TRANSIENT ANALYSIS OF STATIC SCHERBIUS DRIVE

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Certificate

This is the Bonafide Record of the Project titled

Transient Analysis of Static Scherbius Drive

done by

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-THE CLASS

synopsis

SYNOPSIS

Numerical solutions of equations describing the transient performance of a scherbius drive system are investigated using generalised machine theory approach in the synchronously rotating reference frame. A digital computer program is developed for the numerical evaluation of the current, speed and torque following a disturbance in the system. The components were analysed using the Runge Kutta fourth order method.

nomenclature

NOMENCLATURE

R_s	-	Stator resistance
X_s	-	Stator reactance
Rr	-	Rotor resistance referred to stator
Xr	-	Rotor reactance referred to stator
R_f	-	Resistance of the filter circuit
X_f	-	Reactance of the filter
X_{12}	-	Mutual Reactance between stator and rotor referred to stator
L_f	-	Filter Inductance
T_e	-	Electro-Magnetic torque
T_L	-	Load Torque
I_{ds}	-	Steady state direct axis stator current
i_{ds}	-	Instantaneous direct axis stator current
I_{qs}	-	Steady state quadrature axis stator current
i_{qs}	-	Instantaneous quadrature axis stator current
I_{dr}	-	Steady state direct axis rotor current (Reffered to Stator.)
i_{dr}	-	Instantaneous direct axis rotor current (Reffered to Stator.)
I_{qr}	-	Steady state quadrature axis rotor current (Reffered to Stator.)
i_{qr}	-	Instantaneous quadrature axis rotor current (Reffered to Stator.)
V_{ds}	-	Steady state direct axis stator Voltage
v_{ds}	-	Instantaneous direct axis stator voltage
V_{qs}	-	Steady state quadrature axis stator voltage
v_{qs}	-	Instantaneous quadrature axis stator voltage
V_{dr}	-	Steady state direct axis rotor Voltage (Reffered to Stator)

V_{qr}	-	Steady state quadrature axis Rotor Voltage (Reffered to Stator.)
v_{qr}	-	Instantaneous quadrature axis Rotor Voltage (Reffered to Stator.)
V_{sm}	-	Stator line to neutral peak Voltage
V_{rm}	-	Rotor neutral to line peak Voltage referred to Stator
V_I	-	D.C. Input inverter voltage
V_R	-	Rectifier Output Voltage
I_{dc}	-	Rectifier current
m	-	Mutual Inductance between phase
M	-	Mutual Inductance
S	-	Slip
p	-	- d/dt - (differential operator)
P	-	No. of poles
ω	-	Electrical angular Velocity of the fundamental component of applied Voltage
ω_s	-	Slip frequency
ω_r	-	Electrical angular Velocity of Rotor
ω_e	-	Electrical Synchronous speed of Rotor
[Z]	-	Impedance Matrix
[X]	-	Reactance Matrix
H	-	Inertia constant
D	-	Damping constant
f	-	Supply Frequency in Hertz
N	-	Speed of the machine in r.p.m.
ϕ	-	Flux per pole

Super Script

-1	-	Inverse of the Matrix
T	-	Transpose of a Matrix

Subscripts

a, b, c	-	Stator line Values
d, q	-	direct and quadrature axis quantity respectively
s, r	-	Stator and rotor quantity respectively
A, B, C	-	Rotor line values

Greek Letters :

α	-	Triggering delay angle of the Thyristor
π	-	3.14159
μ	-	defined in the text
ψ	-	Flux linkage

Subscript

P	-	Notation for direct and quadrature axis Column Matrix

contents

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introduction

CHAPTER - I

INTRODUCTION

Static slip recovery scheme is a popular variable speed drive mainly due to its low cost, simple control circuitry and comparatively high efficiency.

During sudden changes in the operating conditions the drive experiences a transient condition during which the torque may reach high values imposing undue strain on the mechanical parts.

The most important transients from the electro-mechanical view point are - Currents, Speed, and Electro-mechanical torque.

These transients occur mainly due to :

i) Sudden applications of Voltages or Switching transients

ii) Sudden application of load

iii) Sudden increase and decrease in the applied Voltage

iv) Sudden increase and decrease of the firing angle of the inverter.

The aim of this project is the theoretical investigation of these transients.

