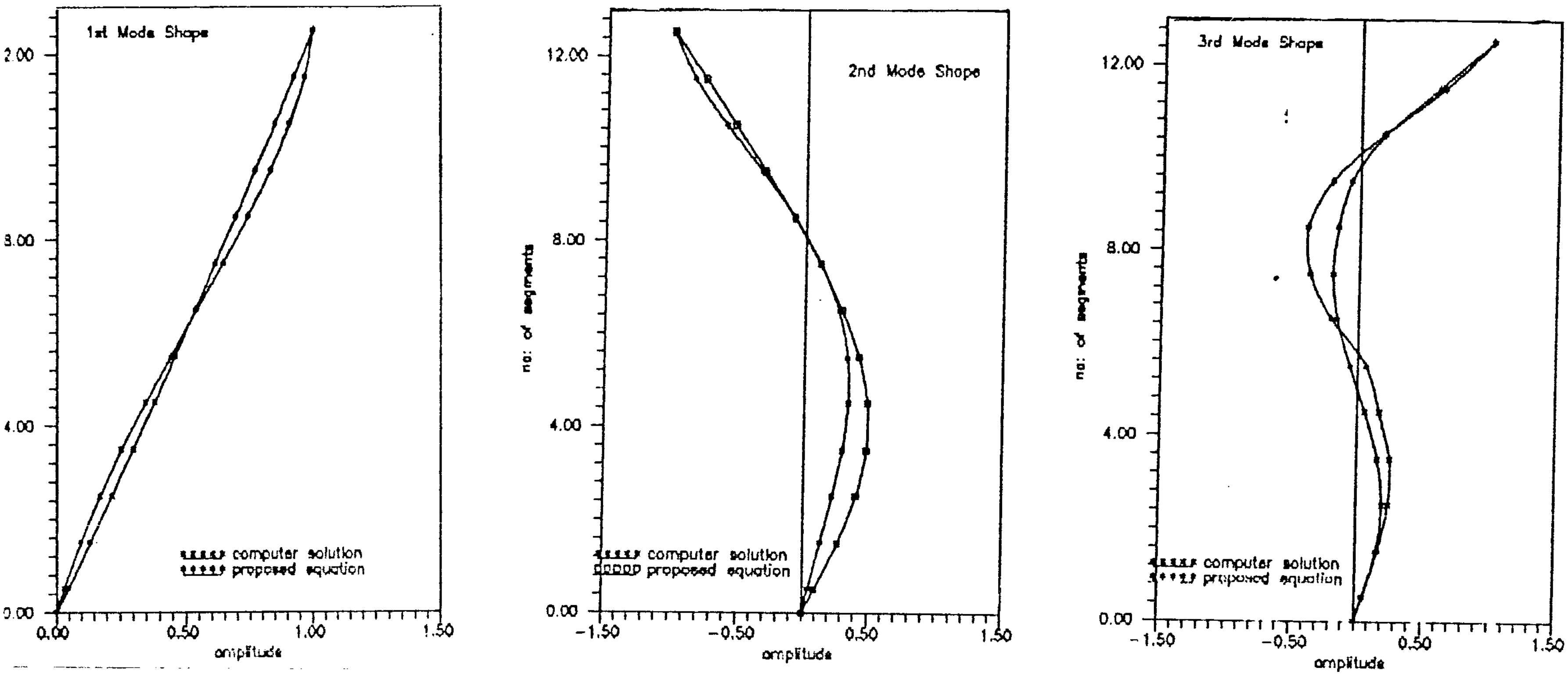


Fig. 4.2 Three Mode Shapes of Earth Dam

CHAPTER 5

COOLING TOWER

5.1 GENERAL

Even in the most advanced fossil fuel thermal (nuclear) power plants, only about 42 (35)% of the generated heat is converted into electric energy. The remaining 58 (65)% is discharged into the environment through the smokestack, the thermal unit and the cooling water circuit. Cooling towers have proven to be indispensable tools for minimizing the thermal pollution of rivers, lakes and sea shores by the waste heat of electric power plants, being able to reconcile aspects of environmental protection, initial and operating costs and reliability of power generation. It serves the purpose of cooling the water in thermal and nuclear power stations. They are classified according to the means by which air is supplied, such as mechanical draught and natural draught towers. In mechanical draught tower air is sucked into the tower by fans provided at top or bottom. If the air is sucked by the fan at top through louvers at bottom it is called an induced draught. When the air is forced in by a fan at the bottom and discharges through the top, it is known as forced draught. The tower must be tall for sufficient bouyancy and must have a large cross section because of the low rate at which air is circulated. Hyperbolic tower is the best example in this category in which the outside hyperbolic shell create a draught of air in similar way to a chimney. The divergent portion at the top helps the air to escape into the atmosphere.

5.2 HISTORIC DEVELOPMENT OF NATURAL DRAUGHT COOLING TOWERS

The well known hyperbolic shape of cooling tower was introduced by two Dutch engineers Van Iterson and Kuyper, who in 1914 constructed the first hyperboloidal tower of 355 m height. Soon capacities and tower heights increased and around 1930 tower heights of 65 m had been reached. The first such buildings to reach higher than 100 m were the towers of the High Maruham Power Station in Britain. The highest German cooling towers, located at the Gundremmingen nuclear power plant, are 165 m high. Still higher towers have been designed, but it is doubtful if they will actually be constructed in the forseable future. Fig. 5.1 shows the highest German cooling tower and Fig. 5.2 shows the historic development of natural draught cooling towers.

Towers are distinctive structures in view of the hyperbolic shape and large size combined with very small shell thickness, appreciably less in proportion than that of an egg shell. Owing to considerable height and comparatively less total weight, the tower is highly sensitive to the effect of lateral loads such as wind and earthquake motion. While the lowest natural frequencies of natural draught cooling towers lie very close to the peak of the energy spectrum for seismic effects, but far away from that of wind, the responses due to both loadings and their design methods differ considerably, where as a quasi-static analysis may be sufficient for the response to wind loading, the analysis of seismic effects requires a complete dynamic analysis.

5.3 RESEARCH SIGNIFICANCE

In the first part of this work a simplified procedure is adopted for finding the fundamental period and in the second part dynamic analysis is done for the cooling tower. Using the results obtained by computer solution a simple method is proposed to get the eigen pairs as accurately as possible.

5.4 SALIENT FEATURES OF THE TOWER

The tower consists essentially of an outside hyperbolic shell, and internal cooling fill at the bottom of the shell, a cold water basin into the cold water falls from the fill and is stored for recirulation through the plant, and a water distribution system. Fig 5.3 shows the section of cooling tower. The shell diameter at the base, throat and top, shell

ight, air inlet height etc are based on thermic design
nsiderations.

The profile of the tower before 1970 was based on a
ngle hyperbola without any offset from the tower axis. The
quation of hyperbola is

$$x^2/a^2 - z^2/b^2 = 1 \qquad (5.1)$$

where x and z are measured from the origin situated at
he throat as shown in Fig. 5.4 and a and b are constants.

