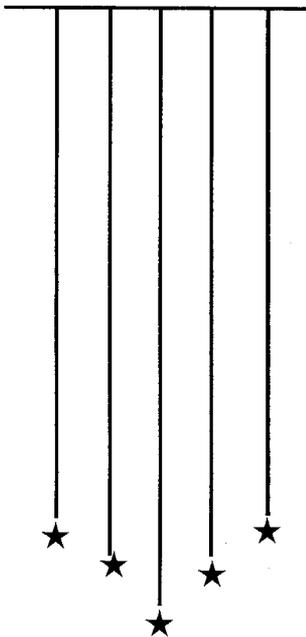
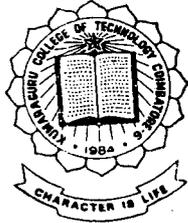


SATELLITE ATTITUDE ERROR AND RATE ESTIMATION USING KALMAN FILTER

PROJECT REPORT



2000 - 2001

P-1371

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IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE AWARD OF THE DEGREE OF
BACHELOR OF ENGINEERING IN
ELECTRONICS & COMMUNICATION ENGINEERING
OF BHARATHIAR UNIVERSITY

Department of Electronics & Communication Engineering
Kumaraguru College of Technology
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CERTIFICATE

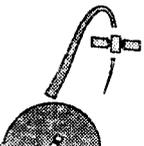
This is to certify that the dissertation entitled “**SATELLITE ATTITUDE ERROR AND RATE ESTIMATION USING KALMAN FILTER**” is the bonafide work carried out at ISRO Telemetry, Tracking and Command Network (ISTRAC), Bangalore by

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The essence of success is dedication to one's duty, but there are people who work behind the scene for realization and betterment of an endeavour undertaken. They do not reap the rewards of success. But surely without them, duty undertaken would be incomplete.

Our sincere gratitude goes to our principal Dr. K.K. Padmanabhan M.Sc. (Engg), Ph.D., for his support rendered throughout the project.

We extend our hearty gratitude to Prof .M. Ramasamy M.E., MISTE, MIEEE (USA), M.I.E., our head of the department for his invaluable guidance and constant encouragement throughout the project.

Our sincere thanks to our internal guide Mr. P.M Senthilkumar M.E., MISTE, for his valuable guidance, innovative ideas and helpful criticism throughout the project.

We thank our external guide Mr.M.Pitchaimani, Satellite Operations Director, IRS 1-B and IRS 1-C Satellites, ISTRAC, for his continuous encouragement and timely guidance. His reassuring personality always enlivened us whenever we got stuck up during the progress of our project.

We thank Mr. N. Lakshmi Narayana, HRD officer, ISTRAC, who permitted us to do the project in this prestigious organization.

The help and cooperation from all the staff members, non-teaching staff of the Department of Electronics and Communication Engineering and the students are gratefully acknowledged.

Remote sensing is one of the important applications of artificial satellites. Current remote sensing satellites are designed to have high resolution and low error. A periodic evaluation of performance is a necessity to understand the degradation setting in the elements of the control loop of the satellite. In this project two techniques namely Spectral Analysis and Kalman Filter are used to find the deviation angles and rates of the camera in the control loop.

In practical environment, a low pass filter's cut-off frequency for filtering noise is not clearly known but the noise level in the observed signal is known. In this project an effort is made to design a filter using FFT techniques, in which noise level is taken as input and this is converted into an equivalent magnitude in the spectrum for filtering.

Kalman Filter is selected because it can be used for online applications. Filtering is done assuming the system is linear and dynamic. By knowing the initial state, state noise variance and measurement noise covariance, the filter equations are recursively applied to estimate the state model using the initial conditions and current measurements. Since lot of assumptions are dumped in the filter equations, the parameters namely the covariance of the error and state noise matrix are properly tuned to avoid divergence.

The project is realized in MATLAB environment and the project has been successfully implemented at ISTRAC, Bangalore.

Acknowledgement

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Introduction

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- *Motivation and Objective*

1.1 Organization History

Under the Ministry of Science and Technology, Government of India, Department of space (DOS) is formed. Under DOS, the Indian space research organization was set up in 1972 for fulfilling the dreams of Indian space program. Other organizations under the control of DOS are ADRIN, DRDL, MCF, LPSC, NRSA and PRL.

The DOS / ISRO headquartered at Bangalore providing overall direction to the technical, Scientific and Administrative functions of below eight units. The functions of these units are as follows

1. VSSC – Vikram Sarabai Space Center, Tiruvananthapuram, engaged in design and development of satellite launch vehicle technology.
2. SHAR - Sriharikota High Altitude Range. Sriharikota is the main operational base of ISRO equipped with sophisticated launching pad facilities.
3. ISAC - ISRO Satellite Center, Bangalore, engaged in design and development of payloads.
4. SAC - Space Application center, Ahmedabad, is ISRO's main applications R&D Center with activities in telecommunications and developing of payloads for Space Crafts.

5. LPSC - Liquid Propulsion System Center, Mahendragiri, engaged in launch vehicle propulsion systems and engine testing.
6. ISTRAC - ISRO Telemetry Tracking and Command network, Bangalore is responsible for establishing operating ground stations for all Space Crafts.
7. NRSA - National Remote Sensing Agency, Hyderabad, engaged in developing and utilizing modern remote sensing techniques and providing training and other operational supports to various end users.
8. DECU - Development and Educational Communication Unit, Ahmedabad, works in cooperation with Doordarshan / I&B Ministry, is involved in research and for providing technical services for educational TV programs with special emphasis in INSAT facilities.

The DOS has setup the National Natural Resources Management Systems to build complete infrastructure for providing operational support service on a national basis to various end users, concerned with natural resources management and augmentation.

1.2 ISTRAC

Commands are necessary to operate satellites. In order to transmit proper commands, information about the satellite's location and condition are needed. Telemetry, Tracking and command provide the means of monitoring and controlling satellite operation. These three functions are

usually integrated into a single subsystem & separated from main communications.

Telemetry is the means by which measurements made at a distance are transmitted to an observer. Tracking is observing and collecting data to plot the moving path of an object. Command is the means by which control is established and maintained. A command originated at Satellite Control Center is sent via landlines to an earth station, and is transmitted to the satellite. Commands are used to control the satellite's onboard equipment status, Operational modes, position and orientation. Discrete commands of fixed pulse duration are used to change the status of the equipment, such as turning it ON/OFF.

1.3 Project Requirements

Artificial satellites of the earth have many potential applications. One of the applications is the remote sensing of the earth and space. LANDSAT of USA, SPOT of France, ADEOS of Japan, RADSAT of Canada, ERS of space agency and IRS of India are some of the remote sensing satellites. Most of the remote sensing satellites is placed in the sun-synchronous orbit, which regresses (drifts), from west to east at the rate of 1 deg /day along with the sun. Advantage of such an orbit is that the illumination condition of

a place at particular latitude is always constant which is a major requirement for image analysis.

Every remote sensing satellite has two major portions with respect to its hardware structure. They are payload and mainframe. Payload is the scientific instrument portion of the satellite. Depending on the purpose, the scientific instrument or a payload may be a Charged – coupled device or microwave radar for earth imaging and an X – ray or a gamma ray monitor for space exploration. Mainframe is the major portion of the satellite body, which comprises 70 % of its weight and volume. It contains many subsystems, which are called as power, telemetry, telecommand, reaction control, thermal control, sensors and attitude control systems. Cameras of current remote sensing satellites are designed to give high resolution and high clarity images.

1.4 Motivation and Objective

The satellite control center at ISTRAC presently controls about 5 Remote Sensing satellites. There are about 1000 parameters monitored in each satellite to assess the health of the satellite over a 32 sec period. Of the 1000 parameters, about 400 parameters show the status of some system/modules in terms of ON/OFF, selected/ non-selected as response. About 600 parameters show numerical numbers, some sampled at 2 sec, 4

sec, 8 sec, 16 sec and 32 sec intervals depending on the criticality. There are about 100 parameters, which form the part of the control loop and they are monitored from sensors, Control algorithms and actuators.

Many times it is required to know the performance of the control loop in terms of pointing accuracies. The pointing accuracies are conventionally defined as the camera deviation angles θ degree and its deviation rate in degree /sec. The corresponding design natures are 0.05 deg and 3×10^{-4} deg/sec. There are always requirements to verify this as part of periodical performance evaluation. This is done mainly to understand the degradation setting in the elements of control loops namely sensors, controllers and actuators. This requires analysis of sensor measurements, which are the inputs to the control loop.

Although various methods are available at Satellite Control Center (SCC), they are all written in FORTRAN and are available in mainframe systems. So, it was felt to redo the work using

1. Kalman Filtering Technique
2. PC based MATLAB

This project deals with two of the three attitude errors, namely, Roll error and pitch error.

Kalman filter technique was selected because of its potential to handle multiple channels simultaneously (for filtering) and also to estimate some of the related parameters (augmented states) at the same time. In this project, six channels of data are considered. They are

- ES1 Roll error
- ES1 Pitch error
- ES2 Roll error
- ES2 Pitch error
- ES Roll normal error = $\frac{\text{ES1 Roll error} + \text{ES2 Roll error}}{2}$
- ES Pitch normal error = $\frac{\text{ES1 Pitch error} + \text{ES2 Pitch error}}{2}$

All these channels carry the camera pointing information in the noisy environment. It is designed to noise filter the 6 channels of data and also compute their rate of variation simultaneously. The Kalman filter described for their project will consider all the 6 channels together for noise filtering and estimate their rate at the same time which is very difficult to realize in classical Digital filter methods. For the purpose of showing the difficulties, the same problem is dealt with frequency domain technique also.

Simulation Studies

- *Basic Definitions*
- *Terminologies of Signal Processing*
- *Sampling Process*
- *Spectrum Representation*
- *Case Study*
- *Noise Generation*
- *Noise Addition*
- *Noise Filtering Algorithm*

2.1 Basic Definitions

Signal

A signal is defined as any variable that carries or contains some kind of information that can be conveyed, displayed or manipulated.

Examples of Signal are: 1. Speech, Sound and Music Signals.

2. Biomedical signal such as ECG, EEG, etc.

3. Video, Image and RADAR signals.

System

A system is defined as a physical device that performs an operation on a signal to produce the required output.

Examples of System are: 1. Filter to remove noise & interference.

2. Quantizer to convert discrete to digital signal.

Signal Processing

Signal processing is defined as any operation that is done on a signal in order to improve or retrieve the information content from the signal.

2.2 Terminologies of Signal Processing

Continuous Signal Frequency

The frequency of a continuous signal is defined as the number of repetitions that the signal experiences for a unit time. It is denoted by F .

The angular frequency Ω is expressed in radians / second.

$$\text{i.e. } \Omega = 2\pi F$$

$$\text{The range of } \Omega \text{ is } -\infty < \Omega < \infty$$

$$\text{The range of } F \text{ is } -\infty < F < \infty$$

Discrete Signal Frequency

The frequency of a discrete signal is defined as number of cycles that is present in a sample. It is denoted by f . The angular frequency ω for discrete signal is expressed in radians / sample [1].

$$\text{i.e. } \omega = 2\pi f$$

$$\text{The range of } \omega \text{ is } -\pi < \omega < \pi$$

$$\text{The range of } f \text{ is } -1/2 < f < 1/2$$

Notations

$x(t)$ → Continuous time signal

$x(n)$ → discrete time signal

$X(\omega)$ → discrete time signal's continuous frequency spectrum

$X(k)$ → discrete time signal's discrete frequency spectrum

Ω → Angular frequency of the continuous time signal

ω → Angular frequency of the discrete time signal

f → frequency of the discrete time signal

F → frequency of the continuous time signal

f_s → sampling frequency

N → Number of samples

Sinusoidal Signal

One of the most important and basic signal that is used in DSP for the purpose of testing a system is the sinusoidal signal which is characterized by the presence of single frequency.

The most general mathematical formula for a sinusoidal time signal is given by

$$x(t) = A \cos (\omega_0 t + \varphi) = A \cos (2\pi f_0 t + \varphi) , \text{ where}$$

A is the amplitude, the scaling factor that determines how large the cosine signal will be.

φ is the phase shift, which is expressed in radians.

f_0 is the frequency which represents the number of repetitions that the signal experiences for a unit second [2].

2.3 Sampling Process

Most of the signals practically encountered are continuous. Due to the advantages of discrete signal processing over analog signal processing, the analog signal is converted to discrete signal. The process of converting

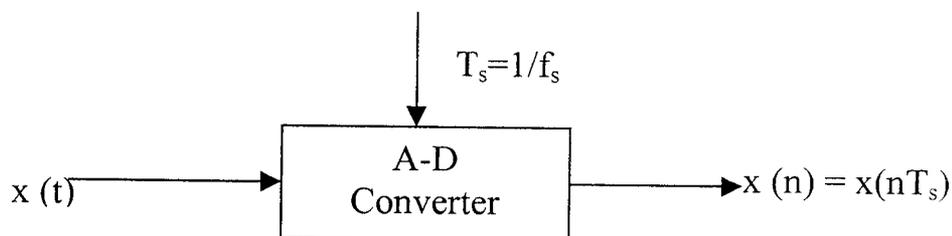
a continuous time signal to a discrete time signal by selecting the values of the continuous signal at discrete instants is called sampling [1].

Let $x(t)$ be a continuous signal. The discrete version can be obtained by sampling the continuous time signal at frequency f_s .

The discrete time signal is given by

$$x(n) = x(nT_s), \quad -\infty < n < \infty$$

The above process can be diagrammatically shown as follows



2.3.1 Selection of Sampling frequency

Shannon and Nyquist theorem

The selection of sampling frequency is given by Shannon and Nyquist where by the original continuous time signal can be reconstructed from the discrete version if and only if, the sampling frequency is greater than or equal to twice the maximum frequency that is present in the continuous time signal.

This can be mathematically given by $f_s \geq 2 * f_{\max}$.

If the sampling frequency is selected such that it does not satisfy the Nyquist criterion then the problem of aliasing and folding arises. There are three possible ranges in which sampling frequency lies. They are

Case 1: $0 \leq f_s \leq f_0$

If f_s lie between the above ranges, then the problem of aliasing occurs. Consider two signals with same amplitude and same phase. The frequencies of the signals are related by $f_1 = f_0 + lf_s$, where l is an integer. When the signals are sampled at a frequency f_s then aliasing occurs.

$$x_1(n) = A \cos(2\pi f_0 n T_s + \varphi)$$

$$x_2(n) = A \cos(2\pi f_1 n T_s + \varphi)$$

$$= A \cos[2\pi(f_0 + lf_s)nT_s + \varphi]$$

$$= A \cos[2\pi f_0 n T_s + 2\pi lf_s n T_s + \varphi]$$

$$= A \cos[2\pi f_0 n T_s + \varphi]$$

$$= x_1(n)$$

The above phenomenon by which a continuous time sinusoidal signal of higher frequency acquires the identity of a sinusoidal sequence of lower frequency by the process of sampling is called aliasing.

Case 2: $f_0 \leq f_s < 2f_0$

If f_s lie between the above ranges, then the problem of folding occurs. Consider two signals with same amplitude and same phase. The frequencies of the signal are related by $f_1 = -f_0 + lf_s$, where l is an integer.

$$x_1(n) = A \cos(2\pi f_0 n T_s + \phi)$$

$$x_2(n) = A \cos[2\pi (f_0 + lf_s) n T_s + \phi]$$

$$= A \cos[2\pi (f_0 + lf_s) n T_s + \phi]$$

$$= A \cos[-2\pi f_0 n T_s + 2\pi l f_s n T_s + \phi]$$

$$= A \cos[2\pi f_0 n T_s + \phi]$$

$$= x_1(n)$$

As the name 'folding' indicates the frequencies f_0 and $f_s - f_0$ are mirror images with respect to $f_s/2$.

Case 3: $F_s \geq 2f_0$

If the sampling frequency satisfies the above Nyquist criterion then the continuous time signal can be reconstructed from the discrete time signal. The maximum frequency that can be retrieved is $f_s/2$ [2].

2.3.2 Periodic Signal and Periodicity after Sampling

A signal is said to be periodic with period N if there exists a value N such that

$$x(n+N) = x(n), \quad 0 \leq n < N$$

Let a continuous signal $x(t)$ be periodic and this is sampled at a frequency $f_s = 1/T_s$. A condition is derived so that the discrete time signal obtained after sampling is also periodic.

$$x(n) = x(nT_s) = x(nT_s + T_0) \quad \text{----- (2.1)}$$

$$x(n) = x(n+N) = x(nT_s + NT_s) \quad \text{----- (2.2)}$$

From eqn 2.1 and 2.2 we get,

$$T_0 = NT_s \quad \text{----- (2.3)}$$

For multiple periods, equation 2.3 becomes

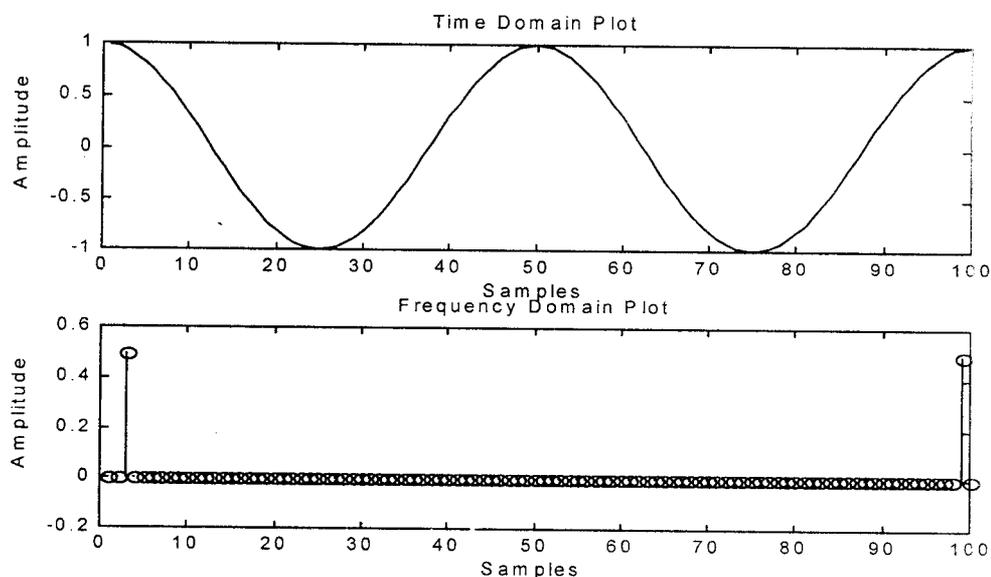
$$MT_0 = NT_s \quad \text{----- (2.4)}$$

$$T_0/T_s = N/M \quad \text{----- (2.5)}$$

This is the condition for the discrete signal to be periodic after sampling [4].

2.4 Spectrum Representation

Till now the representation followed is the variation of the signal with respect to time. Another form of representation is the variation of signal amplitude with respect to frequency. This form of representation is called as spectrum.



2.4.1 Harmonics

A harmonic is defined as a frequency component, which is an integral multiple of the fundamental frequency (f_0).

i.e. $f_k = kf_0$ where f_k is the harmonic of f_0

Consider a signal $x(n)$ having a spectrum $X(\omega)$. Let the spectrum be sampled so that the spectrum becomes discrete.

$$X(\omega) = X\left(\frac{2\pi k}{N}\right)$$

where N is the number of samples and k is an integer from 0 to $N-1$.