The whole project has been divided into three steps.

(1) Preliminary discussion on various types of speed control of 3 phase slip ring induction motor.

speed control in electric drives

CHAPTER - II

SPEED CONTROL OF ELECTRIC DRIVES

2.1 Speed Control of 3 Phase Slip Ring Induction Motor[2]

All alternating current motors without commutators suffer from the disadvantage that they are fundamentally single speed machines, because the magnetic field rotates at synchronous speed. The economic control of the speed of induction motors is, in consequence, much more difficult than that of direct current motors. The various methods of speed control in common use are as follows :

2.2 Rheostatic Speed Control :

This can be applied only to motors with wound rotors. Resistance is included in the rotor circuit, the speed depending on the amount of additional resistance per phase. We know the percentage slip is equal to the percentage rotor copper loss. If the motor is working at a constant torque, the current will be sensibly constant and the drop in speed below synchronism will be proportional to the extra resistance per phase. We thus see that the drop in speed is proportional to the power dissipated in the rheostat, the method therefore being very wasteful if speeds much below synchronism are required. Thus, if the speed is half synchronous speed, half the power supplied to the motor will be wasted in the rheostat. A further disadvantage of the method is that the speed is a function of the external resistance only so long as the load torque remains constant. Consider the two torque/speed characteristics in Fig.1. Curve 1