Now the meridian is shaped by offset at throat which
nfluence the stress distribution in the shell, buckling load
nd natural frequency. Then the equation becomes

$$x = r_o + a\sqrt{1 + z^2/b^2} \qquad (5.2)$$

where r_o is the offset distance. With the limited
nformation available in this respect, here the equation of
the hyperbola without the offset distance (Eqn. 5.1) is
adopted.

In general the ratio of the throat height (h_t) to the
total height (H) in Fig. 5.4 varies from 0.7 to 0.85 and the
ratio of the total height (H) to the bottom diameter (D)
varies from 1.15 to 1.5. The minimum thickness at throat
level is 175 mm. It is assumed that the thickness varies
linearly from throat to bottom and top of the tower

5.5 MODELLING THE COOLING TOWER INTO S.D.O.F SYSTEM

The cooling tower modelled as S.D.O.F system, is a crude approximation. The assumptions made are

a. There exist a perfect fixity between the foundation and the superstructure of the tower.
b. The tower behaves as an elastic system.
c. The second order effect caused by axial shortening due to the self weight of the tower is negligible.
d. The vibration characteristic of the tower is similar to that of a deep cantilever beam having flexural and shearing deformations.
e. The effect of circumferential bending is negligible.

5.5.1 Determination of Spring Factor

The lateral dimension is such that the total deflection of the cooling tower consists of shearing and bending deflection.

The profile of the cooling tower is very complex. Though the curve in elevation is a hyperbola, the diameters at top and throat do not differ appreciably where as the diameter at bottom is about 1.6 times that at throat. To simplify the problem, tower is considered as a circular ring section as shown in Fig. 5.5.

The cross section area of the cooling tower

$$A = \pi D t \qquad (5.3)$$

where D - mean diameter

t - thickness of the cooling tower

The moment of inertia about x-axis and y-axis

$$I_x = I_y = \frac{\pi D^3 t}{8} \qquad (5.4)$$

5.5.1.1 Shearing Deflection Component

The cooling tower as a tapering cantilever shown in Fig. 5.5 with unit load acting at the top. As the deformation is purely shear the deflection[20]

$$\delta_s = \frac{f_s h}{G A} \qquad (5.5)$$

f_s - form factor is equal to 2 for annular section

G - Shearing modulus of concrete

A - Area of annular section

h - Height from the bottom

Integration between the limits 0 and H and substituting for A

$$\delta_s = \int_0^H \frac{2\, dx}{G \frac{\pi}{4} \left[\left\{ \left(\frac{D-a}{H}\right)x + d \right\}^2 - \left\{ \left(\frac{D-a}{H}\right)x + (d-2t) \right\}^2 \right]}$$

$$= \frac{2H}{\pi(D-a)G} \log\left(\frac{D-t}{a-t}\right) \qquad (5.6)$$

where

a - Top mean diameter

D - Bottom mean diameter

t - Mean thickness

d - Mean diameter at any section

H - height measured from the bottom

5.5.1.2 Bending Deflection Component

For finding the deflection due to bending the well known strain energy expression is used.

i.e. The deflection at any section xx

$$\delta_{xx} = \int \frac{M\, m\, dx}{E\, I} \qquad (5.8)$$

where

M - B.M at any section due to external load

m - B.M at any section due to unit load

I - Moment of inertia of the section

E - Modulus of elasticity

Substituting the values for M, m, E and I and integrating between the limits 0 and H we get

$$\delta_{xx} = \frac{x^2\,dx}{E\,\frac{\pi\,t\,D_x^3}{8}} \qquad (5.8)$$

Mean diameter at any section x-x

$$D_x = \left(\frac{D-a}{H}\right) x + a$$

On integration and simplification we get an expression for deflection due to bending

$$\delta_b = \frac{8\,H^3}{E\,\pi\,t\,(D-a)^3}\left[\log\,(D/a) + 2a\,(1/_D - 1/_a) - \frac{a^2}{2}\,(1/D^2 - 1/a^2)\right] \qquad (5.9)$$

The flexibility influence coefficient

$$\delta = \delta_s + \delta_b \qquad (5.10)$$

The spring factor $k = 1/\delta$ (5.11)

5.5.2 Computation of Mass Matrix

The lumped mass associated with each translational degree of freedom is obtained on pro rata basis. The mass at each station is

$$M = \frac{2\,\pi\,R\,t\,h\,\rho}{9.81} \qquad (5.12)$$

where

- R - internal radius of the shell
- t - the thickness of the shell
- h - the height of one discretised unit
- ρ - mass density of concrete

5.6 NATURAL PERIOD

The natural frequency p of SDOF is given by

$$p^2 = \frac{k}{m} \qquad (5.13)$$

The natural period $T = \dfrac{2\pi}{p}$ (5.14)

The period of the tower is influenced by the height of the tower, diameter at various levels, shell thickness etc. The results are given in Table 5.1.

This is a simplified method for finding the period of cooling tower in design office for preliminary design.

5.7 DETAILED ANALYSIS

Dynamic analysis include the formulation of mass matrix [m] and stiffness matrix [k]. Once these matrices are formulated there are many mathematical eigen value theories for obtaining the eigen pairs using a computer. Here

Jacobi's program was used in the extraction of eigen pairs. Then modal analysis was performed along similar lines described[6].

Using the results obtained from the computer a simple method is proposed to get the eigen pairs as accurately as possible.

5.7.1 Formulation of the Mass Matrix [m]

The mass acts along the various D.O.F. The mass will be a lumped matrix having non-zero diagonal elements.

5.7.2 Formulation of the Flexibility Matrix [δ]

Since the tower is considered as a linear structure, total deformation at any section may be obtained by superposing the shearing and bending deflection. This means

$$[\delta] = [\delta_b] + [\delta_s] \tag{5.15}$$

In Fig. 5.6 unit load is applied at the top. Using mechanics theory, the shearing deflection δ_s, the bending deflection δ_b and the rotation due to flexure θ_b can be found. The results are

$$\delta_s = \frac{2H}{\pi (D-a)G} \log \left(\frac{D-t}{a-t}\right) \tag{5.16}$$

$$\sigma_b = \frac{8 H^3}{E \Pi t (D-a)^3} [\log (D/a) + 2a(1/D - 1/a) - \frac{a^2}{2} (1/D^2 - 1/a^2)] \quad (5.17)$$

$$\theta_b = \frac{8 H^2}{E \Pi t (D-a)^2} [-(1/D - 1/a) + a/2 (1/D^2 - 1/a^2)] \quad (5.18)$$

The above equations are valid for $0 < a < D$

5.8 PROGRAMME OF STUDY

In this study, in all nineteen towers of varying heights were investigated. Convergence occured when the cooling tower was discretised into seven segments. For all the towers eigen pairs were found. Using the same, modal analysis was performed for 5% damping exactly the same procedure described in[6]. For performing modal analysis first three model were considered.

5.9 PROPOSED METHOD

The modal analysis described in the code is a semi automated procedure in which the first step is to obtain the eigen pairs. After getting the eigen pairs, modal analysis can be performed manually very easily. Therefore a simple method is proposed to get the eigen pairs as accurately as possible.