This expression comes out from the fact that the angular frequency range 2π is equally split by N samples with each sample instant k [4].

2.4.2 Fourier series and Phasor

Any signal can be represented by linear combination of complex exponentials that are harmonically related. This kind of representation is called the Fourier series representation [1].

The complex exponential can be written as

$$e^{j\omega t} = \cos(\omega t) + j \sin(\omega t)$$

This implies that any signal can be expressed by a linear combination of sines and cosines. This is the importance of sinusoidal signals in the field of signal processing. This Fourier series can be mathematically expressed as follows.

$$x(n) = \sum_{k=0}^{N-1} C_k * e^{j2\pi kn/N}$$

The term C_k represents the magnitude of the signal at the k^{th} harmonic in the spectrum and C_k is called as the phasor of the Fourier series and this represents the Y-axis of the spectrum with the X-axis being frequency.

Now any periodic signal can be expressed in terms of complex exponentials (or) sinusoidal signals and with a proportionate value of C_k each and every signal can be generated uniquely with the help of these complex exponentials (or) sinusoidal signals and this is the sole reason that the sinusoidal signals are used as test signals.

The above-mentioned Fourier series can be represented in different forms. They are

(i) Exponential form $x(n) = \sum_{k=0}^{N-1} C_k * e^{j2\pi kn/N}$ ----- (2.6)

(ii) Phase form
$$x(n) = \sum_{k=0}^{N-1} A_k \cos(2\pi kn/N + \varphi_k) \quad \text{-----}(2.7)$$

(iii) Sine & Cosine form
$$x(n) = \sum_{k=0}^{N-1} a_k \cos(2\pi kn/N) + b_k \sin(2\pi kn/N) \quad (2.8)$$

2.4.3 Relation between Different Forms

1. Phase form and exponential form

$$\text{Real } C_k = (A_k \cos \varphi_k) / 2$$

$$\text{Imag } C_k = (A_k \sin \varphi_k) / 2$$

2. Phase form and Sine, Cosine form

$$a_k = A_k \cos \varphi_k$$

$$b_k = A_k \sin \varphi_k$$

3. Sine, Cosine form and exponential form

$$a_k = 2 * \text{Real } (C_k)$$

$$b_k = 2 * \text{Imag } (C_k)$$

4. Sine, Cosine form and phase form

$$A_k = \sqrt{a_k^2 + b_k^2}$$

$$a_k = A_k * \cos \varphi_k$$

$$b_k = A_k * \sin \varphi_k$$

2.4.4 Relationship between X (k) and Signal magnitude

Let the sampled spectrum be denoted by X (k) and let x (n) represent the signal [1].

By Fourier series we have

$$x(n) = \sum_{k=0}^{N-1} C_k * e^{j2\pi kn/N} \text{----- (2.9)}$$

By the formula for discrete Fourier transform we have

$$x(n) = 1/N \sum_{k=0}^{N-1} C_k * e^{j2\pi kn/N} \text{----- (2.10)}$$

From equations (2.9) and (2.10),

$$C_k = (1/N) * X(2\pi * k/N) \text{----- (2.11)}$$

If the signal magnitude is represented in sine and cosine form of Fourier series then the relations are

$$a_0 = \frac{X(0)}{N}$$

$$a_k = \frac{2 * \text{real } X(k)}{N}$$

$$b_k = \frac{2 * \text{imag } X(k)}{N}$$

2.5 CASE STUDY

The case study is to analyze the properties, which are previously discussed, and the noise filtering characteristics that is followed in order to remove noise from satellite data. The data equation considered is

$$\begin{aligned}x_c(t) = & 0.0472 \cos (2\pi(200)t+ 1.5077)+ 0.1362 \cos (2\pi(400)t+1.8769) \\ & +0.4884 \cos (2\pi(500)t - 0.185)+ 0.2942 \cos (2\pi(1600)t - 1.4488) \\ & +0.1223 \cos (2\pi(1700)t) \quad \text{-----}(2.12)\end{aligned}$$

As mentioned, the continuous time signal is converted to discrete time signal by sampling process. The sampling frequency is selected so that it satisfies the Nyquist criterion. The maximum frequency in the above input signal is 1700 Hz [2].

$$\text{Nyquist frequency} = 3400 \text{ Hz}$$

A sampling frequency of 4000Hz is chosen which is higher than Nyquist frequency. All the components present in the signal are multiples of $(2\pi*100)$. So the fundamental period of the signal is assumed to be 100Hz. The number of samples present in the signal is given by

$$N = T_o/T_s = F_s/F_o = 4000/100$$

$$N = 40 \text{ samples}$$

By putting $t=n/f_s$ in equation 2.12 we get the discrete signal $x(n)$

$$\begin{aligned} x_c(t) = & 0.0472\cos(2\pi(200)n/f_s + 1.5077) + 0.1362\cos(2\pi(400)n/f_s + 1.8769) \\ & + 0.4884\cos(2\pi(500)n/f_s - 0.185) + 0.2942\cos(2\pi(1600)n/f_s - 1.4488) \\ & + 0.1223\cos(2\pi(1700)n/f_s) \end{aligned} \quad \text{-----(2.13)}$$

Now for the discrete signal also to be periodic, the sampled signal should satisfy the relation

$$T_o/T_s = N/M \Rightarrow T_o/T_s = 40$$

So the resulting signal is also periodic with a period of 40 samples. A N-point DFT (here $N=40$) on the above discrete time signal to get the spectrum. The spectrum has totally 40 points and each point specifies a particular frequency. The frequencies are given by

$$f_k = k*f_s/N, 1 \leq k \leq 40$$

The signal in the above case is real which forces the spectrum to be symmetric. So the magnitude spectrum has an even symmetry and the phase spectrum has an odd symmetry. The center point of the spectrum is $N/2$ (i.e.) 20.

2.6 Noise Generation

Noise is defined as any externally added interference that distorts the required or wanted signal. The noise that affects the signal can be from several sources and this noise can have any type of distribution. i.e. uniform distribution, Poisson distribution, Gaussian distribution or any other distribution. The noise that usually affects our signal of interest is Gaussian or white noise, which has spectral components right from 0 Hz to infinity [4].

The distribution for Gaussian case is given by the following mathematical expression.

$$P(x) = \frac{1}{\sqrt{2\pi\sigma}} \exp \left(-\frac{1}{2} \left(\frac{x - \mu}{\sigma} \right)^2 \right)$$

The probability for a Gaussian distribution with zero mean is given by

$$p(x) = 66.7\% \quad - 1 \leq x \leq 1$$

$$p(x) = 95.3\% \quad - 2 \leq x \leq 2$$

$$p(x) = 99.67\% \quad - 3 \leq x \leq 3$$

The noise that affects our signal of interest is assumed to have a variance of 3σ and the noise level for 3σ is $0.22V_{p-p}$, which has been practically found at ISRO labs.

2.7 Noise Addition

Let the noise generated be $n(n)$ and this noise signal is added with the input signal $x(n)$. The resulting signal $S(n)$ is given by

$$S(n) = x(n) + n(n)$$

The output signal $S(n)$ is a deterministic signal with noise having a variance $0.22 V_{p-p}$.

2.8 Noise filtering Algorithm

1. Accept the input signal $x(n)$ and random noise $n(n)$.
2. Compute the noisy signal $S(n)$.
3. Compute the spectrum of the original and the noisy data. i.e., $X(k)$ and $S(k)$ respectively.
4. The noise level is kept at 3σ and noise peak to peak at $0.22V$.
5. Fix 5σ as noise level, noise cut-off = $(5*0.22) / 6 V$.
6. The spectrum is reset for all components whose values are less than the noise cut off.

$$X_o(\omega) = \begin{cases} X(\omega) & \text{if } \text{Mag}(X(\omega)) > \text{cut-off} \\ 0 & \text{if } \text{Mag}(X(\omega)) \leq \text{cut-off} \end{cases}$$

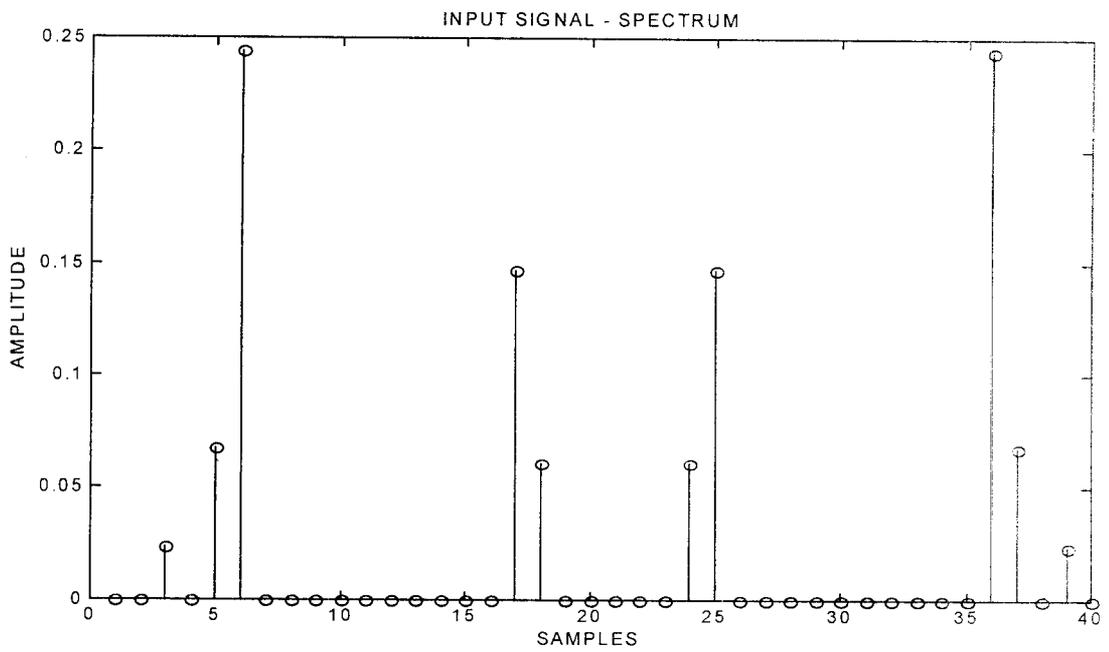
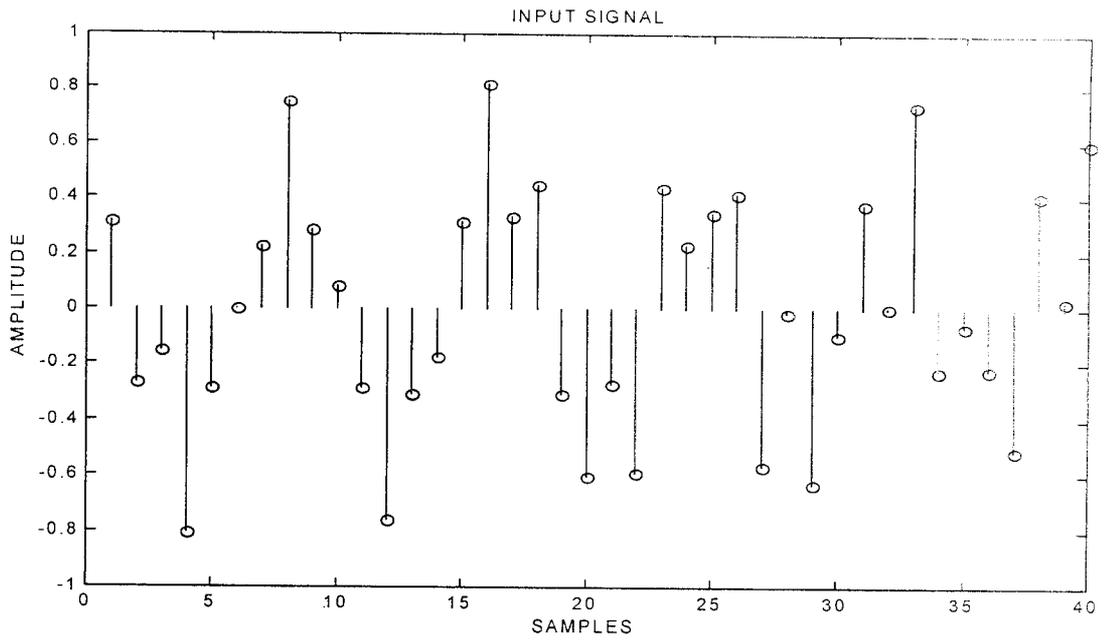
7. Reconstruct the signal with the components of $x_o(\omega)$
8. The reconstructed signal $x_0(n)$ and input signal $x(n)$ are plotted.

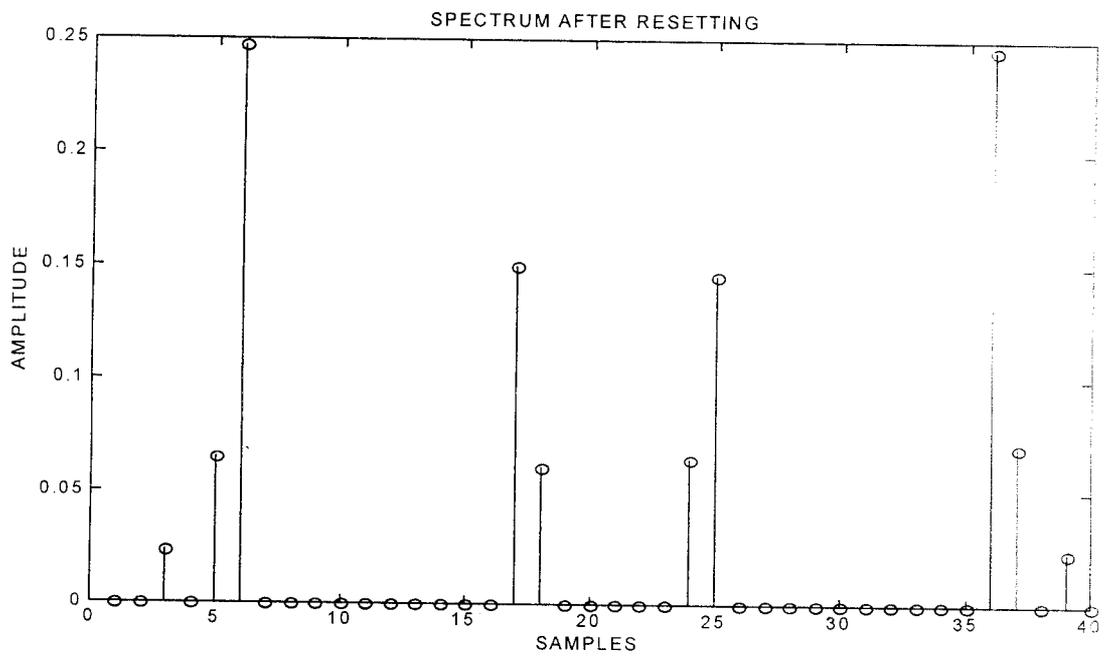
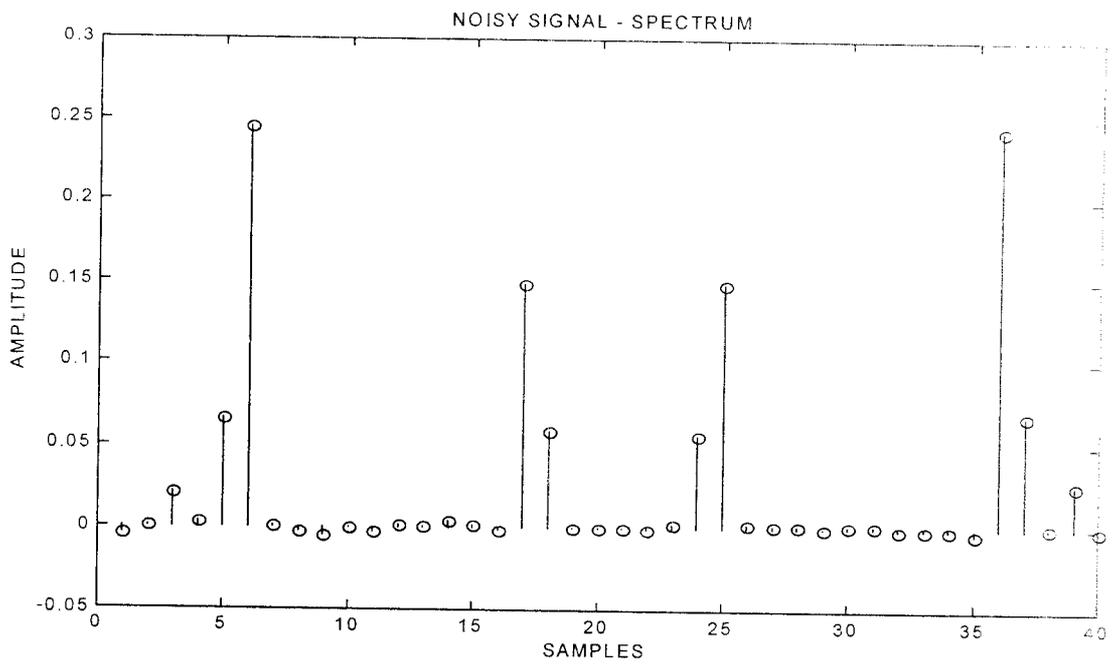
Since this is a simulation case original signal i.e. the known data equation is known. On having a look at the spectrum of the known signal it is clear that there are some spectral components peaking and all the other components are at zero level. The spectrum is symmetric about the point $N=20$.

The spectrum of the noisy signal contains components in all the frequencies and all the components have low amplitude comparatively to the peaking components in the spectrum of the input signal. The components having higher amplitudes in the noisy signal spectrum are those that contribute to the input signal and selection of the higher amplitude spectral components is the principle for reconstruction.

In order to select the higher amplitude components as said above a cut-off is to be fixed. This cut-off is the one that represents the minimum level in the noisy signal's spectrum below which is considered as noise. The components are reset that have entered the spectrum due to noise. The noise that gets added to the signal is usually about 3σ level.

Fix 5σ as noise level and the spectrum is reset for all components whose values are less than the noise cut off. The inverse of the obtained spectrum gives the noise-filtered signal. This signal is now compared with the input signal and on comparison it is clear that the two signals have the same path. The other conclusion is that the reconstructed signal flows through the mean path of the noisy signal. This method is applied to the satellite signal to filter the noise present in the satellite data.





Satellite Data Analysis

- *Satellite Data*
- *Data Extraction*
- *Spectrum Analysis*
- *Error Estimation*
- *Rate Estimation*
- *Practical Difficulties*

3.1 Satellite Data

From the satellite, telemetry data is received in about 1000 channels. These channels contain information about power systems, Sensor systems, Control systems, Thermal systems, Camera subsystems, Telemetry and Telecommand subsystem etc., of the satellite.

The above data represent

- (i) The health of the satellite
- (ii) Position of the satellite in the orbit
- (iii) Orientations angles of the camera towards the earth.

In this project, the three orientations angle errors of the camera are considered. The three orientation angle errors are called as yaw error, roll error and pitch error. When the camera is rotated about the x-axes, due to the satellite environment there is a tilt in the position that reflects as an error and this error is called as yaw error. Similarly, the errors due to the rotation of the camera about the 'y' and 'z' axes are referred as the roll and pitch errors.