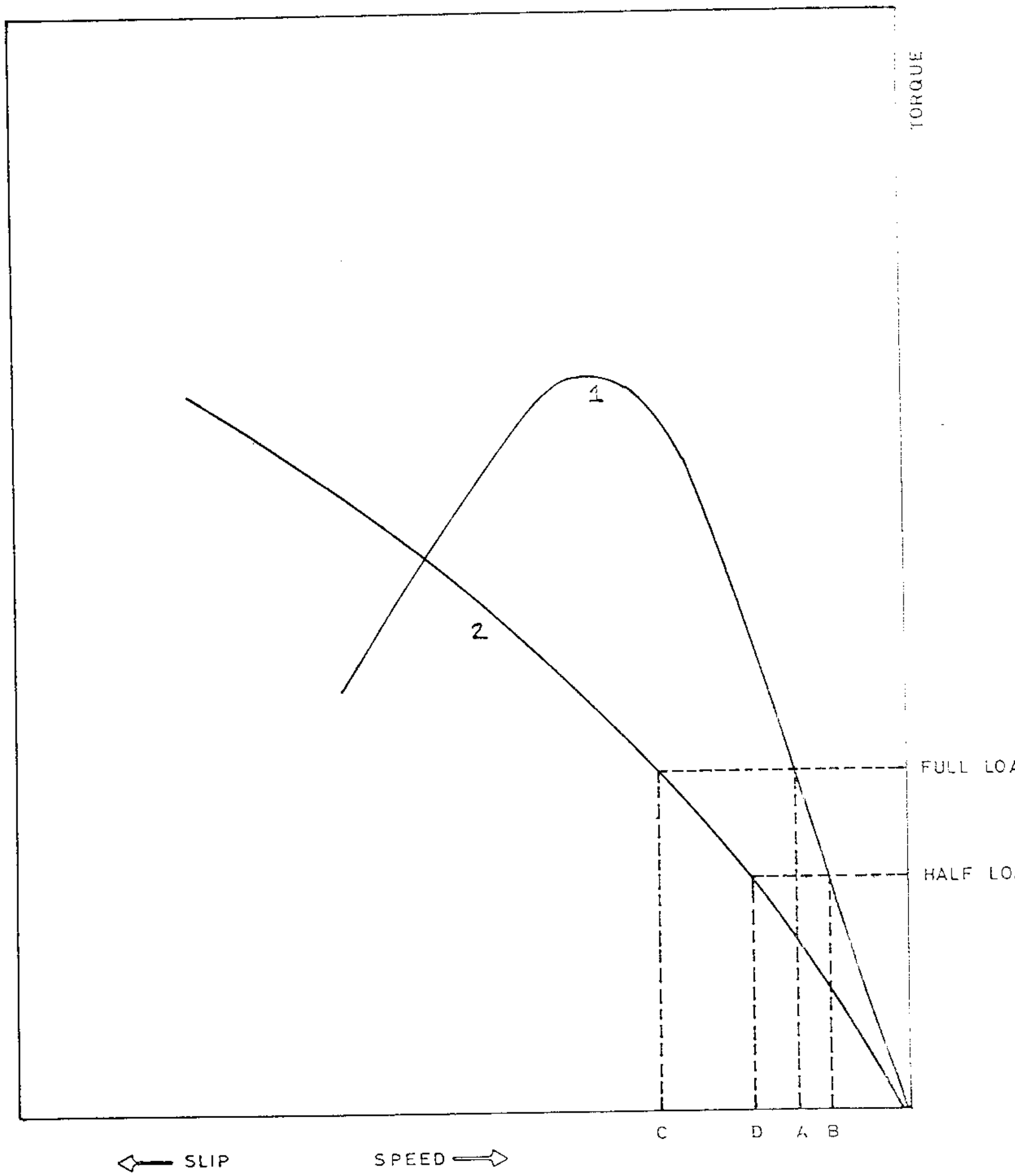


FIG.1. SPEED VARIATIONS WITH CHANGE IN LOAD

compounds to a motor running with the slip-rings short circuited. If the torque falls from, say, full-load to half-load, the change of speed will be AB. If so much resistance is added that the torque/speed characteristic is represented by curve 2, then for the same change in torque the change in speed is CD, which is very much greater than AB.

For very large motors, resistance controllers are almost invariably of the liquid type, but for small and medium size machines metallic resistances with a controller of the bassel type are common.

A starter is in circuit for a short time, generally much less than one minute, and therefore its own thermal capacity will absorb much of the heat generated, a proportion only being radiated. A controller, however, may have some of its resistance in circuit for considerable periods, and so, when equilibrium is reached, all the heat generated has to be dissipated by radiation or some other means, such as an air current in the case of metallic resistances, or water cooling of a liquid resistance.

The close resemblance between the induction motor and the d.c. shunt motor leads to the conclusion that rheostatic control of speed should be possible. This conclusion is in fact supported by the analytical theory leading to the torque speed curves of a motor It will be evident that the variation of rotor resistance requires the use of a wound rotor connected through slip rings to an external rheostat.

With any given resistance in circuit with the rotor, operation will be stable only on those portions of the torque speed curves where the torque decreases as the speed increases.

2.3 Speed Control by Variation of Rotor Voltage :

This method which involves variation of rotor voltage is based on the concept that the energy which would otherwise be wasted in a control rheostat may be usefully returned to the system.

A reversed control voltage represents an additional draft of energy from the supply circuit which can then result in speeds above synchronism.

2.4 The Schrage Brush-Shift Adjustable-Speed Motor :

K.H. Schrage invented this motor. This is a small size induction motor. The special feature is that flexible speed control without the auxiliary devices.

These motors are designed for three phase 50 cycle circuits at 220 and 440V, though two phase motors are available.

Speed Adjustment by Pole-Changing :

The synchronous speed of an induction motor is given by the formula,

$$N = \frac{120.f}{p}$$

Long ago suggested the possibility of obtaining more than one machine speed by providing the stator with windings so arranged that by suitable switching the number of poles could be changed.

2.5 Speed Control by Change of Frequency :

The speed formula N = 120.f/p, indicates the possibility of controlling the speed of an induction mortor by changing the frequency