From the detailed study of frequencies and mode shapes of nineteen cooling towers, the following formulae for the periods of first three modes and the corresponding eigen vectors are obtained.

5.9.1 Period of the Dam

i) Fundamental period, $$T_1 = \frac{158.11}{E_c}\left[\frac{D}{374} - \beta^{8.8}\right] \qquad (5.19)$$

ii) Period of second mode, $T_2 = 0.342\ T_1$ (5.20)

iii) Period of third mode, $T_3 = 0.19\ T_1$ (5.21)

5.9.2 Eigen Vectors of Various Modes

The three mode shapes are shown in Fig. 3.8

i) Fundamental Mode

The expression for the amplitude at any station i is given as

$$a = \left(\frac{x}{H}\right)^{1.15} \qquad (5.22)$$

ii) Second Mode

For portion AB shown in Fig. 3.8

$$a = 0.9 \sin \frac{\pi x}{z} \quad \text{for} \quad 0 < x < z \qquad (5.23)$$

where $z = 0.75\ H$

For portion BC

$$a = -\frac{(y-z)}{(H-z)} \qquad (5.24)$$

ii) Third Mode

For portion AB shown in Fig. 3.8

$$a = 0.9 \sin \frac{2 \pi x}{z} \quad \text{for } 0 < x < z \qquad (5.25)$$

where $z = 0.9\ H$

For portion BC

$$a = \frac{(y-z)}{(H-z)} \qquad (5.26)$$

Using the above expressions, the natural periods and he corresponding modal amplitudes (See Fig. 5.6) may be btained for the first three modes by suitably discretizing he dam.

.10 RESULTS AND DISCUSSIONS

Using the proposed equations for period and modal mplitudes dynamic analysis was performed and compared with igen pairs obtained by computer solution are given in able 5.2, 5.3 and 5.4.

le 5.1 Prediction of eriod of Cooling Tower By SDOF Model

ght)	Bottom Diameter (m)	Top Diameter (average) (m)	Thickness (m)	Period in (sec)
1	174.783	105.494	0.7033	0.538
1	166.087	100.245	0.6867	0.507
1	157.391	94.997	0.6700	0.476
1	148.696	89.748	0.6533	0.445
1	140.000	84.499	0.6367	0.415
1	131.304	79.251	0.6200	0.385
1	122.609	74.003	0.6033	0.356
1	113.913	68.754	0.5867	0.327
1	105.217	63.506	0.5700	0.299
1	96.522	58.257	0.5533	0.271
1	87.826	53.009	0.5367	0.244
1	79.130	47.760	0.5200	0.217
1	70.435	42.511	0.5033	0.191
71	61.739	37.263	0.4867	0.166
1	53.043	32.016	0.4700	0.141
1	44.348	26.769	0.4533	0.116
1	35.652	21.519	0.4383	0.092
31	26.957	16.269	0.4250	0.069
21	18.261	7.277	0.4117	0.045

Table 5.2 COMPARISON OF PERIOD BY PROPOSED METHOD AND COMPUTER SOLUTION OF COOLING TOWER

Height (m)	Period (T_1) sec.		Period (T_2) sec.		Period (T_3) sec.	
	Computer Solution	Proposed Equation	Computer Solution	Proposed Equation	Computer Solution	Proposed Equation
201	0.529	0.526	0.181	0.180	0.101	0.099
191	0.499	0.499	0.170	0.171	0.095	0.095
181	0.469	0.446	0.160	0.162	0.089	0.090
171	0.440	0.419	0.150	0.152	0.083	0.085
161	0.411	0.392	0.140	0.143	0.077	0.079
151	0.383	0.392	0.130	0.134	0.072	0.075
141	0.355	0.365	0.120	0.125	0.066	0.069
131	0.327	0.339	0.110	0.116	0.061	0.064
121	0.300	0.312	0.101	0.107	0.055	0.059
111	0.273	0.285	0.091	0.098	0.050	0.054
101	0.246	0.258	0.082	0.088	0.045	0.049
91	0.220	0.232	0.073	0.079	0.040	0.044
81	0.194	0.205	0.064	0.070	0.035	0.039
71	0.169	0.178	0.056	0.061	0.030	0.034
61	0.144	0.151	0.047	0.052	0.026	0.029
51	0.119	0.125	0.039	0.043	0.020	0.024
41	0.095	0.098	0.031	0.033	0.017	0.019
31	0.071	0.071	0.023	0.024	0.012	0.014
21	0.045	0.056	0.016	0.019	0.008	0.011

Table 5.3 COMPARISON OF EIGEN VECTORS BY PROPOSED METHOD AND COMPUTER SOLUTION OF COOLING TOWER

Sl. No.	Station Number From Top	Weight in kN	First Mode Shape		Second Mode Shape		Third Mode Shape	
			Computer Solution	Proposed Equation	Computer Solution	Proposed Equation	Computer Solution	Proposed Equation
1	1	165946.82	1.000	1.000	-1.000	-1.000	1.000	1.000
2	2	181011.61	0.8539	0.8252	-0.3803	-0.3846	-0.2723	-0.333
3	3	196076.41	0.6858	0.6551	0.2879	0.2154	-1.0823	-0.8934
4	4	211141.25	0.5094	0.4907	0.7659	0.6971	-0.6380	-0.5212
5	5	226206.03	0.3378	0.3333	0.9078	0.9000	0.4821	0.3965
6	6	241270.84	0.1824	0.1852	0.7010	0.7407	1.0521	0.9000
7	7	256335.60	0.0529	0.0524	0.2560	0.2850	0.5199	0.4605

Table 5.4 Comparison of Shear Values for 201 m Height Cooling Tower

tation No.	By Computer Solution Shear KN	By Proposed equation Shear KN
1	18110.95	18543.60
2	32313.34	32435.60
3	43367.54	43359.21
4	52611.70	52330.72
5	60286.81	59779.36
6	66237.38	65804.94
7	68685.06	68355.01