We are concerned about the processing of Roll & Pitch errors coming from the conical earth sensor. There are two earth sensors ES1 & ES2, each of these gives the roll and angles. So a set of four signals is obtained with respect to these sensors. In addition to the two pitch and two roll errors the processed ES-Roll and processed ES-Pitch are also considered.

$$\text{Processed ES Roll} = \frac{\text{ES1 Roll} + \text{ES2 Roll}}{2}$$

$$\text{Processed ES2 Pitch} = \frac{\text{ES1 Pitch} + \text{ES2 Pitch}}{2}$$

With these two signals, a total of six signals are available and these values are stored at a standard satellite database. All data that are available are noisy due to the satellite environment. The data obtained is given to the control system as inputs for the evaluation of the satellite performance. It is very clear that lower the noise in the channel better is the performance of the control system and vice versa. The performance of the control system is evaluated in terms of pointing errors and rates.

The information available about the satellite data is

$$\text{Period of the signal} = 101 \text{ min } 20 \text{ sec}$$

$$\text{Sampling time} = 4 \text{ sec}$$

$$\text{Sampling Frequency} = 0.25 \text{ Hz}$$

$$\text{Number of Samples} = 1520$$

A sample set of data is shown which contains the year, month, day and time of observation of a sample for the six signals.

IRS-P4 PARAMETER AND Telemetry CHANNEL TABULAR OUTPUT

PAGE : 1

Start Time : 25-APR-2000 08 53 00 End Time : 25-APR-2000 12 06 00 STATION : ALL Datatype : BO

DD	MM	YYYY	HH	MM	SS	FID	ES1 SL	ROL	ER	ES2 SL	ROL	ER	PROC SL	ESROL	ES1 SL	PCH	ER	ES2 SL	PCH	ER	PROC SL	ESPCH	
							16				18				20				19				21
25	APR	2000	08	53	03	02	0.040400			0.026800			0.012600		-0.12480			-0.15600			-0.19110		
			08	53	07	04	0.044600			0.085600			0.126000		-0.19110			-0.19500			-0.17940		
			08	53	11	06	0.036200			0.052000			0.042000		-0.15210			-0.14040			-0.16380		
			08	53	15	08	0.061400			0.010000			0.033600		-0.13650			-0.17940			-0.15990		
			08	53	19	10	0.074000			0.010000			0.029400		-0.16380			-0.14040			-0.15600		
			08	53	23	12	0.061400			0.056200			0.046200		-0.18330			-0.16770			-0.17550		
			08	53	27	14	0.002600			0.073000			0.004200		-0.14040			-0.15210			-0.11700		
			08	53	31	00	-0.01000			-0.03200			0.037800		-0.12480			-0.22620			-0.19500		
			08	53	35	02	0.006800			0.047800			0.042000		-0.13260			-0.16770			-0.14040		
			08	53	39	04	0.023600			0.039400			0.012600		-0.06240			-0.18330			-0.15210		
			08	53	43	06	0.006800			0.056200			0.046200		-0.11700			-0.15990			-0.15990		
			08	53	47	08	-0.01840			0.026800			-0.00840		-0.14040			-0.17160			-0.12090		
			08	53	51	10	-0.00160			0.035200			0.037800		-0.14040			-0.17940			-0.15210		
			08	53	55	12	0.002600			0.068800			0.008400		-0.17550			-0.13260			-0.14040		
			08	53	59	14	-0.01840			0.031000			0.050400		-0.10140			-0.10530			-0.13260		
			08	54	03	00	-0.01840			0.047800			0.025200		-0.18720			-0.13650			-0.15600		
			08	54	07	02	0.019400			0.098200			0.054600		-0.08190			-0.16770			-0.09750		
			08	54	11	04	0.011000			0.094000			0.025200		-0.11310			-0.22620			-0.14820		
			08	54	15	06	0.032000			0.098000			0.054600		-0.07800			-0.23790			-0.12090		
			08	54	19	08	-0.03520			0.064600			0.050400		-0.08580			-0.22230			-0.18720		
			08	54	23	10	-0.02680			0.052000			0.058800		-0.13260			-0.17160			-0.14430		
			08	54	27	12	0.019400			0.060400			0.008400		-0.13650			-0.21060			-0.12480		
			08	54	31	14	0.002600			0.098200			-0.00840		-0.12870			-0.15990			-0.14040		
			08	54	35	00	-0.08980			0.022600			-0.02520		-0.07410			-0.17550			-0.08190		
			08	54	39	02	0.006800			0.056200			-0.00420		-0.05460			-0.16770			-0.13650		
			08	54	43	04	-0.02260			0.081400			0.025200		-0.09750			-0.16770			-0.10140		

08 54 47 06	-0.00580	0.077200	0.033600	-0.06630	-0.14040	-0.11700
08 54 51 08	-0.00580	-0.00680	0.046200	-0.11310	-0.12480	-0.10920
08 54 55 10	-0.04780	0.098200	0.00000	-0.08190	-0.14430	-0.07020
08 54 59 12	-0.00580	0.085600	0.021000	-0.05850	-0.13260	-0.18720
08 55 03 14	-0.06040	0.089800	-0.01680	-0.10920	-0.15210	-0.12870
08 55 07 00	-0.01420	0.068800	0.012600	-0.06630	-0.17160	-0.13260
08 55 11 02	-0.03520	0.018400	-0.01680	-0.07800	-0.14040	-0.11700
08 55 15 04	-0.10660	0.035200	0.008400	-0.05850	-0.14820	-0.10530
08 55 19 06	-0.08140	0.052000	-0.00420	-0.07410	-0.15600	-0.09750
08 55 23 08	-0.03520	0.115000	0.012600	-0.07800	-0.13650	-0.12480
08 55 27 10	-0.08980	0.115000	0.012600	-0.00390	-0.16770	-0.09750
08 55 31 12	-0.06040	0.005800	-0.00840	-0.05460	-0.14430	-0.09750
08 55 35 14	-0.01420	0.060400	0.016800	-0.07410	-0.16380	-0.10140

IRS-P4 PARAMETER AND Telemetry CHANNEL TABULAR OUTPUT

PAGE : 2

Start Time : 25-APR-2000 08 53 00 End Time : 25-APR-2000 12 06 00 STATION : ALL Datatype : BO
 Change in value : N

DD	MM	YYYY	HH	MM	SS	FID	ES1 SL	ROL	ER	ES2 SL	ROL	ER	PROC SL	ESROL	ES1 SL	PCH	ER	ES2 SL	PCH	ER	PROC SL	ESPCH
							16	18			20			17			19			21		
08	55	39	00	08	55	14	-0.01420	0.060400	0.016800	0.016800	-0.07410	-0.16380	-0.10140	-0.07410	-0.16380	-0.10140	-0.07410	-0.16380	-0.10140	-0.07410	-0.16380	-0.10140
08	55	43	02	08	55	00	0.020600	0.005800	0.016800	0.016800	-0.11700	-0.19110	-0.11700	-0.11700	-0.11700	-0.19110	-0.11700	-0.11700	-0.19110	-0.11700	-0.11700	-0.19110
08	55	47	04	08	55	02	-0.06460	0.052000	-0.00840	-0.00840	-0.14430	-0.12480	-0.14430	-0.14430	-0.12480	-0.14430	-0.12480	-0.14430	-0.12480	-0.14430	-0.12480	-0.14430
08	55	51	06	08	55	04	-0.02680	0.047800	-0.02940	-0.02940	-0.10140	-0.20670	-0.10140	-0.10140	-0.20670	-0.10140	-0.10140	-0.20670	-0.10140	-0.10140	-0.20670	-0.10140
08	55	55	08	08	55	06	-0.04780	0.089800	0.046200	0.046200	-0.10530	-0.18330	-0.10530	-0.10530	-0.18330	-0.10530	-0.10530	-0.18330	-0.10530	-0.10530	-0.18330	-0.10530
08	55	59	10	08	55	08	-0.08980	0.060400	0.021000	0.021000	-0.07410	-0.17160	-0.07410	-0.07410	-0.17160	-0.07410	-0.07410	-0.17160	-0.07410	-0.07410	-0.17160	-0.07410
08	56	03	12	08	56	10	-0.02260	0.119200	0.037800	0.037800	-0.07410	-0.14040	-0.07410	-0.07410	-0.14040	-0.07410	-0.07410	-0.14040	-0.07410	-0.07410	-0.14040	-0.07410
08	56	07	14	08	56	12	-0.08560	0.031000	-0.00840	-0.00840	-0.09360	-0.16770	-0.09360	-0.09360	-0.16770	-0.09360	-0.09360	-0.16770	-0.09360	-0.09360	-0.16770	-0.09360
08	56	11	00	08	56	14	-0.01420	0.102400	0.046200	0.046200	-0.10530	-0.13260	-0.10530	-0.10530	-0.13260	-0.10530	-0.10530	-0.13260	-0.10530	-0.10530	-0.13260	-0.10530
08	56	15	02	08	56	11	-0.01000	0.119200	0.033600	0.033600	-0.07020	-0.14040	-0.07020	-0.07020	-0.14040	-0.07020	-0.07020	-0.14040	-0.07020	-0.07020	-0.14040	-0.07020
08	56	19	04	08	56	15	-0.05200	0.064600	-0.01260	-0.01260	-0.02340	-0.16770	-0.02340	-0.02340	-0.16770	-0.02340	-0.02340	-0.16770	-0.02340	-0.02340	-0.16770	-0.02340
08	56	23	06	08	56	19	-0.03940	0.068800	0.068800	0.068800	-0.11310	-0.13260	-0.11310	-0.11310	-0.13260	-0.11310	-0.11310	-0.13260	-0.11310	-0.11310	-0.13260	-0.11310
08	56	27	08	08	56	23	-0.05620	0.119200	0.012600	0.012600	-0.05460	-0.15600	-0.05460	-0.05460	-0.15600	-0.05460	-0.05460	-0.15600	-0.05460	-0.05460	-0.15600	-0.05460
08	56	31	10	08	56	27	-0.04780	0.060400	0.008400	0.008400	-0.04290	-0.14820	-0.04290	-0.04290	-0.14820	-0.04290	-0.04290	-0.14820	-0.04290	-0.04290	-0.14820	-0.04290
08	56	35	12	08	56	31	-0.03940	0.005800	0.012600	0.012600	-0.08190	-0.15600	-0.08190	-0.08190	-0.15600	-0.08190	-0.08190	-0.15600	-0.08190	-0.08190	-0.15600	-0.08190
08	56	39	14	08	56	35	0.011000	0.110800	0.008400	0.008400	0.011700	-0.09360	0.011700	0.011700	-0.09360	0.011700	0.011700	-0.09360	0.011700	0.011700	-0.09360	0.011700
08	56	43	00	08	56	39	-0.04360	0.085600	0.008400	0.008400	-0.09360	-0.12480	-0.09360	-0.09360	-0.12480	-0.09360	-0.09360	-0.12480	-0.09360	-0.09360	-0.12480	-0.09360
08	56	47	02	08	56	43	-0.02260	0.073000	0.071400	0.071400	-0.19110	-0.11310	-0.19110	-0.19110	-0.11310	-0.19110	-0.19110	-0.11310	-0.19110	-0.19110	-0.11310	-0.19110
08	56	51	04	08	56	47	-0.05200	0.068800	-0.02520	-0.02520	-0.11700	-0.07800	-0.11700	-0.11700	-0.07800	-0.11700	-0.11700	-0.07800	-0.11700	-0.11700	-0.07800	-0.11700
08	56	55	06	08	56	51	-0.07300	0.094000	0.021000	0.021000	-0.12090	-0.12090	-0.12090	-0.12090	-0.12090	-0.12090	-0.12090	-0.12090	-0.12090	-0.12090	-0.12090	-0.12090
08	56	59	08	08	56	55	-0.02260	0.119200	0.021000	0.021000	-0.06630	-0.12480	-0.06630	-0.06630	-0.12480	-0.06630	-0.06630	-0.12480	-0.06630	-0.06630	-0.12480	-0.06630
08	56	59	08	08	56	59	-0.01420	0.047800	0.000000	0.000000	-0.05850	-0.12870	-0.05850	-0.05850	-0.12870	-0.05850	-0.05850	-0.12870	-0.05850	-0.05850	-0.12870	-0.05850
08	57	03	10	08	57	03	-0.05200	0.022600	0.029400	0.029400	-0.06240	-0.13650	-0.06240	-0.06240	-0.13650	-0.06240	-0.06240	-0.13650	-0.06240	-0.06240	-0.13650	-0.06240
08	57	07	12	08	57	07	-0.08980	0.077200	0.046200	0.046200	-0.12090	-0.21840	-0.12090	-0.12090	-0.21840	-0.12090	-0.12090	-0.21840	-0.12090	-0.12090	-0.21840	-0.12090
08	57	11	14	08	57	11	-0.06460	0.089800	0.025200	0.025200	-0.12090	-0.16770	-0.12090	-0.12090	-0.16770	-0.12090	-0.12090	-0.16770	-0.12090	-0.12090	-0.16770	-0.12090

08 57 15 00	-0.00160	0.014200	-0.00840	-0.01560	-0.09750	-0.12090
08 57 19 02	-0.14020	0.052000	-0.01260	-0.03510	-0.17160	-0.12090
08 57 23 04	-0.05620	0.068800	0.00000	-0.10530	-0.12480	-0.11310
08 57 27 06	-0.03520	0.089800	0.008400	-0.06240	-0.13650	-0.13260
08 57 31 08	-0.01840	0.089800	-0.00840	-0.06240	-0.12870	-0.09750
08 57 35 10	-0.03520	0.102400	0.004200	-0.08580	-0.10140	-0.08580
08 57 39 12	-0.10240	0.123400	-0.02940	-0.03120	-0.12090	-0.09750
08 57 43 14	0.006800	0.043600	0.004200	-0.00780	-0.11700	-0.08970
08 57 47 00	-0.08980	-0.00680	0.008400	-0.05850	-0.10920	-0.04290
08 57 51 02	-0.01840	0.068800	0.012600	-0.10920	-0.13260	-0.07410
08 57 55 04	-0.05200	0.098200	0.016800	-0.08580	-0.15210	-0.08190
08 57 59 06	-0.03100	0.039400	0.016800	-0.04290	-0.13650	-0.10920
08 58 03 08	-0.12340	0.089800	0.029400	-0.03510	-0.08190	-0.11700
08 58 07 10	-0.08980	0.115000	0.00000	-0.09750	-0.07410	-0.09750

IRS-P4 PARAMETER AND Telemetry CHANNEL TABULAR OUTPUT

PAGE : 3

Start Time : 25-APR-2000 08 53 00 End Time : 25-APR-2000 12 06 00 STATION : ALL Datatype : BO
 Change in value : N

DD	MM	YYYY	HH	MM	SS	FID	ES1 SL	ROL	ER	ES2 SL	ROL	ER	PROC SL	ESROL	ES1 SL	PCH	ER	ES2 SL	PCH	ER	PROC SL	ESPCH
							16			18			20		17			19			21	
08	58	11				10	-0.08980			0.115000			0.00000		-0.09750			-0.07410			-0.09750	
08	58	11				12	-0.05620			0.089800			0.004200		-0.08190			-0.15210			-0.10140	
08	58	15				14	-0.07300			0.047800			-0.02100		-0.07410			-0.12870			-0.08190	
08	58	19				00	-0.12340			0.085600			-0.03360		-0.10530			-0.15600			-0.12090	
08	58	23				02	-0.06040			0.098200			0.029400		-0.03900			-0.10530			-0.09750	
08	58	27				04	0.006800			0.043600			0.033600		-0.06240			-0.08580			-0.09750	
08	58	31				06	-0.09820			0.073000			0.004200		-0.02730			-0.12870			-0.10530	
08	58	35				08	-0.08140			0.035200			-0.02940		-0.05850			-0.10140			-0.07410	
08	58	39				10	-0.06880			0.073000			0.021000		0.031200			-0.09750			-0.06240	
08	58	43				12	-0.09820			0.018400			-0.02520		-0.08190			-0.11700			-0.10530	
08	58	47				14	-0.05200			0.052000			-0.00420		-0.02340			-0.10920			-0.07800	
08	58	51				00	-0.07720			0.106600			0.008400		-0.07800			-0.10530			-0.06240	
08	58	55				02	-0.04780			0.052000			-0.00420		-0.05070			-0.14040			-0.07800	
08	58	59				04	-0.08140			0.089800			-0.01260		0.019500			-0.09750			-0.08970	
08	59	03				06	-0.14020			0.085600			-0.00420		0.003900			-0.07020			-0.07020	
08	59	07				08	-0.03520			0.068800			-0.03360		-0.08580			-0.11700			-0.03900	
08	59	11				10	-0.02260			0.119200			-0.02520		-0.02730			-0.11700			-0.11310	
08	59	15				12	-0.11920			0.039400			-0.01680		-0.07020			-0.10530			-0.10530	
08	59	19				14	-0.18220			0.060400			-0.07140		0.011700			-0.07800			-0.07020	
08	59	23				00	-0.07300			0.068800			0.004200		-0.00390			-0.12480			-0.06240	
08	59	27				02	-0.10660			0.094000			-0.02100		-0.01170			-0.12480			-0.07410	
08	59	31				04	-0.10660			0.056200			0.008400		-0.07410			-0.11310			-0.07800	
08	59	35				06	-0.14020			0.060400			-0.06720		-0.04290			-0.07800			-0.06240	
08	59	39				08	-0.11920			0.098200			-0.02940		0.015600			-0.09750			-0.07800	
08	59	43				10	-0.13180			0.136000			-0.00420		-0.07410			-0.08580			-0.06240	

08 59 47	12	-0.12760	0.060400	-0.03780	0.00000	-0.08580	-0.06630
08 59 51	14	-0.17380	0.131800	-0.01680	-0.02730	-0.15210	-0.08970
08 59 55	00	-0.09820	0.089800	-0.04620	-0.05070	-0.13650	-0.09750
08 59 59	02	-0.14440	0.056200	-0.03360	0.015600	-0.10530	-0.03120
09 00 03	04	-0.04360	0.089800	0.016800	-0.06630	-0.06630	-0.07800
09 00 07	06	-0.13180	0.005800	-0.01680	0.003900	-0.10530	-0.01950
09 00 11	08	-0.07720	0.144400	-0.00840	0.00000	-0.10920	-0.07410
09 00 15	10	-0.04360	0.064600	0.025200	-0.02340	-0.07410	-0.06630
09 00 19	12	-0.08980	0.026800	-0.02940	0.011700	-0.08580	-0.10140
09 00 23	14	-0.10240	0.068800	-0.00840	-0.01560	-0.09360	-0.08970
09 00 27	00	-0.11500	0.077200	-0.00420	-0.02730	-0.07800	-0.06240
09 00 31	02	-0.15700	0.064600	-0.00420	-0.04290	-0.13650	-0.05070
09 00 35	04	-0.19480	0.056200	-0.04200	-0.04680	-0.12090	-0.03120
09 00 39	06	-0.10240	0.039400	0.004200	-0.03120	-0.07410	-0.12870

3.2 Data Extraction

One of the difficult portion in our project is the data extraction. The database output is arranged page by page. The data is available in around 10 columns and after a particular number of rows, some lines of header information about satellite ID, ground station ID, orbit number etc. are included. This data runs for about 4000 samples.

The data is extracted from the standard satellite database by using a MATLAB program, which accepts these data as input and gives the required columns as output. Each column represents the satellite signal of a particular channel. The programming is done after the deletion of comments from the database.