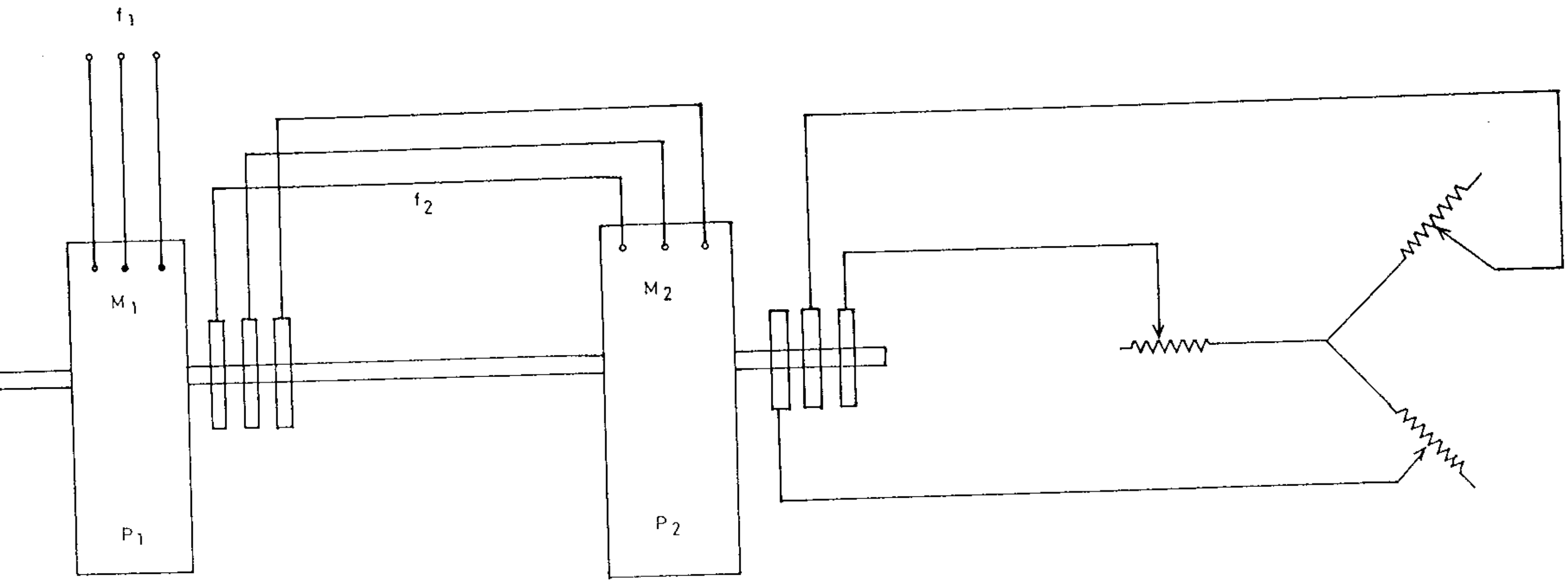


FIG. 2. CONCATENATION OF TWO INDUCTION MOTORS

of the supply circuit; but in as much as this method requires a separate variable speed synchronous alternator for each motor to be independently controlled, it has been applied only in the case of ship propulsion, where each of the driving motors, direct connected to a propèllor shaft, is supplied by its own turbine driven generator.

2.6 Concatenation, or Tandem Control of Speed :

As shown in Fig. 2 two wound rotor induction motors are mechanically coupled either directly or through gears and are electrically connected in concatenation (sometimes referred to as "in tandum" or "in Cascade").

slip recovery schemes

CHAPTER - III

SLIP POWER RECOVERY SCHEMES[3]

The speed of an induction motor can be controlled by controlling its slip. However to simple voltage schemes in which slip power is dissipated in machine rotor, slip recovery schemes converts slip power to D.C. power which is then; inverted and returned to A.C. supply to to implement this technique the motor requires slip rings. This basic control technique is known as Kramer or Scherbius Drive.

3.1 The Conventional Kramer System :

This method utilises a commutator machines, direct connected to the main motor. The slip frequency impressed upon the stator winding of the auxillary machine develops a magnetic field rotating in space at the corresponding slip speed and because of the presence of the commutator the rotor end and current will likewise have slip frequency regardless of the actual speed of the shaft.

Adjustment of the regulating transformer controls the magnitude of the rotor emf, hence also the speed of the main motor.

At speeds below synchronism, the surplus energy in the rotor of the main motor is absorbed by the auxillary machine acting as a motor, thereby causing the auxillary machine to absorb part of the mechanical load and to that extent relieving the main motor.