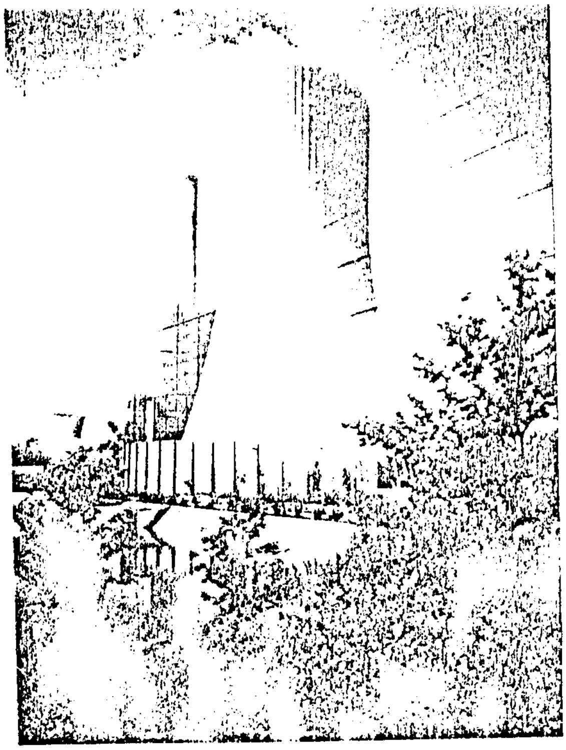


Fig. 5.1 Cooling Tower Herne IV, Germany

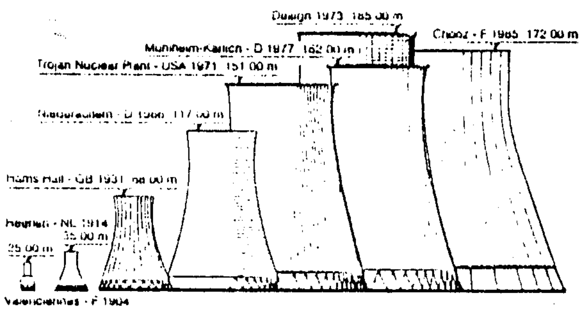


Fig. 5.2 Historic development of Natural Draught Cooling Towers

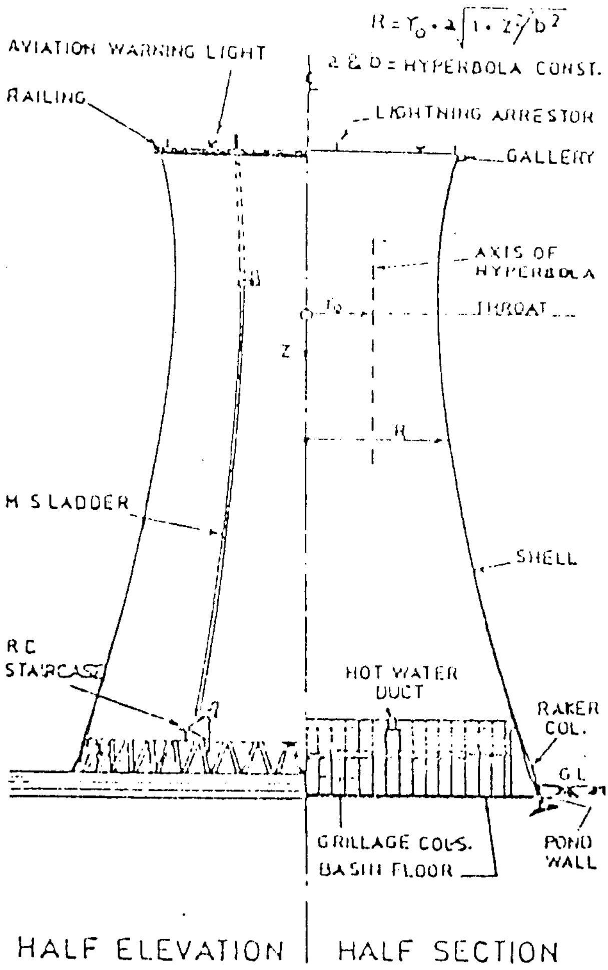


Fig. 5.3 Typical Cross Section of Natural Draught Cooling Tower

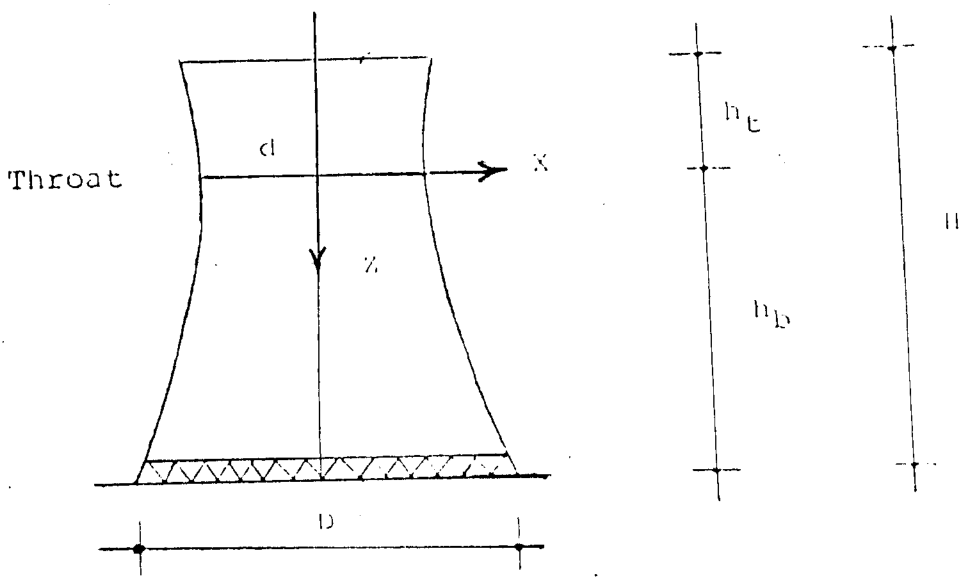


Fig. 5.4 Simplified Profile of Cooling Tower

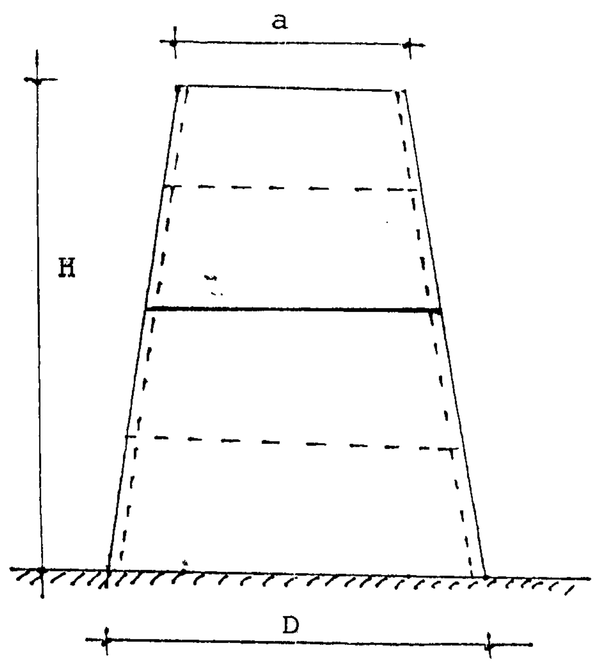


Fig. 5.5 Equivalent Circular Ring Section

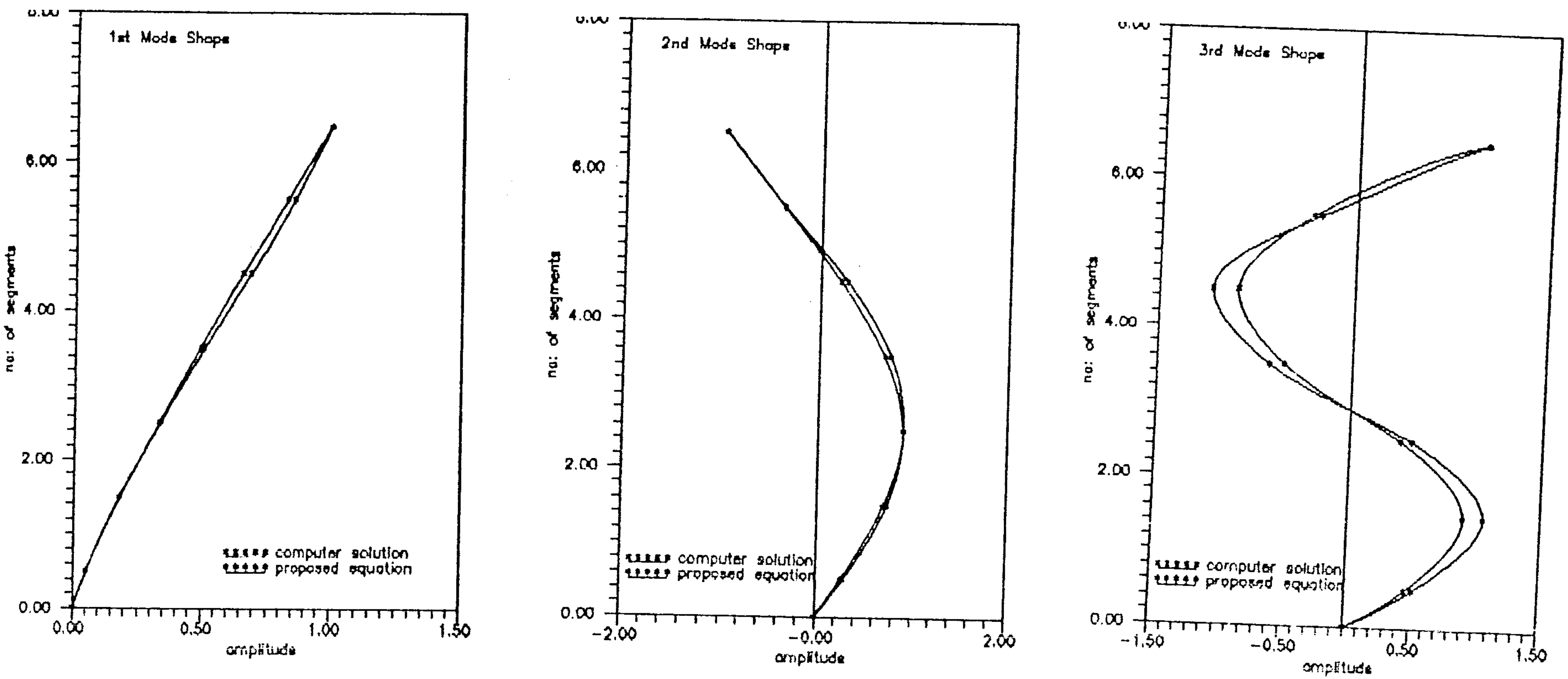


Fig. 5.6 Three Mode Shapes of Cooling Tower

CHAPTER 6

VIERENDEEL GIRDER

6.1 GENERAL

In 1896, a Belgian engineer, Professor Arthur Vierendeel, suggested a method of constructing an 'open' web girder with rigid joints, comprised of a top and a bottom chord with vertical members only between the booms. This type of girder is known as a Vierendeel Girder.

Unlike a truss with diagonal members which are generally designed for direct stresses only, the Vierendeel Girder members are subjected to bending, axial and shear stresses. Many uses can be made of this type of rigid frame, especially for bridges, as in Belgium, though in Great Britain Vierendeel Girders are commonly seen in church, school and industrial structures where clerestorey lighting is required and the absence of diagonal frame members is desirable.

Typical forms of Vierendeel Girders[3] are shown in Fig. 6.1.

6.2 RESEARCH SIGNIFICANCE

The Vierendeel Girder is a statically indeterminate structure. Here an approximate method of analysis is done to find the fundamental frequency and compared with the results obtained from the computer solution. An equation is proposed to find the fundamental frequency of the Vierendeel Girder.

All stable structures may be classified as determinate or indeterminate. This traditional terminology is unfortunate because it may imply that the latter category of structure cannot be analyzed. In fact, it means only that indeterminate structures require more than the equations of equilibrium for their analysis. Other conditions, such as strain compatability are required.

Statically indeterminate structures are those structures which cannot be analysed with the help of equations of static equilibrium alone. These structures are also known as hyperstatic structures. For the analysis of these structures it becomes necessary to consider the deformation of the structure because the equations of statics alone are not sufficient for the solution of the problem. In the case of statically indeterminate structures, the number of unknowns is greater than the number of independent equations derived from the conditions of static equilibrium. Additional equations, based on the compatability of deformations must be written inorder to obtain a sufficient number of equations for the determination of unknowns. The number of these additional equations necessary for the solution of the problem, is known as the degree of static indeterminancy or degree of redundancy of the structure.