3.3 Spectrum Analysis

3.3.1 Real Signal

The signals that are received from the satellite is real which forces the spectrum to be symmetric about the point $N/2$.

$$\text{i.e. } |X(k)| = |X(N-k)|$$

where k lies between 0 to $N/2$

3.3.2 Noise Filtering

The spectrum is found by calculating the N point FFT on the samples obtained from the data extracted. Here, N represents the number of sample points in both time scale and frequency scale.

The physical frequencies that are presented in the signal are given by

$$f_k = kf_s/N \text{ where } k = 1, 2, \dots, 1520$$

$$f_k = \frac{k}{4 \times 1520}$$

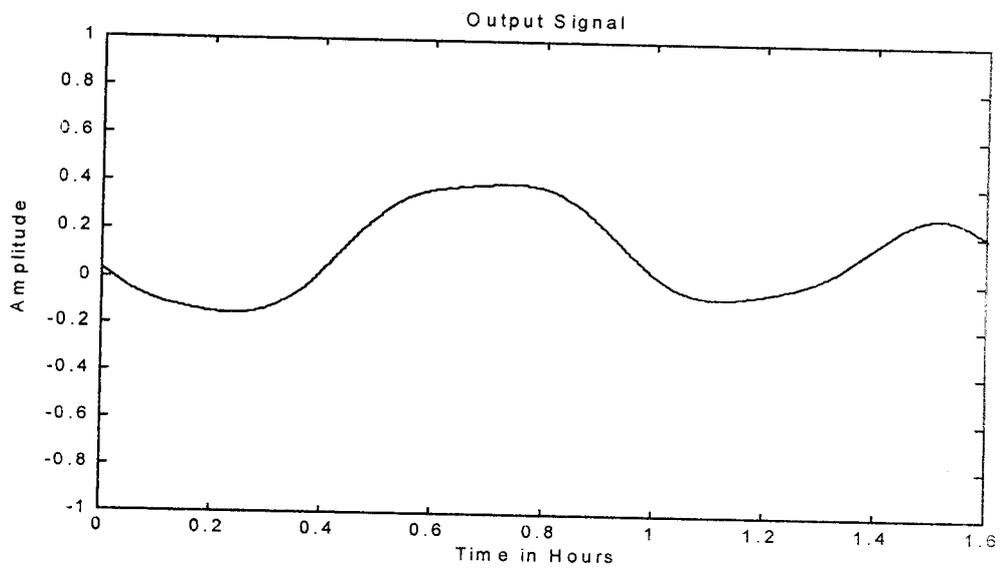
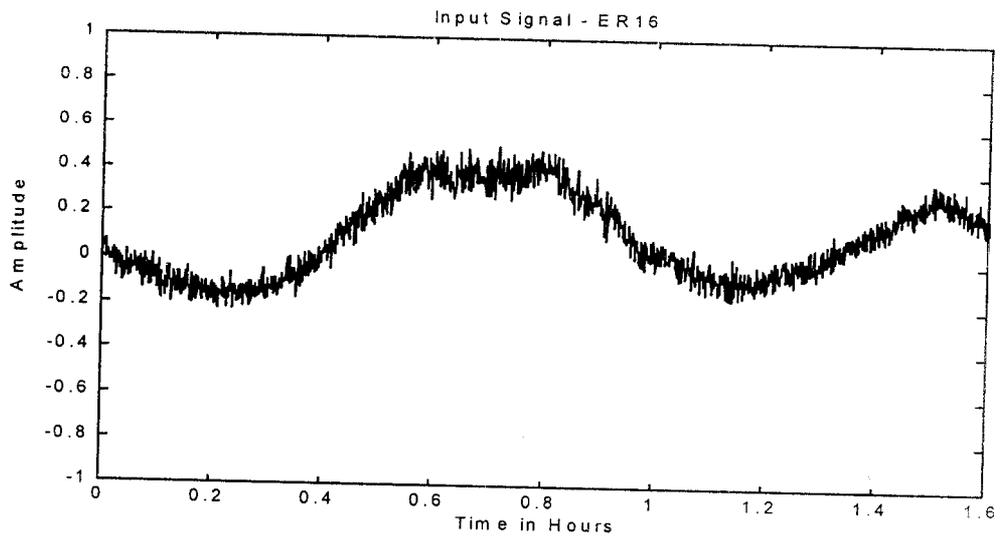
From sampling theorem the frequencies that can be retrieved from the data are 0 Hz to $f_s/2$ Hz.

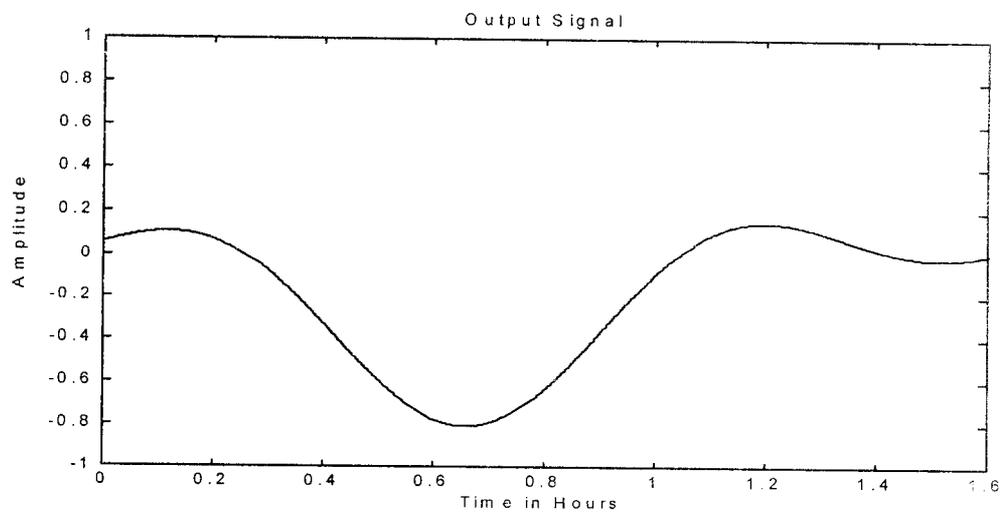
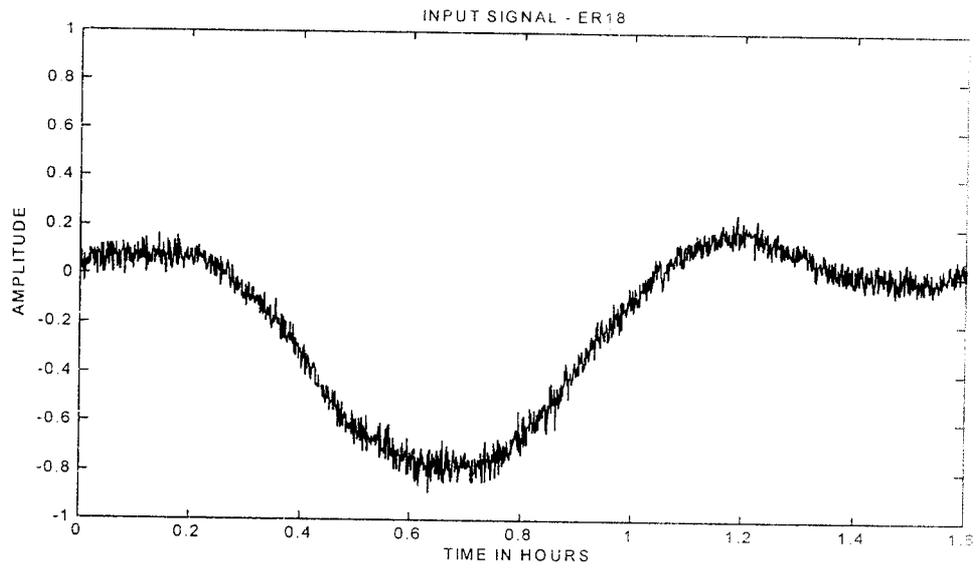
$$\text{Sampling frequency} = 1/4 \text{ Hz}$$

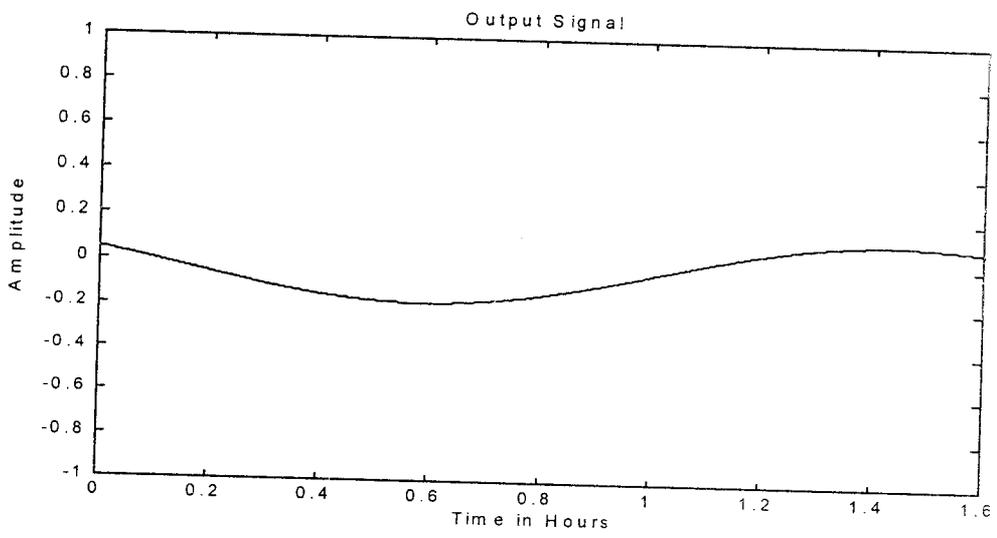
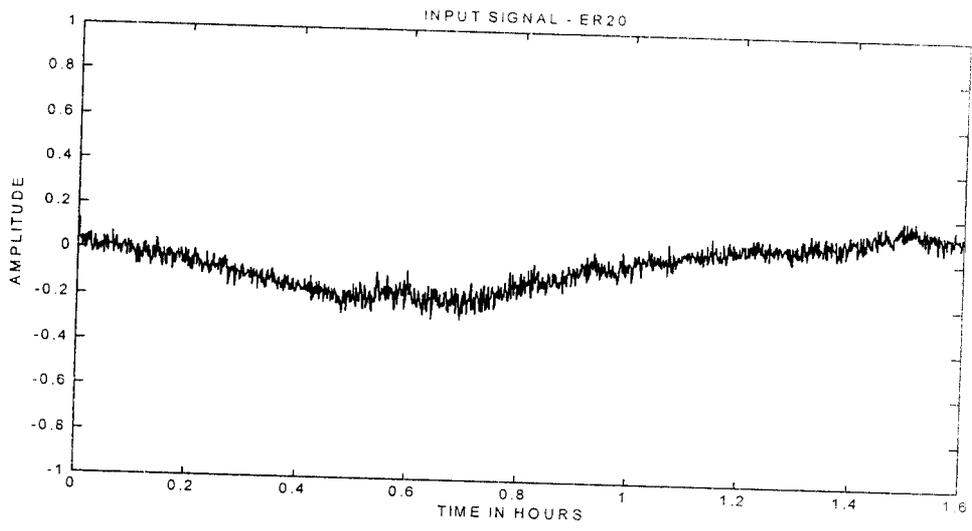
$$\text{Maximum frequency that can be reconstructed} = 1/8 \text{ Hz}$$

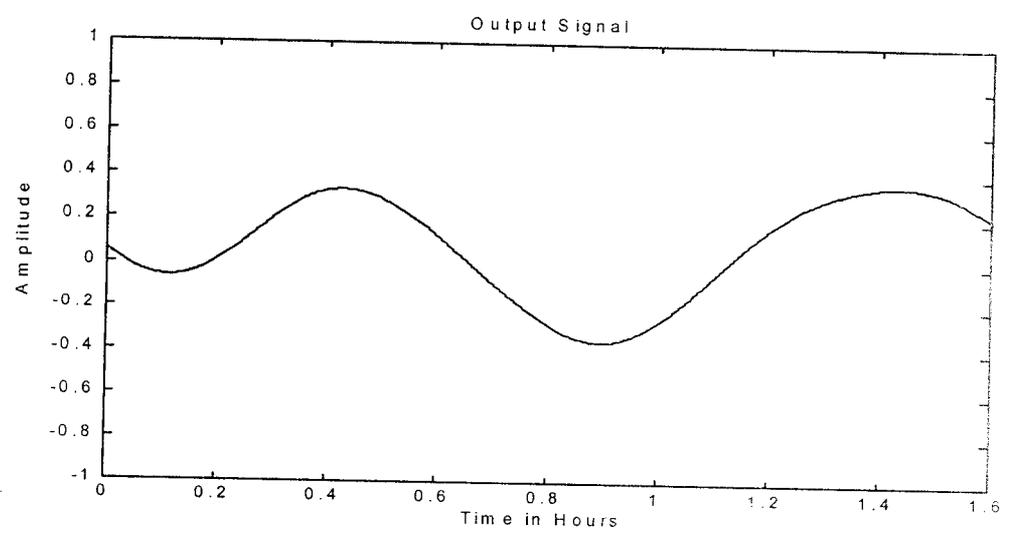
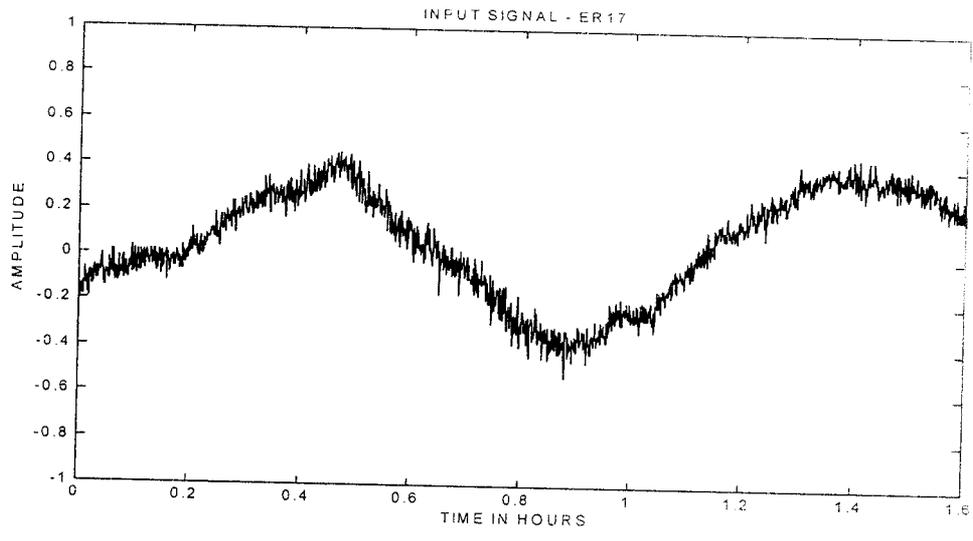
The spectrum is observed for the components that are peaking out. Only certain components in the spectrum have a significant value with respect to the error band. The spectral components having values lying within the error band are reset.

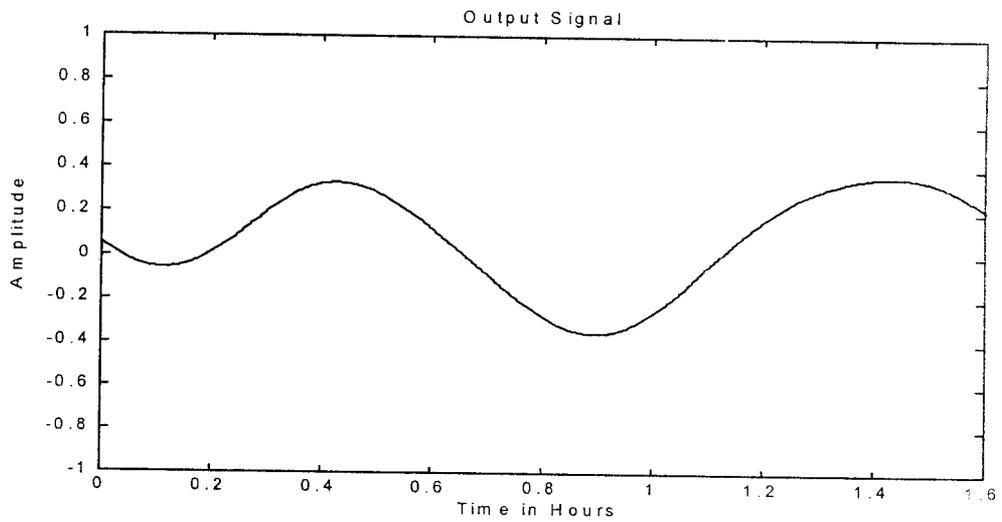
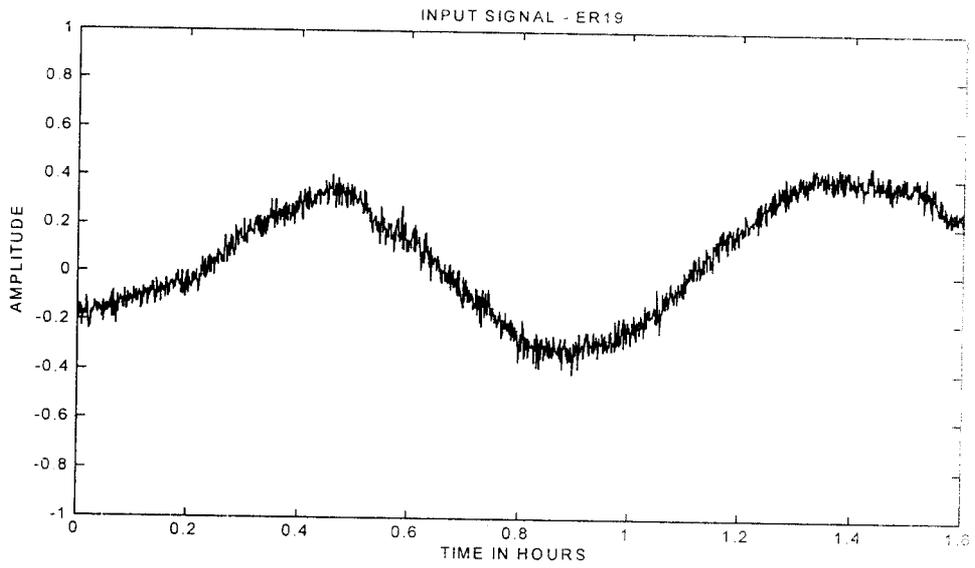
The frequency spikes having values greater than the error cut-off's are used to reconstruct the signal. These frequencies and their corresponding

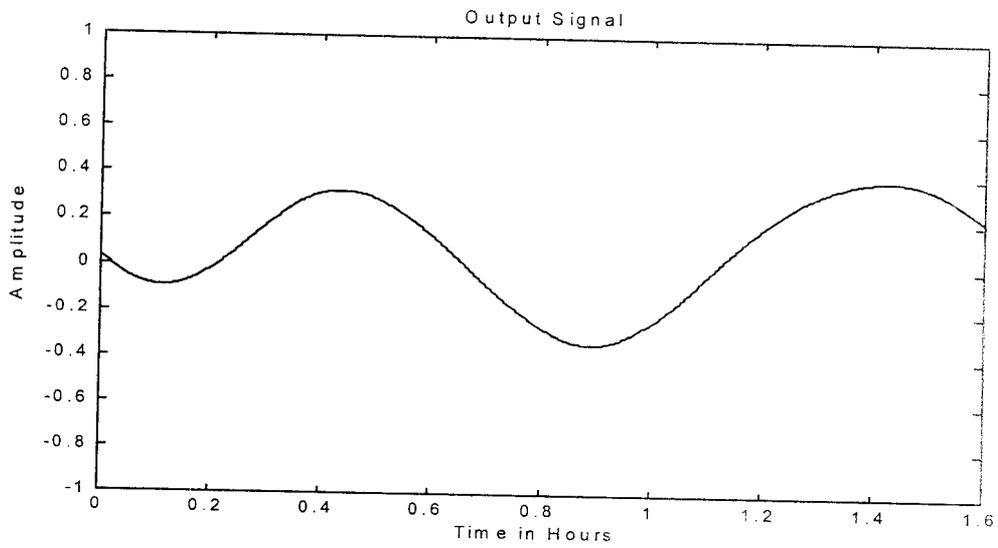
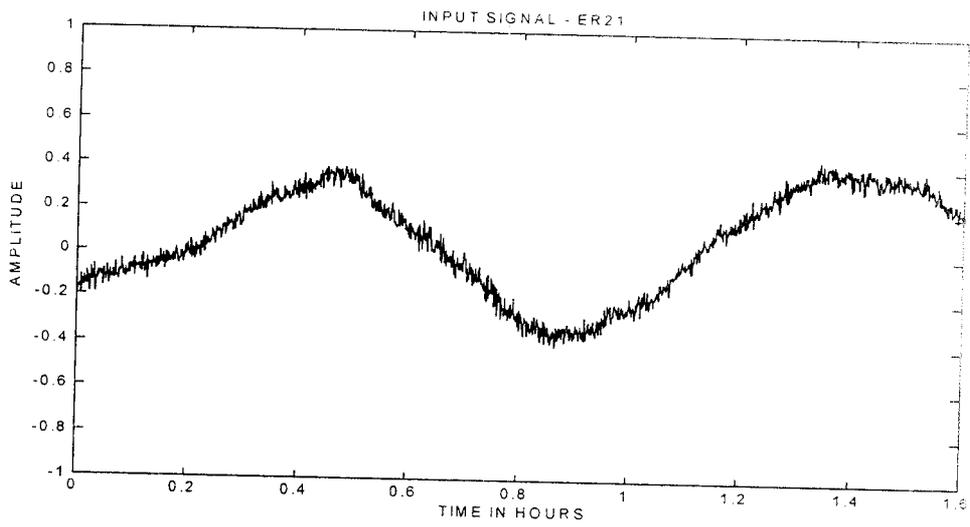












amplitudes are used in sine and cosine form of fouries series for reconstruction. The above-obtained signal represents the noise free signal.