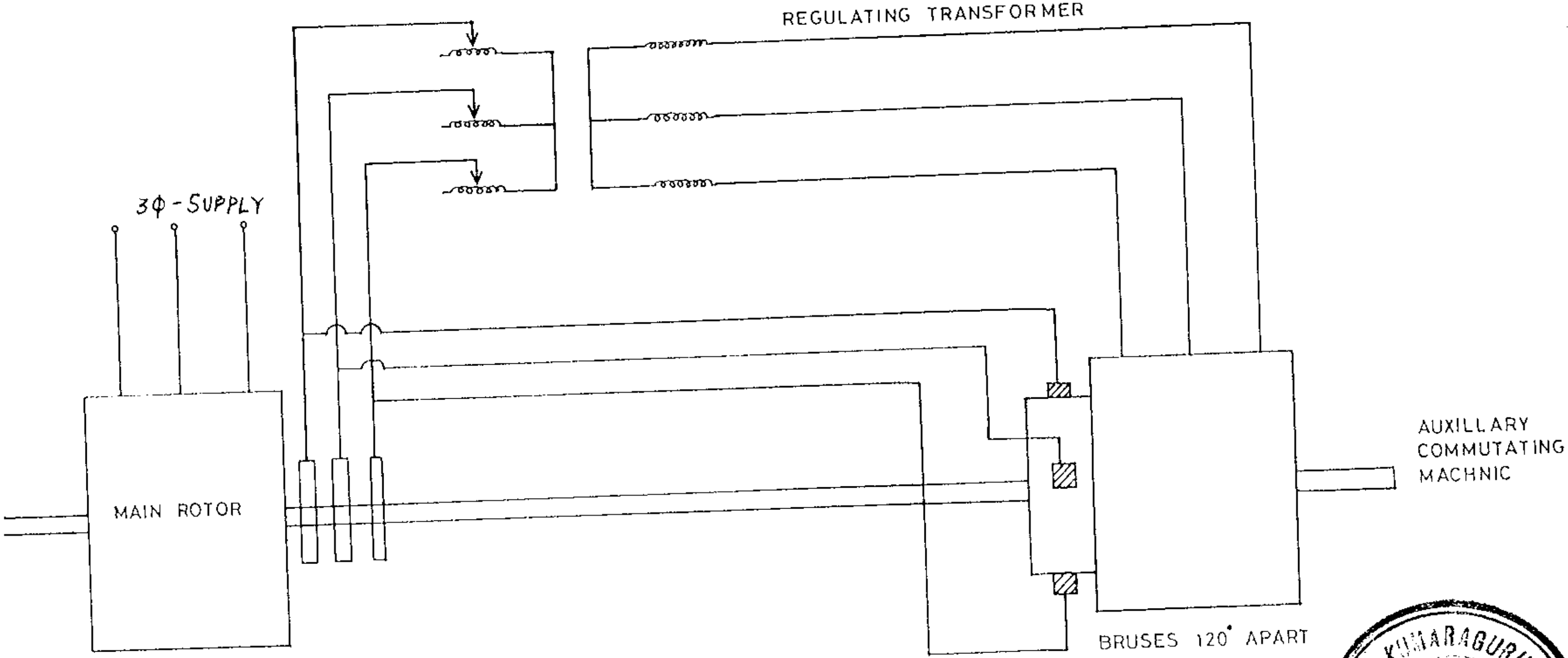


FIG. 3. KRAMER SYSTEM OF SPEED CONTROL

The size of the auxillary machine is determined by the amount speed adjustment required; for example, if the speed of the set is to be reduced x percent below synchronism, its rating must be x per cent of that of the main motor.

Theoretically, this arrangement permits power factor control by shifting the brushes of the commutating machine, but practically this feature is restricted by the circumstance that commutation difficulties limit the possible range of range of brush shift. It is possible to overcome this difficulty by means of commutating fields or by speacial phase combinations in the machine itself. The chief disadvantage of this system is that the commutated machine must be designed for the same speed as the main motor, it becomes practically impossible to design a correspondingly high speed commutating machine.

3.2 The Conventional Scherbius System :

The commutating machine, excited at slip frequency from the rotor of the main motor, develops a brush voltage of slip frequency which is injected into the rotor circuit of the main motor and so serves to regulate the speed of the latter. The commutating machine is direct connected to an induction motor supplied from the main line, so that its speed departs from a fixed value only to the extent of the slip of the auxillary induction machine.

Adjustment of the taps of the regulating transformer varies the excitation of the commutating machine, thereby varying the voltage developed in its rotor and so controlling the speed of the main motor. If, for instance, the regulating transformer is so adjusted as to reduce the speed of the main motor, the surplus secondary energy of the latter drives the commutating machine as a motor, and the auxillary induction machine then becomes an induction generator which returns most of this surplus energy as electrical energy to the supply circuit. This should be contrasted with the action of the Kramer System, which returns the surplus energy in the form of mechanical energy to the load.