For Vierendeel Girders the statical indeterminancy include external and internal indeterminancy. The external indeterminancy is reckoned just like beams using the three conditions of equilibrium, V = 0, M = 0 and H = 0. The internal indeterminancy is determined using the axium 3* number of closed loops. Therefore the total statical indeterminancy is the sum of external and internal indeterminancy.

6.3 APPROXIMATE METHOD OF ANALYSIS

The Vierendeel Girder shown in Fig. 6.2 is a statically indeterminate structure. The structure is externally determinate and the internal indeterminancy is 9. To make this a determinate one, internal hinges are provided at the mid point of all the members. Thus an additional equation, moment of all the forces around that hinge must be zero can be applied in addition to the three overall equations of equilibrium.

6.3.1 Computation of Deflection

Unit load is applied at the point and the reactions are determined. To satisfy the equilibrium condition at each joint the whole girder is analysed. Then the principal diagonal elements of the flexibility matrix is obtained by the strain energy equation $\int \frac{M\,m\,dx}{EI}$. By adding the deflection of all the members we get the total deflection when the unit load is applied at that station.

6.3.2 Calculation of Weight

By assuming suitable member dimension, span, spacing, slab thickness, live load and snow load, the total load per translational D.O.F. can be determined.

The steps for the determination of deflection and weight is exemplified by an illustrative example. See Appendix III.

6.3.3 Computation of Fundamental Frequency

Knowing the deflection and the mass the fundamental frequency is obtained using Dunkerley's equation.

This equation gives a lower bound solution to the fundamental frequency. The main concept in this method is that, for a stable system the frequencies corresponding to each degree of freedom are real, distinct and well apart spaced from each other. The significant advantage of

Dunkerley's equation is it requires only the principal flexibility element. The classical Dunkerley's equation is of the form

$$1/p^2 = \sum m_i \delta_{ii} \tag{6.1}$$

The frequency obtained by Dunkerley's equation is increased by a factor 1.14, since it gives only lower bound solution.