With the help of spectrum analysis technique we can derive the expressions for (i) Error Estimation

(ii) Rate Estimation

3.4 Error Estimation

The error estimation is done with the help of error equation. The error equation gives the reconstructed signal in terms of sine and cosine form of the Fourier series. This equation consists of frequency components that constitute the data [3]. The error equation is given below

$$X(n) = 2 * \left[\frac{a_0}{2} + a_k(1) \cos\left(\frac{2\pi n}{N}\right) + b_k(1) \sin\left(\frac{2\pi n}{N}\right) + a_k(2) \cos\left(\frac{2\pi n \cdot 2}{N}\right) + \right. \\ \left. b_k(2) \sin\left(\frac{2\pi n \cdot 2}{N}\right) + \dots + a_k\left(\frac{N}{2}\right) \cos\left(\frac{2\pi n \cdot N/2}{N}\right) + b_k\left(\frac{N}{2}\right) \sin\left(\frac{2\pi n \cdot (N/2)}{N}\right) \right] \quad (3.1)$$

Since the signal is real, only N/2 components are used to reconstruct the signal and to compensate this a factor of two is included in the above equation.

3.5 Rate Estimation

3.5.1 Differentiation of Fourier series

The estimation of rate for the given signal is carried out by differentiation of the error expression. The resulting expression is called as the rate equation [3].

On differentiating (3.1), we get

$$\begin{aligned} \text{Rate (n)} = \frac{d}{dn} [x(n)] = & 2 * \frac{2\pi n}{N} \left[-a_k(1) \sin \left[\frac{2\pi n}{N} \right] + b_k(1) \cos \left[\frac{2\pi n}{N} \right] \right. \\ & - 2 * a_k(2) \sin \left[\frac{2\pi n \cdot 2}{N} \right] + 2 * b_k(2) \cos \left[\frac{2\pi n \cdot 2}{N} \right] + \dots \\ & \left. - N/2 a_k \left[\frac{N}{2} \right] \sin \left[\frac{2\pi n(N/2)}{N} \right] + N/2 b_k \left[\frac{N}{2} \right] \cos \left[\frac{2\pi n(N/2)}{N} \right] \right] \end{aligned}$$

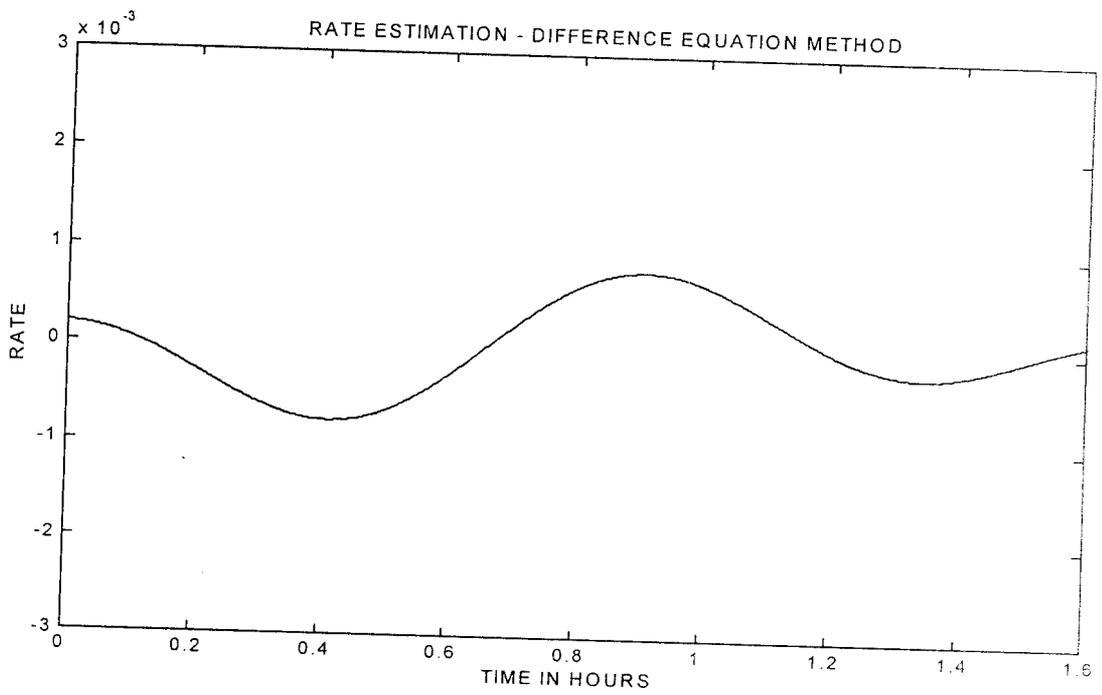
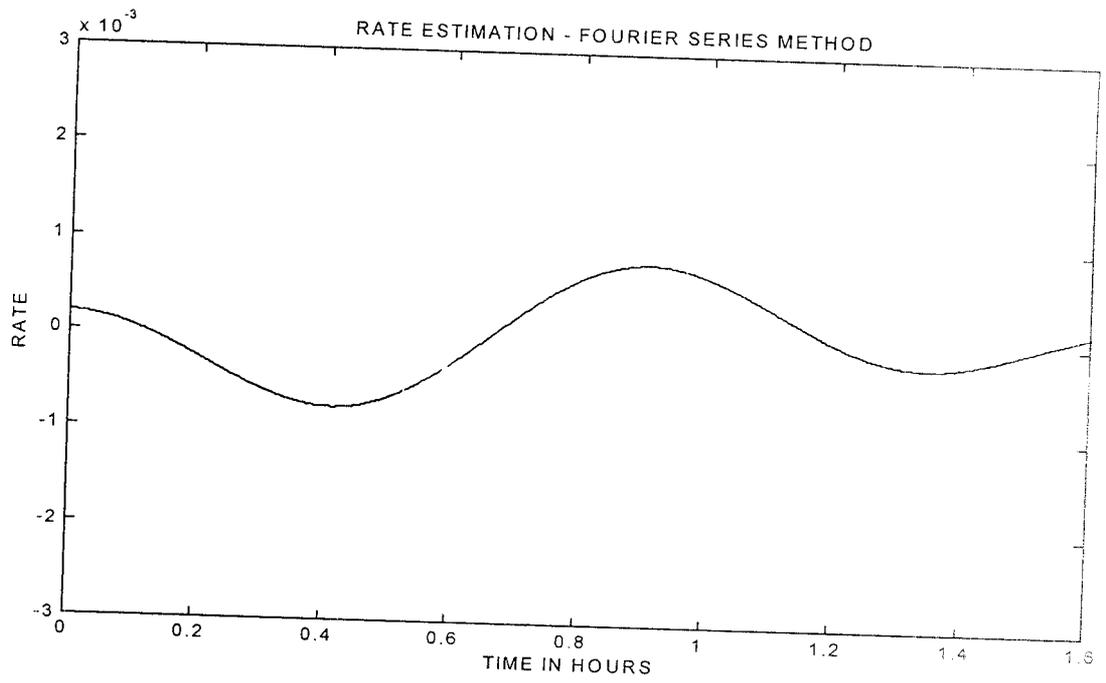
----- (3.2)

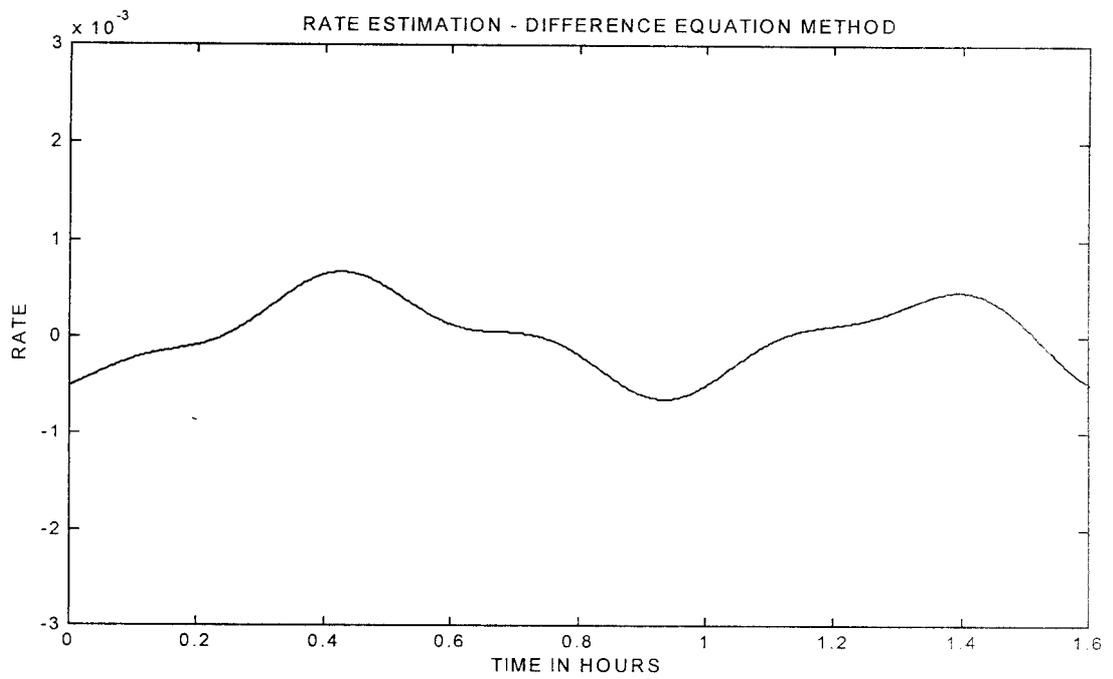
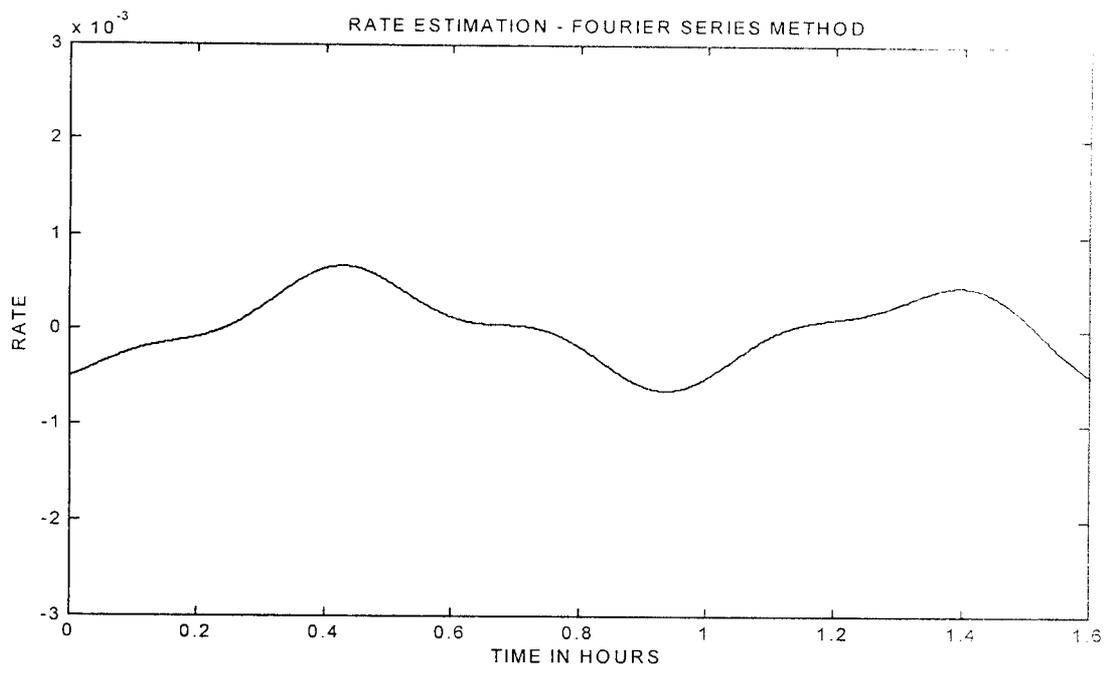
3.5.2 Difference Equation

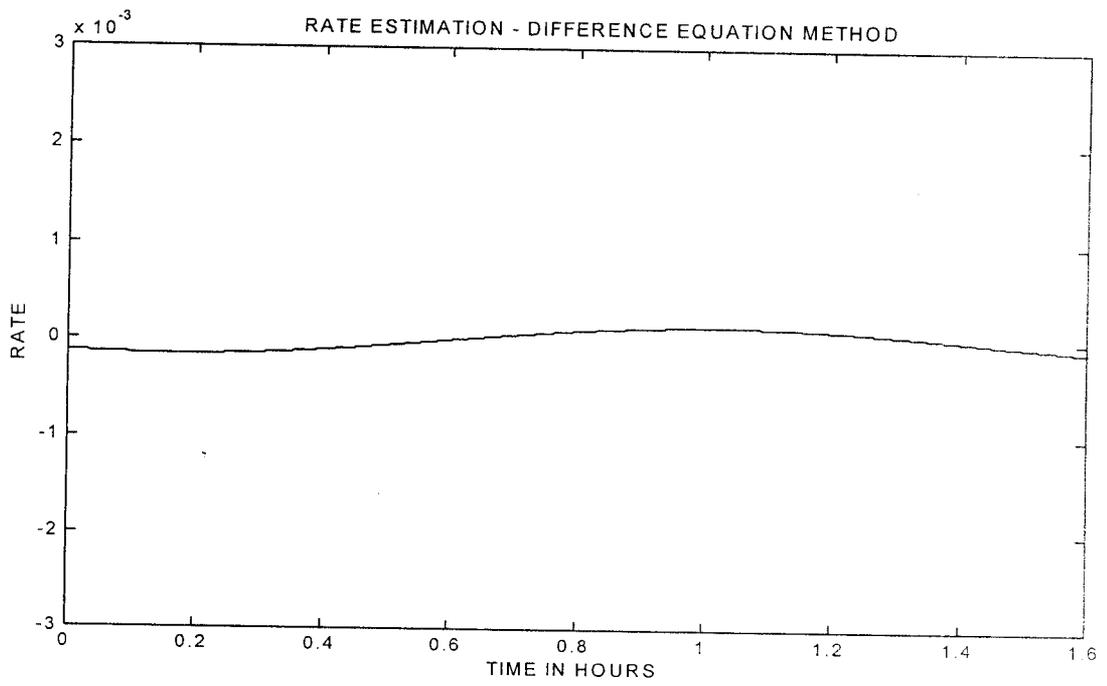
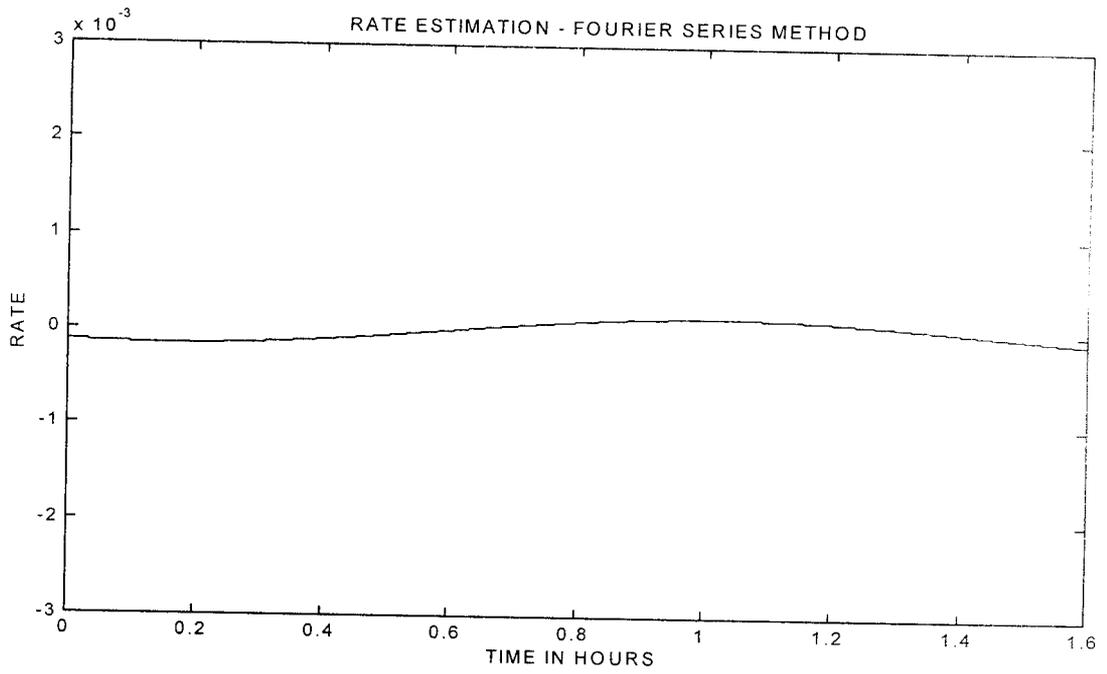
The rate of the given signal is determined using the difference equation method. In this method, rate is calculated as the difference between the subsequent samples. The rate for the last sample alone is calculated using the first and last sample. The first sample is used because the signal is periodic.