For any given setting of the regulating transformer, the speed of the main motor will remain substantially constant regardless of load variation. This can be shown in the following manner.

The voltage induced in the rotor of the main motor is SE, where E may be taken as substantially constant if the primary leakage impedance drop is small. If the leakage impedance drop in the rotor of the main motor is ignored, a voltage proportional to SE, and having slip frequency, is thus impressed upon the stator of the commutating machine, which then develops a magnetic field, Φ_c, rotating in space at a speed corresponding to the slip frequency S_f.

$$a\ (S.E) = 4.44\ K_{b1} \cdot K_{p1}\ (S_f)\ N.\Phi_c$$

where
- a - cross section
- K_{b1} - spread factor } of winding
- K_{p1} - pitch factor

If the small variation of speed of the regulating set is neglected, the voltage (of slip frequency) developed in the rotor of the commutating machine also remains nearly constant.

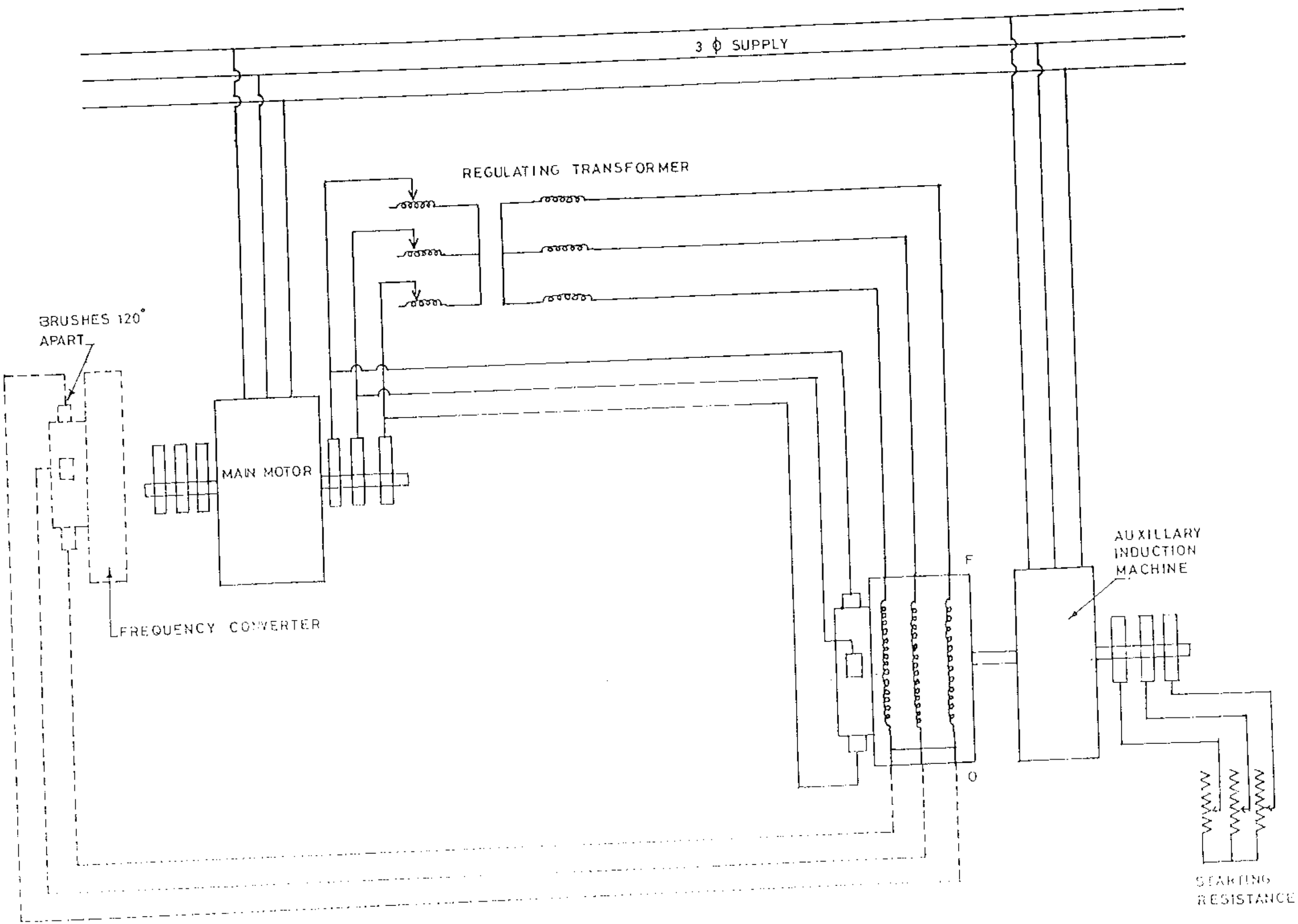


FIG. 4. SCHERBIUS SYSTEM OF SPEED CONTROL

For the sake of simplicity, the Fig.4 indicates only a single poly phase winding of shunt type on the stator of the commutating machine. In actual practice, because of the large magnitude of the rotor current which must flow through the brushes of this machine, it is necessary to provide neutralizing or compensating windings to overcome, the otherwise unduly large reactance drop.

When the circuit diagram has the form shown in full lines in figure the set is adapted to speed control in the range below synchronism.

3.3 Static Kramer Drive :

Instead of wasting the slip power in the rotor circuit resistance, it can be converted to 50 HZ ac and pumped back to the line through a thyristor converter cascade is known as a Static Kramer drive. The original Kramer Drive System used a rotary converter instead of a diode rectifier and fed power to a dc motor coupled to the same induction machine shaft. The slip power in this principle is converted to mechanical power, which contributes partially to the mechanical power output of the induction machine shaft.

The static kramer system is popular in large power pump and compressor type drives where the range of speed variation is usually limited. The drive system is not only efficient but the converter power rating is low, because it has to handle only the slip power. This power rating becomes lower for a more restricted speed range near the synchronous speed. The additional advantages are that the drive system has dc-machine like characteristics and the control circuit is simple.