Based on the results obtained by the approximate method of analysis, a graph is plotted with number of bays along the x-axis and frequency along the y-axis (See Fig. 6.3). An equation is proposed to find the fundamental frequency of vierendeel girder.

$$P = \frac{14.48}{n_b^{1.24}} \tag{6.2}$$

where P - fundamental frequency

n_b - number of bays

Knowing the fundamental frequency, Period T is obtained by the relation

$$T = \frac{2\pi}{P} \tag{6.3}$$

6.4 DESCRIPTION OF COMPUTER PROGRAM

The program 'subspace iteration'[2] was used for finding the fundamental frequency. Structure stiffness and mass matrix is obtained by using the available program.

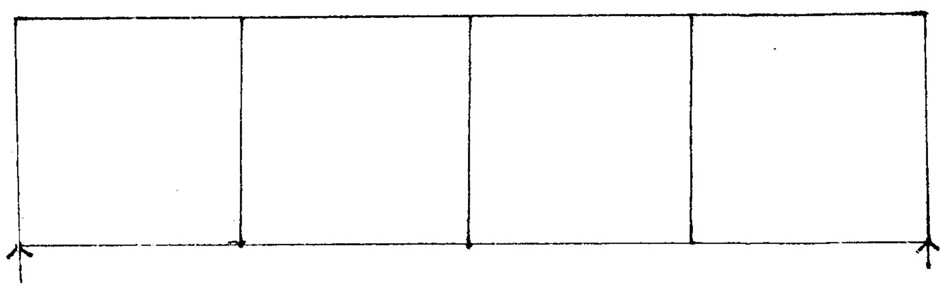
The data to be fed to the computer are number of nodes, degrees of freedom at each node, co-ordinates at each node, number of elements, dimensions of beam and column elements. Obtain the element stiffness matrix first then assembled to get the banded structural stiffness matrix. Mass matrix is calculated on pro rata basis. Using these stiffness matrix and mass matrix the smallest eigen values and corresponding eigen vectors in the generalized eigen problem are solved by the subspace iteration method. The lowest eigen value gives the fundamental frequency and hence the fundamental period.

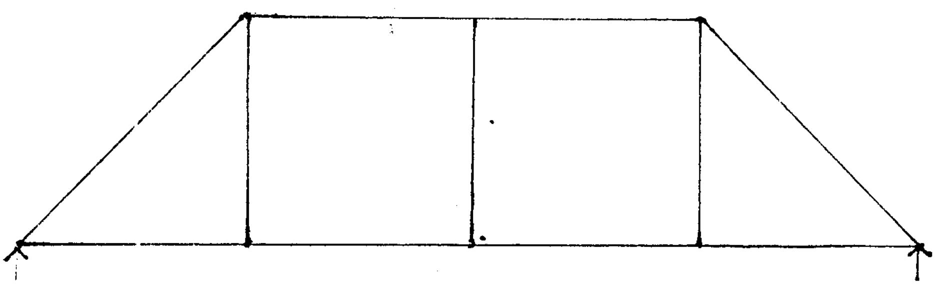
6.5 RESULTS AND DISCUSSIONS

The results obtained using the proposed equation and the computer solution is given in the Table 6.1. The fundamental mode shape is shown in Fig. 6.4. The period obtained using the computer solution is more, the reason is the axial deformation is also included.

Table 6.1 Comparison of Periods Obtained by Approximate Method, Proposed Equation and Computer Solution

No. of bays	Period from approximate method	Period from proposed equation	Computer Solution
2	0.160	0.168	0.321
3	0.288	0.277	0.345
4	0.416	0.394	0.499
5	0.497	0.520	0.658
6	0.675	0.652	0.820
7	0.750	0.736	0.984
8	0.935	0.930	1.151
9	1.195	1.288	1.320
10	1.639	1.538	1.492