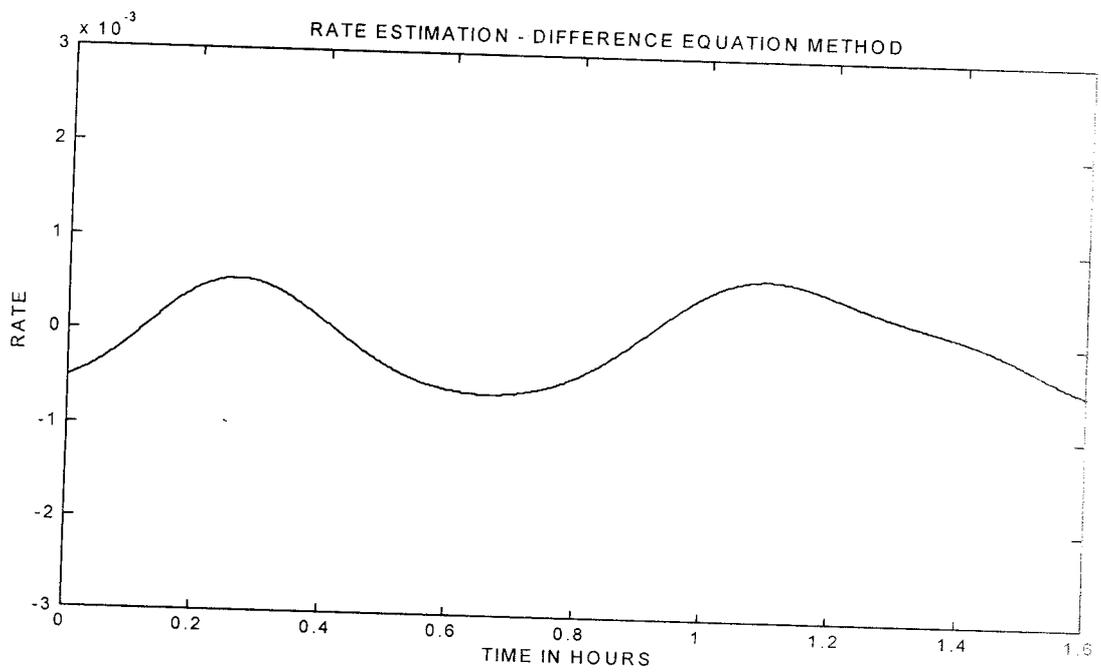
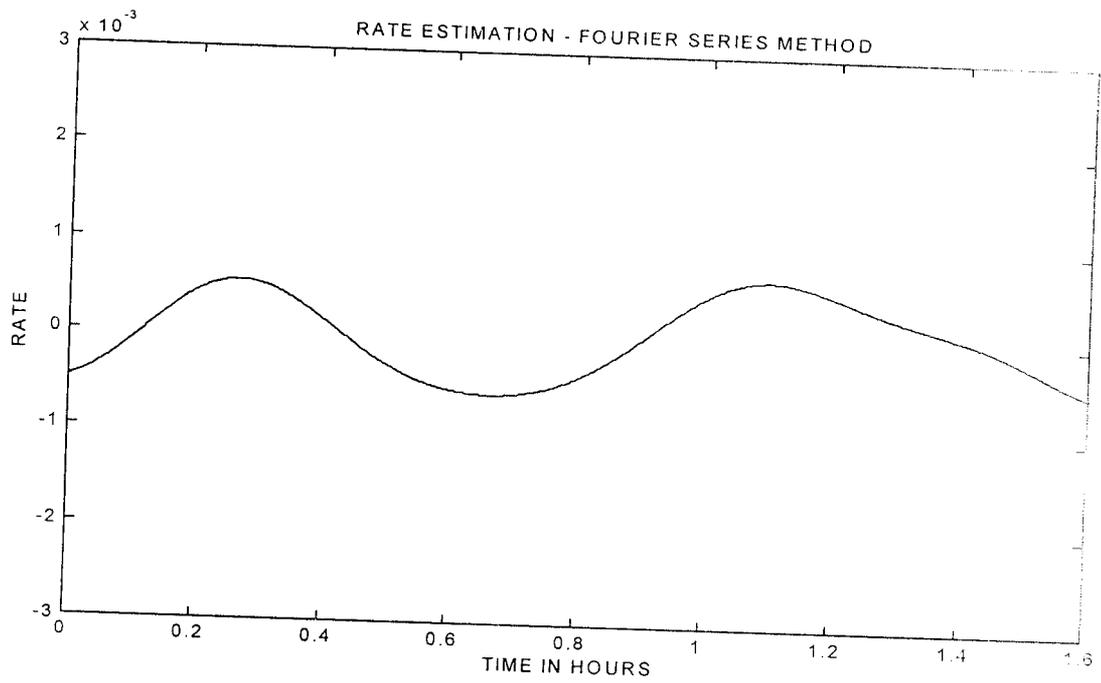
$$\text{RATE (n)} = x(n+1) - x(n), n \neq N$$

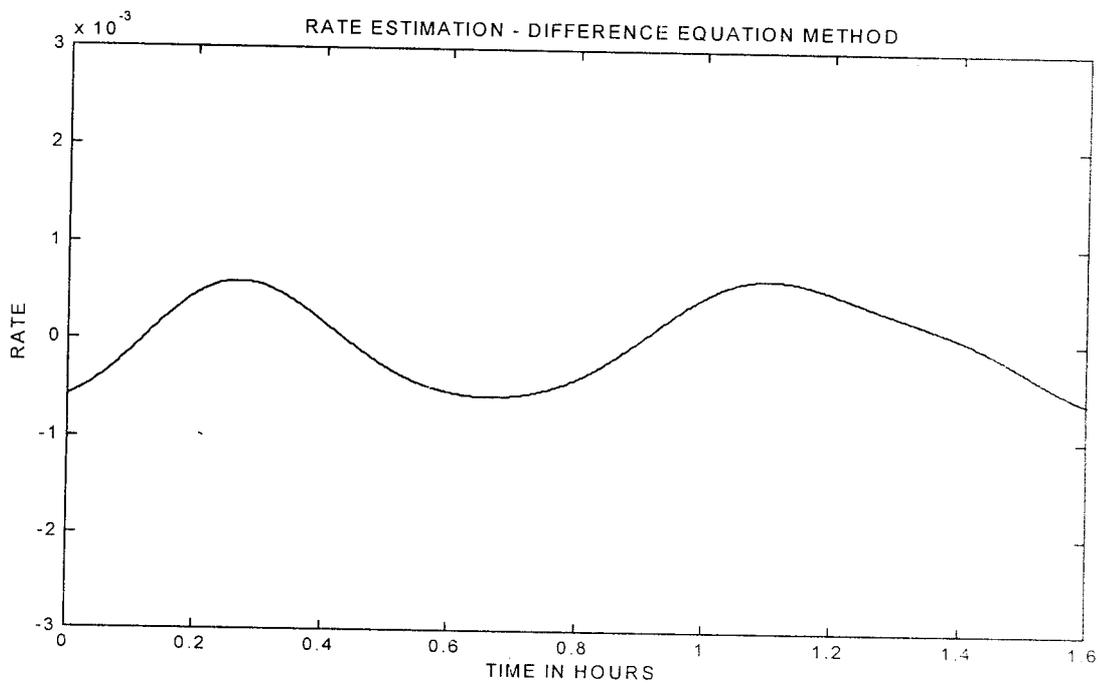
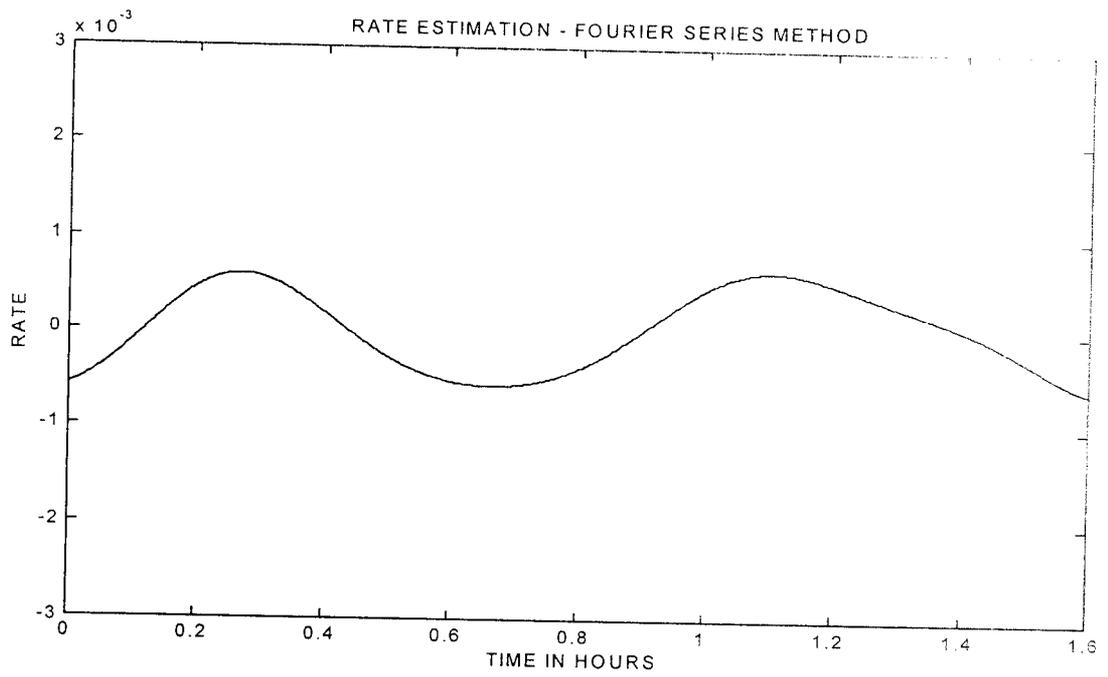
$$\text{RATE (N)} = x(1) - x(N)$$

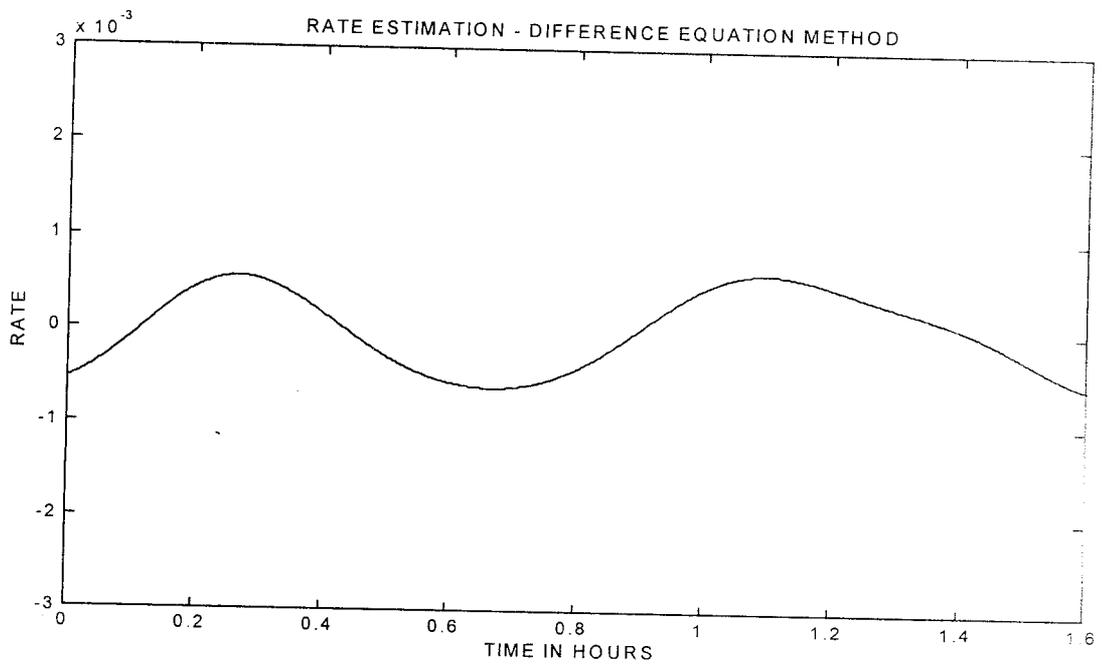
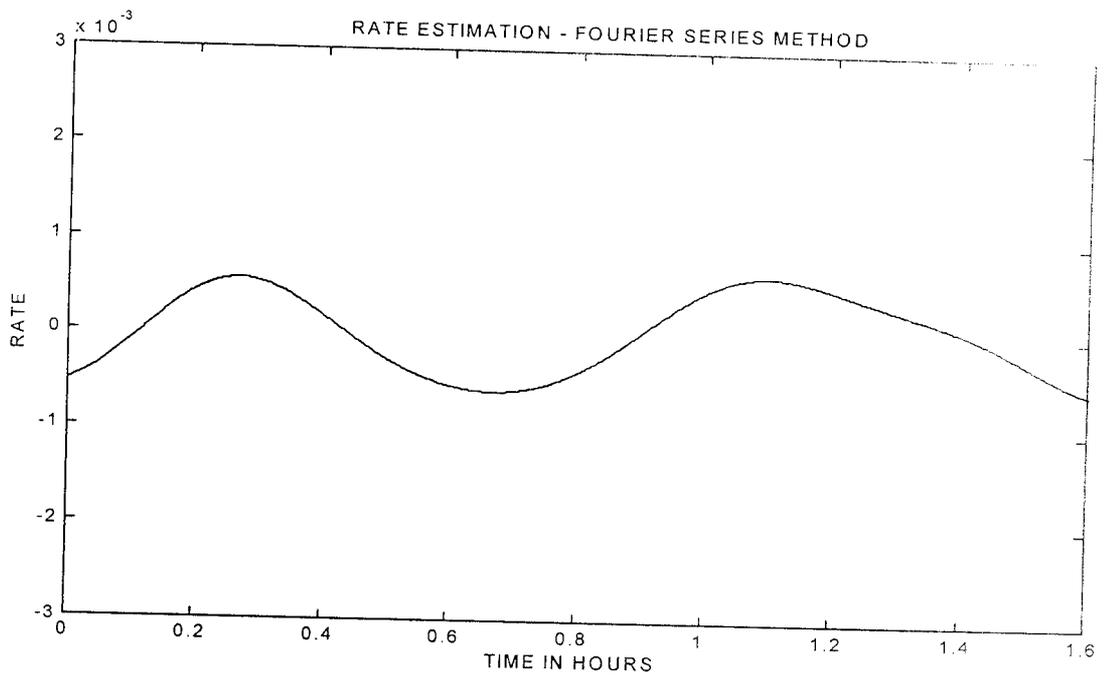












3.5.3 Comparison of Rates

The rate determined by the methods

1. Differentiation of Fourier series
2. Difference Equation

are plotted. It is observed from the rate plots that satellite drift are in the order of 10^{-3} .

3.6 Practical Difficulties

1. The spectral coefficients are obtained through fft function of MATLAB. These values differ from the original spectral coefficients by a factor N.

Let $X(k)$ be the spectral values found by taking fft

C_k be the original spectral coefficients

$$\therefore X(k) = N * C_k$$

2. In MATLAB, the spectrum is determined using complex form of Fourier series and the reconstruction is done with the sine and cosine form of Fourier series.

$$x(n) = \sum_{-N}^N c_k e^{-j2\pi kn/N}$$

$$x(n) = \frac{a_0}{2} + \sum_{-N}^N a_k \cos\left(\frac{2\pi kn}{N}\right) + b_k \sin\left(\frac{2\pi kn}{N}\right)$$

The relation between C_k and (a_k, b_k) is given by

$$C_k = (a_k - ib_k) / 2 \quad \text{----- (3.3)}$$

$$a_k = 2 * \text{real}(C_k), b_k = 2 * \text{Imag}(C_k) \text{ and } C_0 = a_0 \quad \text{----- (3.4)}$$

If we use the above set of equations a minus factor is not taken into account. Therefore a negative sign is included in the sine and cosine form of Fourier series.

$$X(n) = \frac{a_0}{2} + \sum_{-N}^N a_k \cos\left(\frac{2\pi kn}{N}\right) - b_k \sin\left(\frac{2\pi kn}{N}\right)$$

3.7 Conclusion

- (i) From the plots it is clear that the output signal follows the mean path of the input noisy signal.
- (ii) The band in which the error lies is in the 3σ level of noise.
- (iii) The rate of the satellite signals is in the range $-1 * 10^{-3}$ to $+1 * 10^{-3}$.
- (iv) This method does not account for the noise removal at the frequency spikes.
- (v) Since this method requires the spectrum, this cannot be used for online applications.

Kalman Filter

- *Introduction*
- *Definition*
- *Features*
- *Description of Kalman Filter*
- *Derivation of Matrices*
- *Filter Divergence*
- *Filter Tuning*
- *Filter Performance*

4.1 Introduction

Kalman gave a powerful and new means of statistical estimation for stochastic processes. This became the famous Kalman filter later. Kalman filter is nothing but a recursive solution to Gauss least squares problem. The estimate made from noisy measurements is unbiased and has minimum variance. Generalization of Weiner filter resulted in the realization of Kalman filter [5].

4.2 Definition

The Kalman filter is a recursive, mean square error filter, which uses a dynamic model for time development of the system and a model of sensor measurement to obtain the most accurate estimate possible of the system using a linear estimator based on the present and past measurements [9].

4.3 Features

The main features of Kalman filter are

- 1) Kalman posed the problem in a general framework that had a unifying influence on known results. Further, one can analyze the behavior of the estimates within the general framework and thereby obtain significant insights into the results obtained from computational studies.

- 2) The Kalman filter equations provide an extremely convenient procedure for digital computer implementation. One can very easily write a computer program for the Kalman Algorithm without much understanding of the theory, which led to its development [6].

4.4 Description of Kalman Filter

4.4.1 Filter Equations

The Kalman filter is a recursive, mean square error filter for the computation of $X^{\text{hat}}(k+1/k+1)$, $K = 0, 1 \dots N$. The system that is considered is composed of two essential components

- State
- Measurement

General Equations:

The state is given by

$$X^{\text{hat}}(k+1) = \phi x(k) + W(k) \quad \text{----- (4.1)}$$

and the measurement data are related to the state by

$$Z(k+1) = H x(k+1) + V(k+1) \quad \text{----- (4.2)}$$

The equations (1) and (2) are called as the state and the measurement equations respectively.