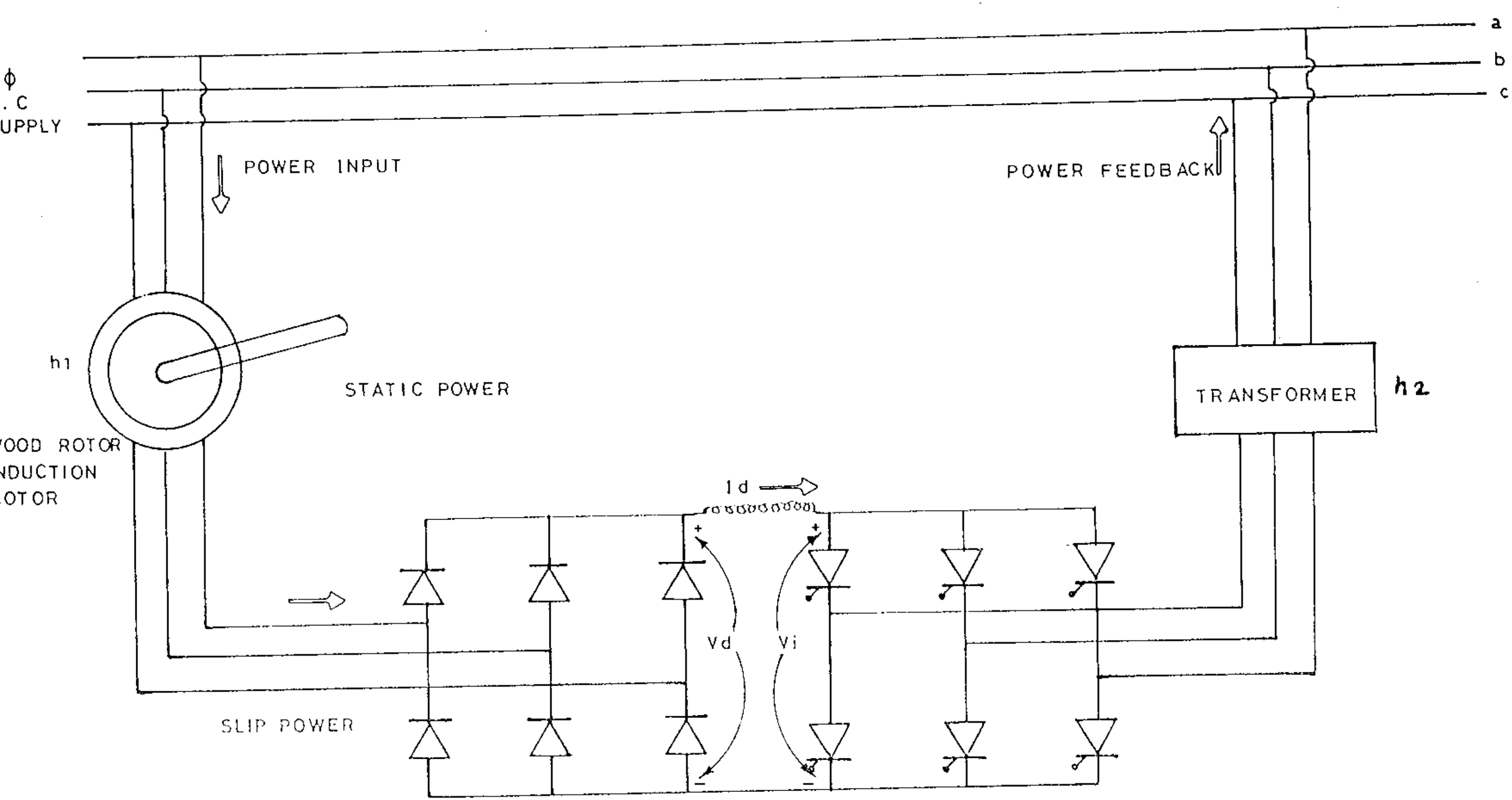


FIG.5. STATIC KRAMER DRIVE SYSTEM

The air gap flux of the machine is established by the stator supply and it remains practically constant if stator drops and supply voltage fluctuation are neglected. Ideally, the rotor current is a six-stepped wave in phase with the rotor phase voltage, if the dc link current I_{dc} is considered harmonic free, and commutation overlap angle of the diode rectifier is neglected.

In steady state operation, the rectified slip voltage V_R and the inverter voltage V_I will balance for a certain dc current I_{dc}.

The voltage V_R will be proportional to slip and the current I_{dc} will be proportional to torque. The simplified speed and torque expressions can be derived as follows. Neglecting the stator and rotor drops, the voltage V_R is given by

$$V_R = 1.35/n_1 \; S.V_{sm} \qquad \ldots (1)$$

where n_1 is the stator - to - rotor turns ratio of the machine,

V_{sm} the stator line voltage, and S the per unit slip.

The inverter terminal voltage V_I is given as

$$V_I = 1.35/n_2 \; . \; V_2 \,|\cos\alpha| \qquad \ldots (2)$$

where

n_2 is the transformer line side-to-inverter ac side turns ratio and α is the inverter firing angle, which is in the range 90° to 180°.

Since V_R and V_I must balance in the ideal case, equations (1) and (2) give