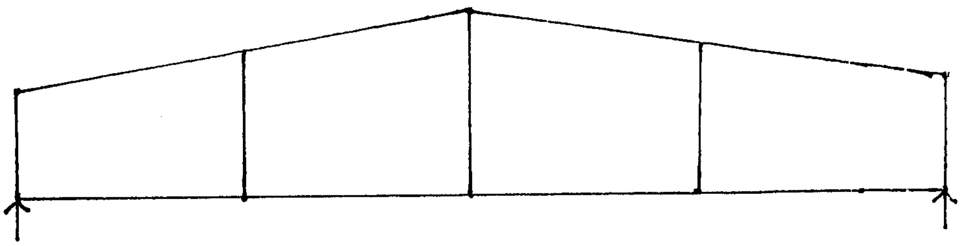


Fig. 6.1 Typical Forms of Vierendeel Girders

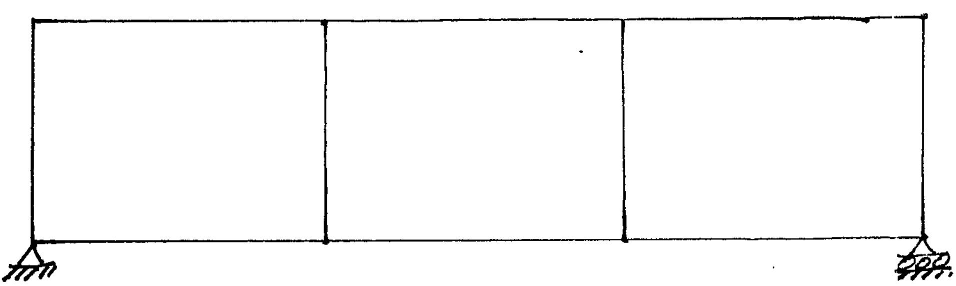


Fig. 6.2 vierendeel Girder with 3 - Bays

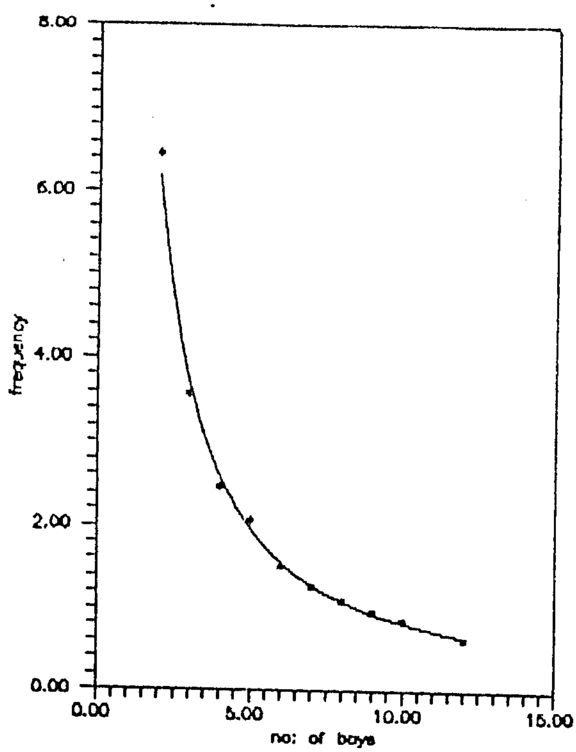


Fig. 6.3 Number of Bays Vs Freequency of Vierendeel Girder

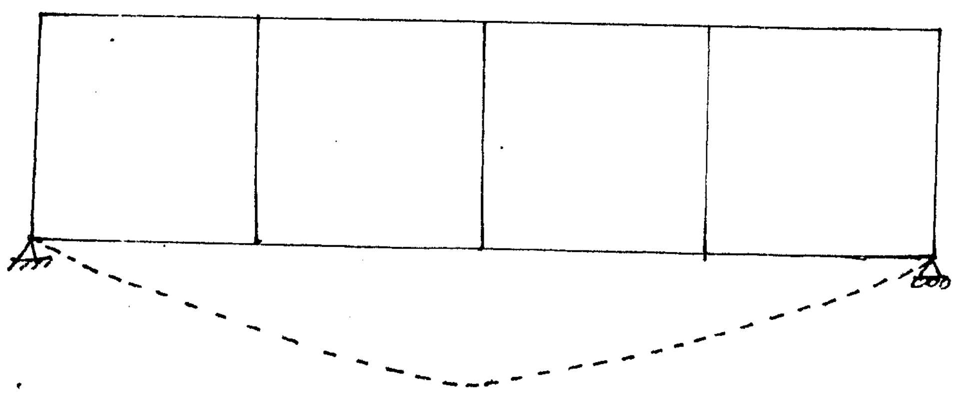


Fig. 6.4 Fundamental Mode Shape of a 4 - Bay Vierendeel Girder

CHAPTER 7

CONCLUSION AND SCOPE OF FURTHER STUDIES

7.1 CONCLUSION

Modal analysis of special structures like gravity dam, earthdam, cooling tower were studied and the following conclusions are made.

Natural period of vibration is a logical index used to obtain the base shear and base moment during preliminary design. In this thesis two approximate mathematical models are put forward for reckoning the natural period of gravity dam. The concepts involved in the two models are exceedingly simple and amply attest the accuracy of code equation.

The primary factor involved in the dynamic analysis is the determination of eigen pairs of the dam. Here a method is proposed based on the theory of mechanics of materials for establishing the stiffness matrix. Whatever be the type of analysis employed to formulate the stiffness matrix, use of a computer is inevitable for finding the eigen pairs. A simple hand computation procedure is put forward to estimate the periods of first three modes and the corresponding eigen vectors for gravity dam and earth dam. The prediction of the proposed formulae is in fair agreement with the computer solution. Eventhough the method is approximate, the modal nalysis solution is reasonably good compared with exact nalysis.

The design of cooling tower requires the fundamental period for earthquake design. Here a method is proposed based on the elementary principles of strength of materials and structural dynamics. Dynamic analysis is done, using these results a hand computation procedure is proposed for the determination of eigen pairs. The results obtained are in good agreement with the computer solution.

For Vierendeel Girder an equation is proposed to find the fundamental frequency. Approximate method of analysis is done and based on the results obtained by this method an equation is proposed to find its natural period. The results obtained by subspace iteration method and by proposed equation are compared.

7.2 SCOPE OF FURTHER STUDIES

For gravity dam and earth dam, dynamic behaviour of dam - reservoir system can be studied by treating the system as a coupled one. For cooling tower finite element analysis can be done and for Vierendeel Girder rigorous dynamic analysis can be carried out.

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APPENDIX I

The following data are available for a gravity dam

Total height of the dam H	=	310 m
Top width of the dam t	=	10 m
Bottom width b	=	220 m
Width at mid height a	=	115 m
Mid height of the dam h	=	155 m
Poisson's ratio μ	=	0.15
Young's modulus of concrete	=	2.5×10^7 KN/m^2
E	=	$2G(1+\mu)$
Shear modulus, G	=	$\frac{E}{2.3}$
Acceleration due to gravity g	=	9.81 m/sec^2
Unit weight of concrete	=	24 KN/m^3

Solution

Substituting a,b and h in Eqn. 3.10 the shear contribution due to unit load is given by

$$= 1.1491/G$$

Substituting a,b and h in Eqn. 3.15 the flexural contribution becomes

$$= 0.9650/G$$

Therefore, flexibility influence coefficient

$$= \quad = 2.1141/G$$

Mass of the dam $m = \dfrac{115 \times 310 \times 24 \times 1}{9.81}$

$$= 87217 \text{ KN-sec}^2/\text{m}$$

Now $p^2 = \dfrac{1}{m} = \dfrac{2.5 \times 10^7}{2.3 \times 87217 \times 2.1141}$

$$= 58.95$$

$$p = 7.68 \text{ rad/sec.}$$

Natural period $T = \dfrac{2\pi}{p} = \dfrac{2\pi}{7.68}$

$$= 0.818 \text{ sec.}$$

Using code procedure $T = \dfrac{5.55 H^2}{B} \quad \dfrac{W_m}{g E_s}$

$$= 0.758 \text{ sec.}$$

FOR TDOF MODEL

Solution

First the bottom segment 1 is considered, for this segment

$a = 167.5$ m

$b = 220.0$ m

$h = 77.5$ m

Substituting the above in Eqn. 3.10 gives the shearing contribution

$$\delta'_{11} = 0.4825/G$$

Substituting in Eqn. 3.15 gives the flexural contribution

$$\delta''_{11} = 0.0906/G$$

Therefore the principal flexibility influence coefficient $\delta_{11} = \delta'_{11} + \delta''_{11}$

$$= \left[\frac{0.4825}{G} + \frac{0.0906}{G}\right] = \frac{0.5731}{G}$$

Considering the segment 2 in the top, the details are as follows :

$$a = 62.5 \text{ m}$$

$$b = 220 \text{ m}$$

$$h = 232.5 \text{ m}$$

Substituting the above in Eqn. 3.10 and Eqn. 3.15 respectively

$$\delta'_{22} = 2.229/G$$
$$\delta''_{22} = 4.805/G$$

The flexibility influence coefficient

$$\delta_{22} = \delta'_{22} + \delta''_{22}$$

$$= \left[\frac{2.229}{G} + \frac{4.805}{G}\right] = \frac{7.034}{G}$$

Mass of the segment 1, m_1 $= \dfrac{24 \times 167.5 \times 155}{9.81}$

$= 63516.82$ KN sec^2/m

Mass of segment 2, m_2 $= \dfrac{24 \times 62.5 \times 155}{9.81}$

$= 23700$ KN sec^2/m

The Dunkerley's eqn. is

$$\frac{1}{p^2} = [m_1 \delta_{11} + m_2 \delta_{22}]$$

$$p^2 = 53.52$$

$$p = 7.315 \text{ rad/sec}$$

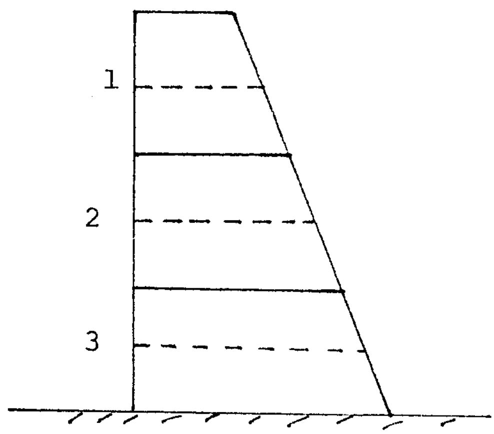
The natural period T $= \dfrac{2\pi}{7.315} = 0.858$ sec

The corrected period T $= \dfrac{0.858}{1.14} = 0.753$ sec

APPENDIX II

Data same as given in Appendix I

The cross section of a dam discretized into three egments is shown below. There are three stations (1) (2) nd (3) shown ringed on the sight side where the ranslational degrees of freedom x_1, x_2 and x_3 are located.