Prediction Equations:

$$\text{State} \quad : X^{\text{hat}}(k+1/k) = \phi X^{\text{hat}}(k/k) \quad \text{----- (4.3)}$$

$$\text{Measurement: } Z^{\text{hat}}(k+1/k) = H X^{\text{hat}}(k+1/k) \quad \text{----- (4.4)}$$

The mean square estimate is given by

$$X^{\text{hat}}(k+1/k+1) = X^{\text{hat}}(k+1/k) + K_k [Z(k+1) - Z^{\text{hat}}(k+1/k)] \quad \text{----- (4.5)}$$

where Kalman filter gain K_k is given by,

$$K_k(k+1) = P(k+1/k) H^T(k+1) [H(k+1) P(k+1/k) H^T(k+1) + R(k+1)]^{-1} \quad \text{----- (4.6)}$$

Prediction of State Error Covariance matrix: The Matrix $P(k+1/k)$ is the covariance of the error in the predicted estimate.

$$P(k+1/k) = [Q(k+1/k) P(k/k) \varphi(k+1/k) + \varphi(k)] \text{ -----(4.7)}$$

Estimation of State Error Covariance matrix: The matrix $P(k/k)$ is the covariance of the error in the observed estimate and is given by,

$$P(k/k) = (I - K_k(k) H(k)) P(k/k-1) (I - K_k(k) H(k)) + K_k(k) R(k) K_k^T(k) \text{ ---(4.8)}$$

The equations (4.3) to (4.8) constitute the Kalman filter equations. By knowing the initial state $X_{0/0}$ and its covariance $P_{0/0}$, state noise variance Q , measurement noise covariance R , measurement is obtained. As soon as measurement is obtained, the estimate state and its covariance can be executed and the cycle is repeated for the next measurement.

It can be seen that one need not store measurement as and when measurement is available it can be processed. Because of its recursive nature Kalman filter is very much suitable for online / real time application. Since the calculation of gain is independent of actual measurement, it can be calculated priory and stored for real time usage. Also covariance can be calculated in advance, which can be used for performance analysis. If the Kalman gain is small, filter follows dynamics and measurements are ignored. Depending on the value of R , gain increases or decreases corresponding measurements are weighted [9].

4.4.2 Notations

The following are the symbols used in the derivation of Kalman filter equations.

X_k = Actual value of the state at k^{th} instant

Q_k = State noise covariance matrix

$X_{k/k-1}$ = Estimate of the state X_k before processing the current measurement, predicted state.

$P_{k/k-1}$ = Covariance of the predicted state error

$X_{k/k}$ = Estimate of the state X_k after processing the current measurement, filtered value.

$P_{k/k}$ = Covariance of the error in estimated state

Z_k = Actual measurement

R_k = Measurement noise covariance matrix

$Z_{k/k-1}$ = Predicted measurement

$k/k-1$ = Estimate at k based on $k-1$ value

K_k = Kalman gain

I = Identity matrix

4.4.3 Assumptions

i) Dynamics:

A discrete time linear dynamical system can be represented as a set of difference equation.

$$X_k = \phi_{k/k-1} X_{k-1} + U_{k-1}$$

where X is the instant at which the process is described, takes integer values 1,2... and corresponds to time t_1, t_2, \dots

X_k = (n x 1) Vector – State of the system

U_{k-1} = (n x 1) Vector – Excitation or control function or input

$\phi_{k/k-1}$ = (n x n) Matrix – State transition matrix

If there are random forces such as wind, gust etc. acting on the system, then U_{k-1} can represent this. One reason for the assumption of Gaussian noise is, when micro noise gets added together it gives Gaussian noise independent of their original distribution and this is given by central limit theorem.

For a moving body, velocity is assumed to be constant and then the state equations for One-Dimensional case can be written as

$$X_k = X_{k-1} + X_{k-1} \tau$$

$$X_k = X_{k-1}$$

where τ is the sampling time.

ii) Measurement Equations:

The measurement equations, which tie measurements with the state, are assumed to be linear.

$$\text{i.e. } Z_k = H_k Y_k$$

Select Roll, pitch, Roll, pitch, Roll, pitch as state vector. The measurement gives Roll and pitch. For this case measurement equations are given by,

$$\begin{pmatrix} \text{Roll} \\ \\ \\ \text{Pitch} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ \\ \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} \text{Roll} \\ \dot{\text{Roll}} \\ \ddot{\text{Roll}} \\ \text{Pitch} \\ \dot{\text{Pitch}} \\ \ddot{\text{Pitch}} \end{pmatrix}$$

Here Roll and Pitch represent the I-derivatives and Roll and Pitch represent the II-derivatives [9].

4.4.4 Derivation of Matrices

System Matrix (H):

For our filtering problem the inputs are roll and pitch angles with their I and II derivatives i.e.,

Inputs = {Roll, Pitch, Roll, Pitch, Roll, pitch}

and the outputs are the filtered roll and pitch angle values

Outputs = {Roll 1, Pitch 1}

$$\therefore H = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

State Transition Matrix (Q):

For our case, we assumed velocity for a moving body to be constant and hence

$$\text{Roll}_k = \text{Roll}_{k-1} + \dot{\text{Roll}}_{k-1} \tau$$

$$\dot{\text{Roll}}_k = \dot{\text{Roll}}_{k-1}$$

$$\text{Pitch}_k = \text{Pitch}_{k-1} + \dot{\text{Pitch}}_{k-1} \tau$$

$$\dot{\text{Pitch}}_k = \dot{\text{Pitch}}_{k-1}$$

$$\begin{pmatrix} \text{Roll} \\ \dot{\text{Roll}} \\ \text{Pitch} \\ \dot{\text{Pitch}} \end{pmatrix}_k = \begin{pmatrix} 1 & \tau & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \tau \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \text{Roll} \\ \dot{\text{Roll}} \\ \text{Pitch} \\ \dot{\text{Pitch}} \end{pmatrix}_{k-1}$$

Measurement Noise Covariance:

The Measurement noise covariance R is given by,

$$\begin{aligned} R &= E [V_k V_k^T] \\ &= V_k V_k^T \\ &= \begin{pmatrix} 0.22/6 \\ 0 \end{pmatrix} [0.22/6 \ 0] \end{aligned}$$

$$R = \begin{pmatrix} 0.0013 & 0 \\ 0 & 0.0013 \end{pmatrix}$$

4.5 Divergence

The performance of the kalman filter under actual operating conditions can be seriously degraded from the theoretical performance indicated by the state covariance matrix. The magnitude of the estimation errors as measured by the determinant of the estimation error covariance matrix is monotonically decreasing function of a number of observations. The errors in fact increase continuously although additional data are being processed. The possibility of such errors is called divergence.

There are several approaches for preventing filter divergence. Divergence occurs when the filter assigns too small a weight to the last measured data. Thus the current data make only a small correction in the

estimate, so small in fact that errors actually grow because of the natural interaction of position and velocity errors. An obvious fix is to more or less arbitrarily increase or decrease the weighting of current data. One such fix involves increasing the state covariance matrix while holding the orbital period uncertainty constant. The frequency and amount of increase must be determined empirically and then it is employed in Kalman filter [6].

In order to prevent divergence in Kalman filter, several techniques are used and the most common method is filter tuning.

4.6 FILTER TUNING

The most difficult portion of our project is tuning the Kalman filter. On deriving the equation for Kalman filter a lot of assumptions such as non-linearity, unmodelled forces etc. are made. All these assumptions are dumped into two constants: P_k the covariance of the predicted state error and Q_k state noise covariance matrix. The constants P_k and Q_k are adjusted to make the filter track perfectly and this process is called tuning. All procedures for Kalman filter tuning are kept as secret. So random changes are made each time to see whether the filter is tracking and these changes have to be done until perfect tracking is observed.

Tuning is done with initial values of P_k and Q_k less than one. These constants are adjusted simultaneously and at each change the output is compared with previous output. If the increase or decrease in P_k or Q_k results in better performance of the filter, then adjustment is made in the same direction. This process is repeated until perfect tracking is obtained.

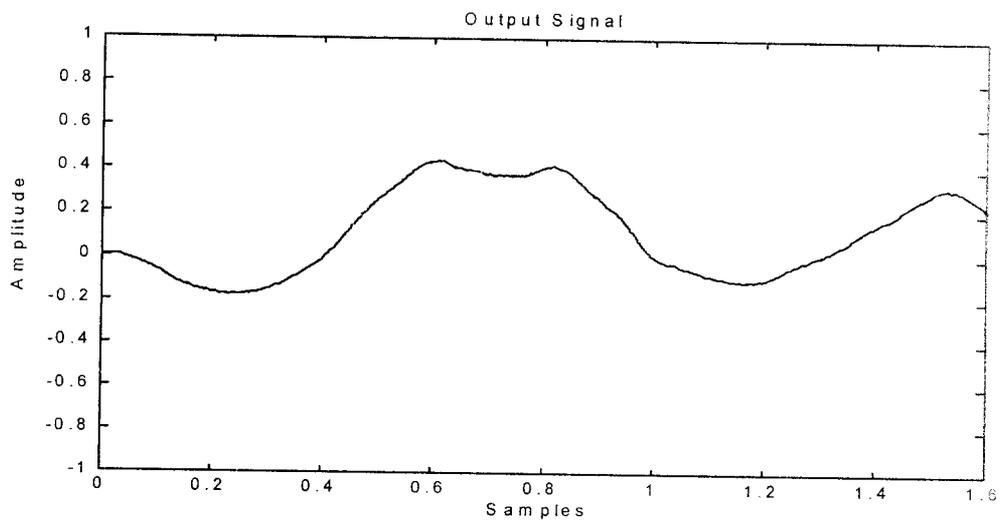
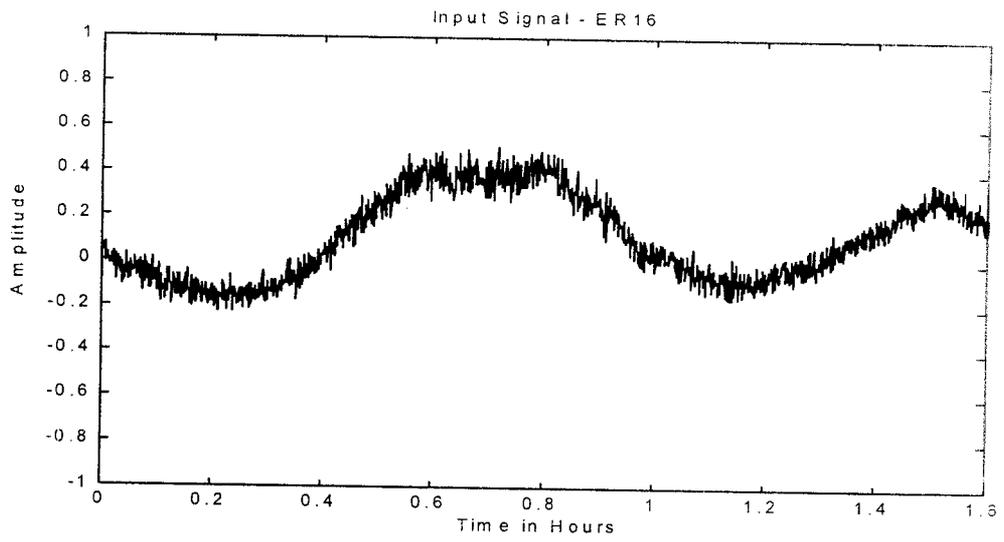
4.7 Filter Performance

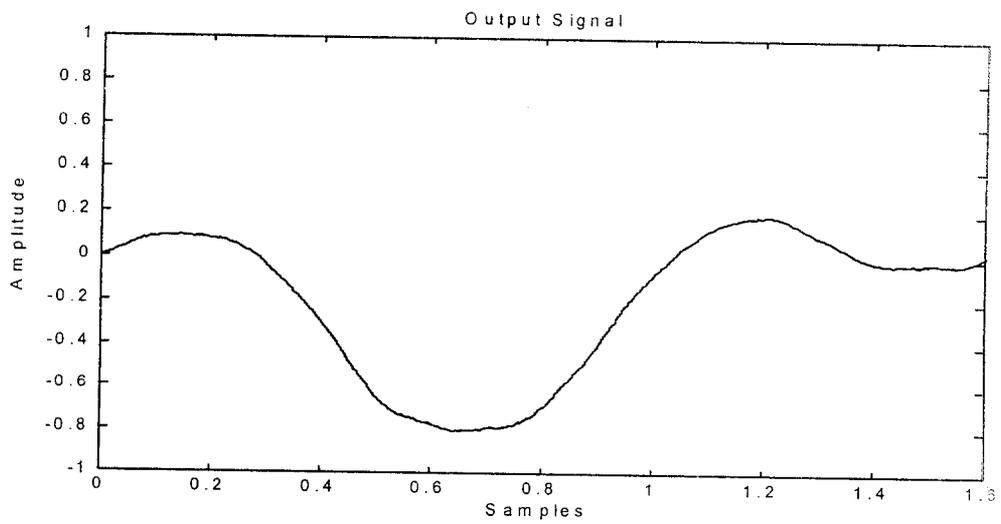
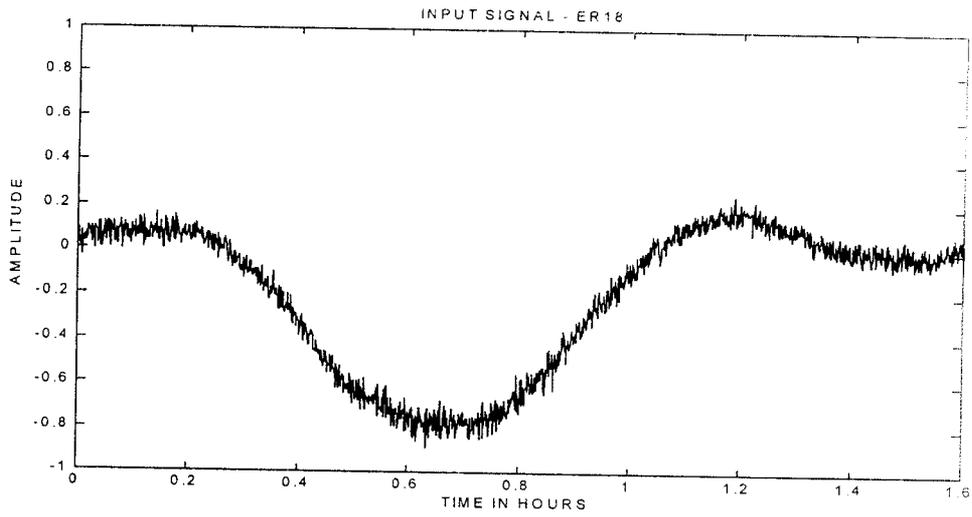
The performance of the Kalman filter is analyzed in terms of covariance of the error in estimated state, Kalman gain and stabilization. The value of covariance of error in estimated state (P_k) after tuning is in the order of 10^{-8} . The noise is reduced by a factor of 100. This can be inferred from the measurement noise covariance matrix (R_k) and final value of the covariance of the error in estimated state.

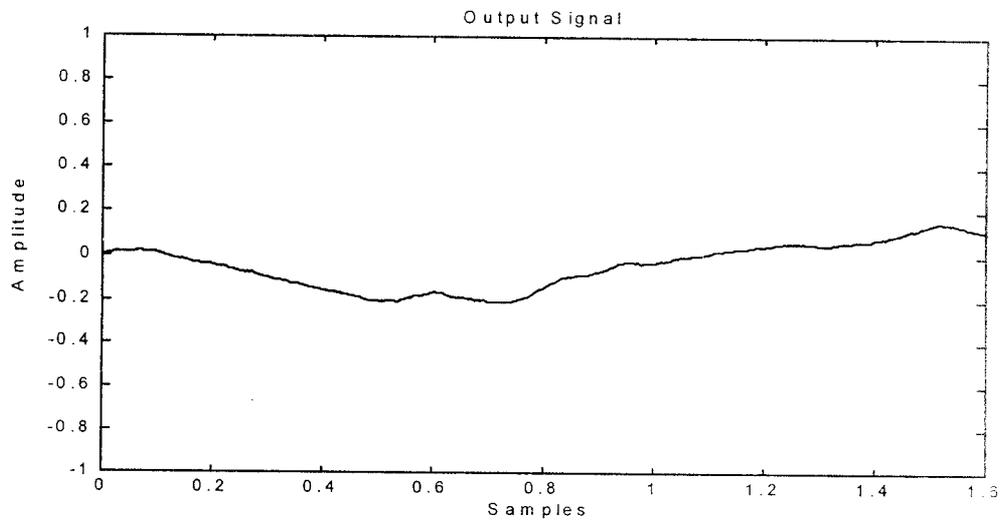
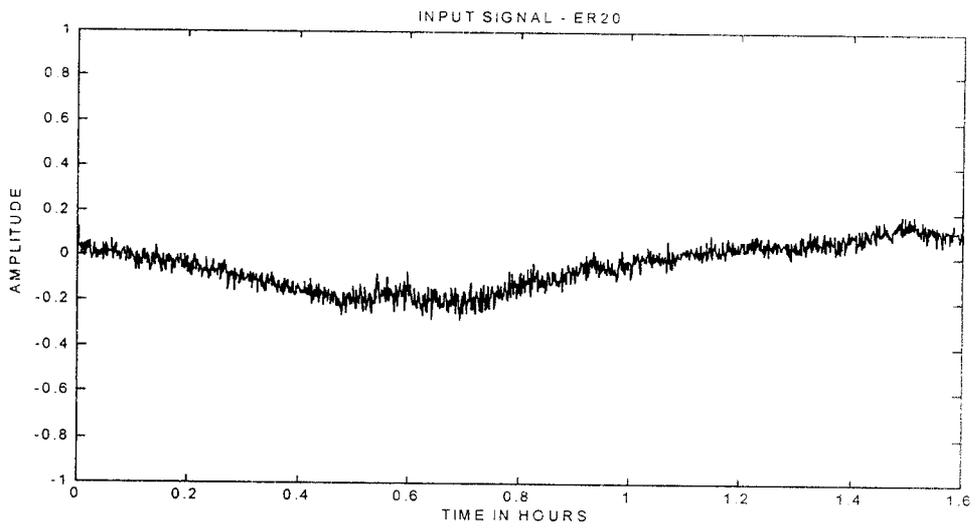
Order of measurement noise covariance matrix (R_k) is 10^{-3}

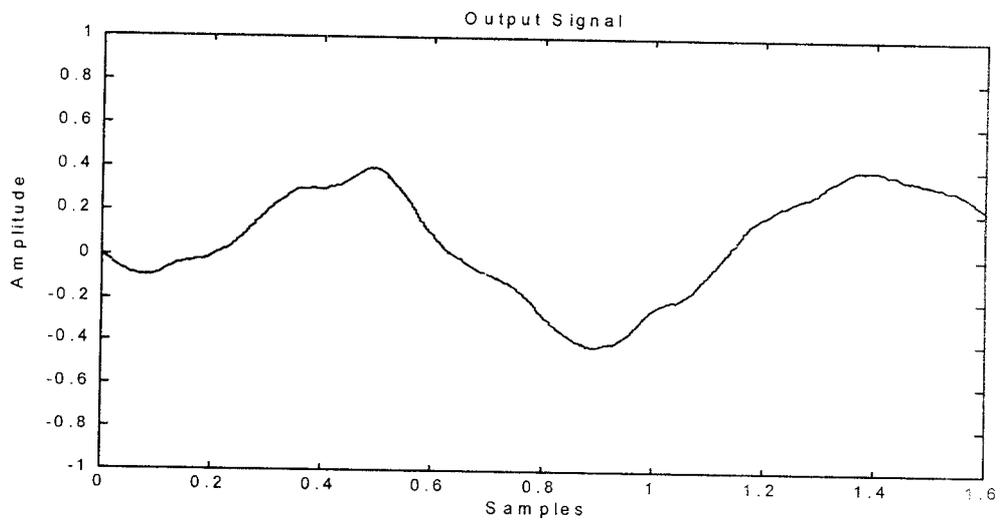
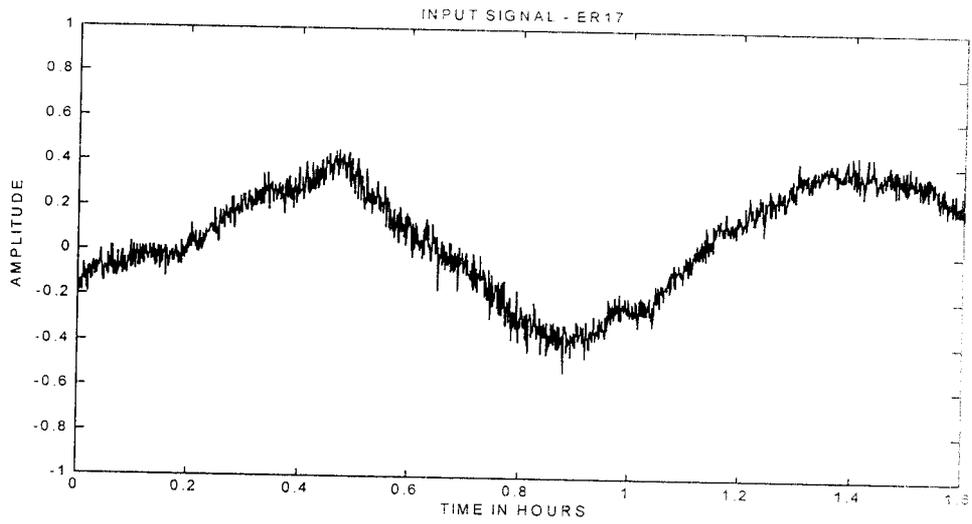
Order of final value of covariance of the error in estimated state (P_k) is 10^{-5}

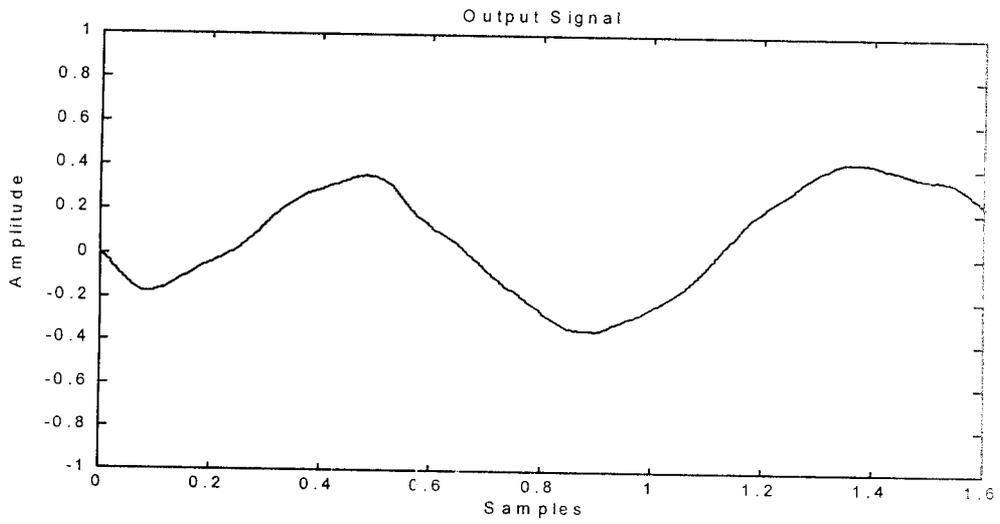
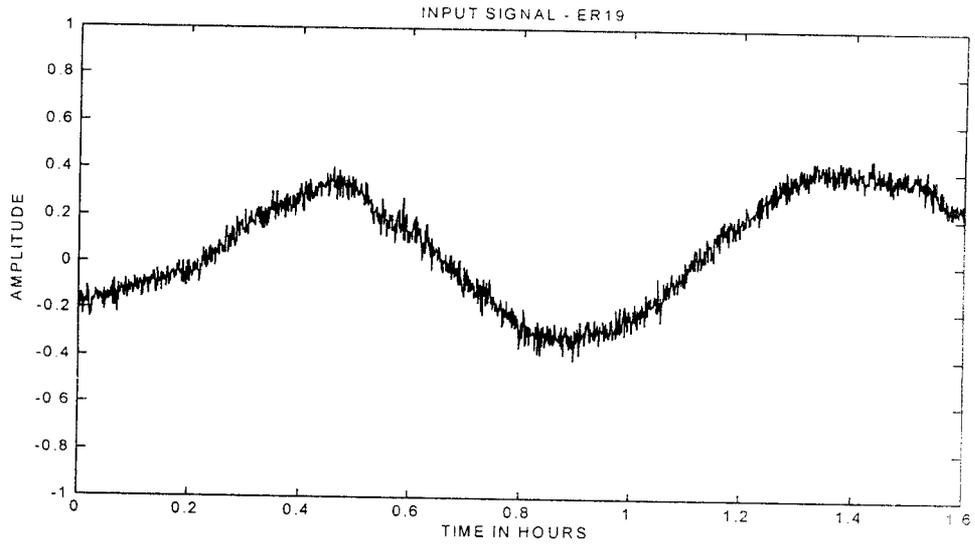
The Kalman gain is constant after 250 samples and from this period the filter parameters are constant.

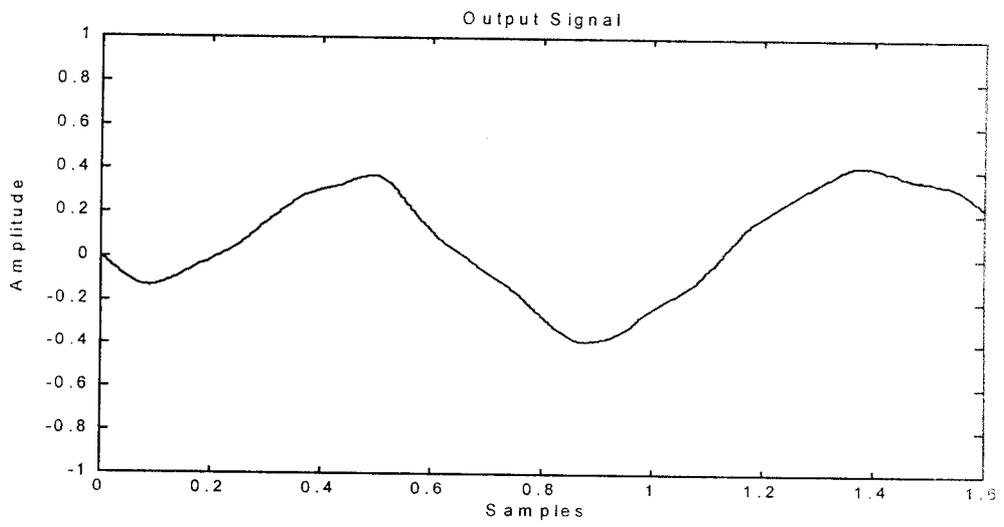
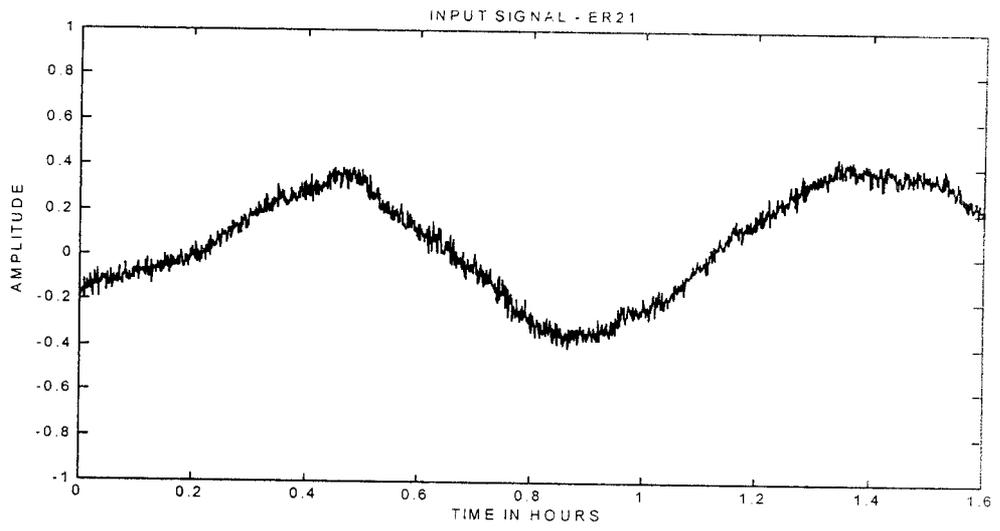












The noise present in the two orientation angles of the Indian Remote Sensing Satellites is filtered using the following two techniques

- 1) Spectrum Analysis
- 2) Kalman Filter

On examining the filtered signals obtained from the above techniques we see that both of them give similar outputs.

From the observation it is clear that the Spectral Analysis technique operating in frequency domain requires processing of spectrum to get the filtered output and hence it cannot be used for real-time applications whereas Kalman filter operating in time domain gives the output immediately after it gets the input processed by the filter equations and hence it is used for real-time applications.

The programs has been proposed to be used at ISTRAC

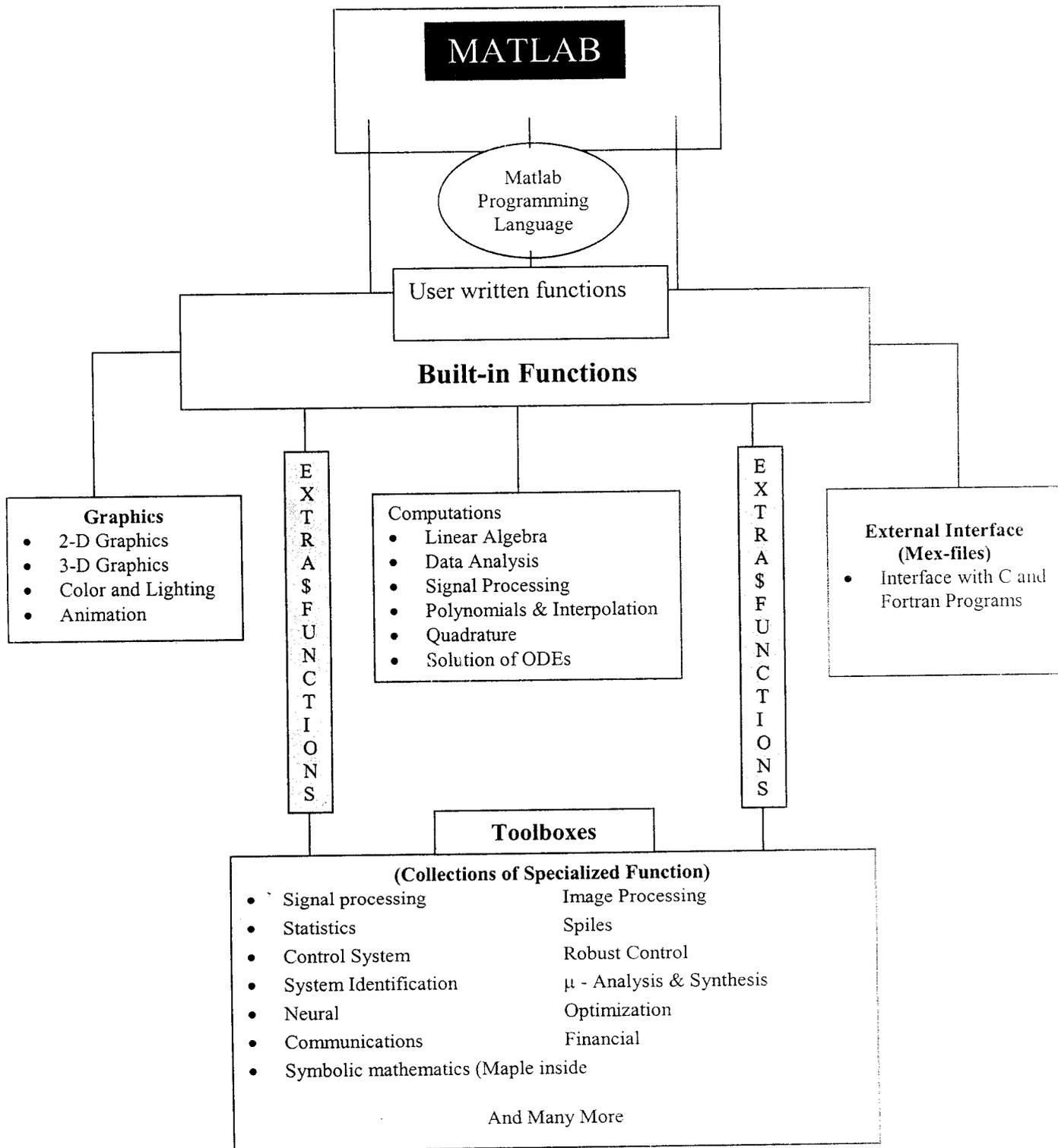
- 1) To understand the camera pointing of the satellite.
- 2) To compare and validate the measurements of onboard gyroscopes to understand the gyro drift.

MATLABTM is a software package for high-performance numerical computation and visualization. It provides an interactive environment with hundreds of built-in functions for technical computation, graphics, and animation. Best of all, it also provides easy extensibility with its own high-level programming language. The name MATLAB stands for MATrix LABoratory.

The diagram in Fig.1.1 shows the main features and capabilities of MATLAB. MATLAB's built-in functions provide excellent tools for linear algebra computations, data analysis, signal processing, optimization, and numerical solution of ODEs, quadrature, and many other types of scientific computations. Most of these functions use state-of-the art algorithms. There are numerous functions for 2-D and #-D graphics as well as for animation. Also, for those who cannot do without their Fortran or C codes, MATLAB even provides an external interface to run those programs from within MATLAB. The user, however, is not limited to the built-in functions; he can write his own functions in the MATLAB language. Once written, these functions behave just like the built-in functions. MAT Lab's language is very easy to learn and to use.

There are also several optional 'Toolboxes' available from the developers of MATLAB. These Toolboxes are collections of functions written for special applications such as Symbolic Computation, Image Processing, statistics, Control system Design, and Neural Networks.

The basic building block of MATLAB is the matrix. The fundamental data-type is the array. Vectors, scalars, real matrices and complex matrices are all automatically handled as special cases of the basic data-type. What is more, you almost never have to declare the dimensions of a matrix. MATLAB simply loves matrices and matrix operations. The built-in functions are optimized for vector operations. Consequently, vectorized commands or codes run much faster in MATLAB.



COMMANDS USED

STEM

Discrete sequence or "stem" plot

STEM (Y) plots the data sequence Y as stems from the x-axis terminated with circles for the data value.

STEM (X, Y) plots the data sequence Y at the values specified in X.

PLOT

Linear plot

PLOT (X, Y) plots vector Y versus vector X. If X or Y is a matrix, then the vector is plotted versus the rows or columns of the matrix, whichever line up.

SUBPLOT

Create axes in tiled positions

SUBPLOT (m, n, p), or SUBPLOT (mnp), breaks the Figure window into an m-by-n matrix of small axes, selects the pth axes for the current plot, and returns the axis handle. The axes are counted along the top row of the Figure window, then the second row, etc.

AXIS

Control axis scaling and appearance

AXIS ([XMIN XMAX YMIN YMAX]) sets scaling for the x- and y-axes on the current plot.

TITLE

Graph title

TITLE ('text') adds text at the top of the current axis.

TITLE('text','Property1', PropertyValue1,'Property2', PropertyValue2...) sets the values of the specified properties of the title.

XLABEL

X-axis label

XLABEL ('text') adds text beside the X-axis on the current axis.

YLABEL

Y-axis label

YLABEL ('text') adds text beside the Y-axis on the current axis.

HOLD ON

Holds the current plot and all axis properties so that subsequent graphing commands add to the existing graph.

HOLD OFF

Returns to the default mode whereby PLOT commands erase the previous plots and reset all axis properties before drawing new plots.

ZOOM

Zoom in and out on a 2-D plot

ZOOM with no arguments toggles the zoom state.

ZOOM (FACTOR) zooms the current axis by FACTOR.

ZOOM RESET clears the zoom out point. When zoom is on, click the left mouse button to zoom in on the point under the mouse. Click the right mouse button to zoom out. Each time you click, the axes limits will be changed by a factor of 2 (in or out).

ZOOM ON turns zoom on for the current figure.

ZOOM OFF turns zoom off in the current figure.

ZOOM XON or ZOOM YON turns zoom on for the x or y axis only.

INPUT

Prompt for user input

`R = INPUT ('Comment')` gives the user the prompt in the text string and then waits for input from the keyboard. The input can be any MATLAB expression, which is evaluated, using the variables in the current workspace, and the result returned in `R`. If the user presses the return key without entering anything, `INPUT` returns an empty matrix.

`R = INPUT ('What is your name','s')` gives the prompt in the text string and waits for character string input. The typed input is not evaluated; the characters are simply returned as a MATLAB string.

FSCANF

Read formatted data from file

`[A, COUNT] = FSCANF (FID, FORMAT, SIZE)` reads data from the file specified by file identifier `FID`, converts it according to the specified `FORMAT` string, and returns it in matrix `A`. `COUNT` is an optional output argument that returns the number of elements successfully read.

`FID` is an integer file identifier obtained from `FOPEN`.

FOPEN

Open file

`FID = FOPEN (FILENAME, PERMISSION)` opens the specified file with the specified `PERMISSION`. If the file is opened for reading and it is not found in the current working directory, `FOPEN` searches down the MATLAB's search path.

FCLOSE

Close file

ST = FCLOSE (FID) closes the file with file identifier FID obtained from an earlier FOPEN. FCLOSE returns 0 if successful and -1 if not.

ST = FCLOSE ('all') closes all open files, except 0, 1 and 2.

FPRINTF

Write formatted data to file.

COUNT = FPRINTF (FID, FORMAT, A...) formats the data in the real part of matrix A (and in any additional matrix arguments), under control of the specified FORMAT string, and writes it to the file associated with file identifier FID. COUNT is the number of bytes successfully written. FID is an integer file identifier obtained from FOPEN. It can also be 1 for standard output (the screen) or 2 for standard error.

FFT

Fast Fourier Transform

FFT (X) is the discrete Fourier transform (DFT) of vector X. If the length of X is a power of two, a fast radix-2 fast-Fourier transform algorithm is used. If the length of X is not a power of two, a slower non-power-of-two algorithm is employed. For matrices, the FFT operation is applied to each column. For N-D arrays, the FFT operation operates on the first non-singleton dimension.

IFFT

Inverse discrete Fourier transform

IFFT (X) is the inverse discrete Fourier transform of X.

IFFT (X, N) is the N-point inverse transform.

1. **John G. Proakis and Dimitris G. Manolakis**, “Digital Signal Processing – principles, algorithms and applications”.
2. **Mc Clellan, Schafer and Yoder**, “DSP First”.
3. **Srinivasa .H and Kesavaraju .V**, “Determination of drift rates of IRS-1A by applying Fourier transform synthesis on errors data”.
4. **Sanjit K. Mitra**, “Digital Signal Processing, a Computer based approach”.
5. **H. W. Sorenson**, “Least-squares estimation from Gauss to Kalman”.
6. **H. W. Sorenson**, “Kalman Filtering, Theory and Application”.
7. **K. V. Ramachandra**, “Estimation of optimum steady- state position, velocity and acceleration from the noisy sampled position data”.
8. **RudraPratap**, “ Getting started with MATLAB 5”.
9. ISRO Reference for Kalman Filter.
10. MATLAB user’s manual.