) Formulation of [δ_s] matrix

The station No. 3 is considered first

$h = 51.666$ m

$a = 185$ m

$b = 220$ m

$G = 1.0869 \times 10^7$ KN/m^2

$$E = 2.5 \times 10^{7} \ KN/m^{2}$$

$$= 0.15$$

$$\delta_{33} = \frac{1.2 \ h}{G \ (b-a)} \ \ln \ [b/a]$$

substituting the above data in Eqn. 3.23

$$\delta_{33} = 2.824 \times 10^{-8} \ m$$

From the knowledge of shearing deflection characteristic, it follows that

$$\delta_{33} = \delta_{23} = \delta_{13}$$

From reciprocal theorem

$$\delta_{23} = \delta_{32}$$

$$\delta_{13} = \delta_{31}$$

In the same way δ_{22} and δ_{11} can be determined

$$\delta_{22} = \delta_{12} = \delta_{21}$$

The matrix $[\delta_s]$ becomes

$$[\delta_s] = 10^{-7} \begin{bmatrix} 2.586 & 1.057 & 0.282 \\ 1.057 & 1.057 & 0.282 \\ 0.282 & 0.282 & 0.282 \end{bmatrix}$$

b) **Formulation of [δ_f] becomes**

The leading elements δ_{11}, δ_{22} and δ_{33} are found by substituting relevant value for a, h and G besides b in Eqn. 3.24.

We get

$$\delta_{11} = 7.336 \times 10^{-7} \text{ m}$$

$$\delta_{22} = 8.883 \times 10^{-8} \text{ m}$$

$$\delta_{33} = 2.356 \times 10^{-9} \text{ m}$$

Using the strength of materials theory, the off-diagonal elements can be found using the concerned rotation O_f.

For example, δ_{13} is found as

$$\delta_{13} = \delta_{33} + 2 \text{ S } \Phi_f$$

Φ_f - slope eat station 3 when unit load is applied at station 3

δ_S - 103.333 m

By substituting

h = 51.666 m

a = 185 m

b = 220 m

E = 2.5×10^7 KN/m^2

in Eqn. 3.25

$$\theta_f = 7.155 \times 10^{-11} \text{ radian}$$

$$\delta_{13} = 2.356 \times 10^{-9} + 7.155 \times 10^{-11} \times 2 \times 103.333$$

$$= 1.714 \times 10^{-8} \text{ m} = \delta_{31}$$

In this way all the elements can be computed

$$[\delta_f] = 10^{-7} \begin{bmatrix} 7.336 & 1.959 & 0.174 \\ 1.959 & 0.888 & 0.0975 \\ 0.174 & 0.0975 & 0.0236 \end{bmatrix} \text{ metres}$$

By adding $[\delta_s]$ and $[\delta_f]$ matrices

$$[\delta] = 10^{-7} \begin{bmatrix} 9.923 & 3.016 & 0.454 \\ 3.016 & 1.945 & 0.380 \\ 0.454 & 0.380 & 0.306 \end{bmatrix} \text{ metres}$$

The above matrix can be inverted to get the lateral stiffness matrix [k]

c) Formulation of Mass Matrix

The mass corresponding to degree of freedom 1

$$m_1 = 1/2 \times 103.33 \times [10 + 80] \times \frac{24 \times 1}{9.81}$$

$$= 11376.15 \text{ KN sec}^2/\text{m}$$

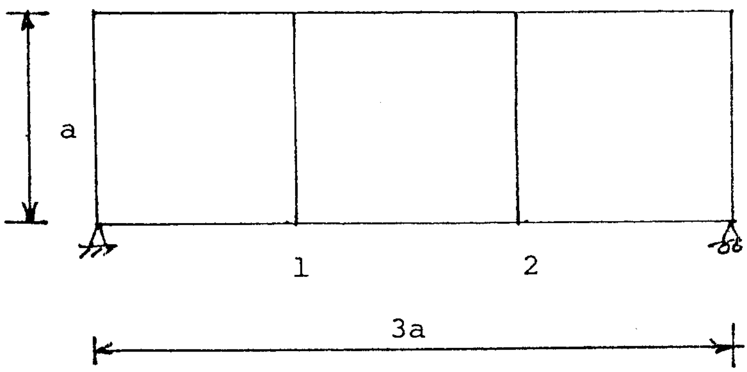
Similarly m_2 and m_3 are found

$$\text{The mass matrix } [m] = \begin{bmatrix} 11376.15 & 0 & 0 \\ 0 & 29072.38 & 0 \\ 0 & 0 & 46768.60 \end{bmatrix}$$

Using the materal stiffness matrix [k] and the mass matrix [m] in Jacobi's program, the eigen pairs are as follows.

Eigen values p^2	Eigen vectors
67.4	1.000
	0.3874
	0.0704
383.3	-1.000
	0.893
	0.399
1073.6	1.000
	-2.003
	3.396

APPENDIX III



3 - bay Vierendeel Girder

Calculation of Deflection (3-Bay)

Unit load is applied at station (1) as shown in Fig. the girder is analysed the deflection

$$\delta_{11} = \int \frac{M\ m\ dx}{E\ I}$$

$$= 8 \int_0^{a/2} \frac{(1/3x)^2\ dx}{E\ I} + 12 \int_0^{a/2} \frac{(1/6x)^2\ dx}{E\ I}$$

$$= \frac{a^3}{19.64\ E\ I}$$

Similarly unit load is applied at Station (2) the deflection

$$\delta_{22} = \frac{a^3}{19.64\ E\ I}$$

Calculation of Weight

Assumed Data

The spacing of the girder	=	4 m
Thickness of the slab	=	9 cm
dimension	=	12 x 12 cm
Dimension of bay	=	2 x 2 m
Live load	=	2.5 KN/m^2
Snow load	=	0.1 KN/m^2

Solution

1. Weight of the girder = 10 x 0.12 x 0.12 x 2 x 24
   = 7 KN

2. Weight of slab/girder = 24 x 0.09 x 4 x 6
   = 51.84 KN say 52 KN

3. Live load for earthquake design is 25% of L.L
   = 2.5 x 6 x 4 x 0.25
   = 15 KN

4. Snow load = 0.1 x 6 x 4 = 24 KN

   Total load = 76.4

   Load/translational D.O.F $= \frac{76.4}{3} = 25.5$ KN

   Mass $= \frac{25.5}{9.81} = 2.6\ KN\text{-}sec^2/m$