$$S = n_1/n_2 \,|\cos\alpha| \qquad \ldots (3)$$

$$\omega_r = \omega_e (1 - |\cos\alpha|) \quad \ldots (4)$$

assuming that $n_1/n_2 = 1$. Equation (3) indicates that ideally speed can be controlled between zero and synchronous speed by controlling the inverter firing angle alpha (α). At zero speed, the voltage V_R is maximum, which corresponds to angle $\alpha = 180°$, and at synchronous speed $V_R = 0$ when $\alpha = 90°$. Again neglecting losses, the following power equations can be written :

$$SP_g = V_I I_{dc} \quad \ldots (5)$$

$$Pm = (1 - S) Pg = Te.\omega n = Te.\omega e (1 - S) \quad \ldots (6)$$

where

Pg - is the air gap power

Pm - the mechanical output power.

Combining the equations (5) and (6)

$$Te = V_I I_{dc}/S \omega e \quad \ldots (7)$$

Substituting equations (2) and (3) gives

$$Te = 1.35 VL/\omega e.n_1 . I_{dc} \quad \ldots (8)$$

Which indicates that the torque is proportional to current I_{dc}

The drive system has nearly the characteristics of a separately excited d.c motor. The air gap flux is constant and the torque is proportional to current I_{dc}. With a higher load torque, I_{dc} will increase and for a fixed V_I, V_R should slightly increase to overcome the dc link drop, indicating a speed drop like a dc machine.

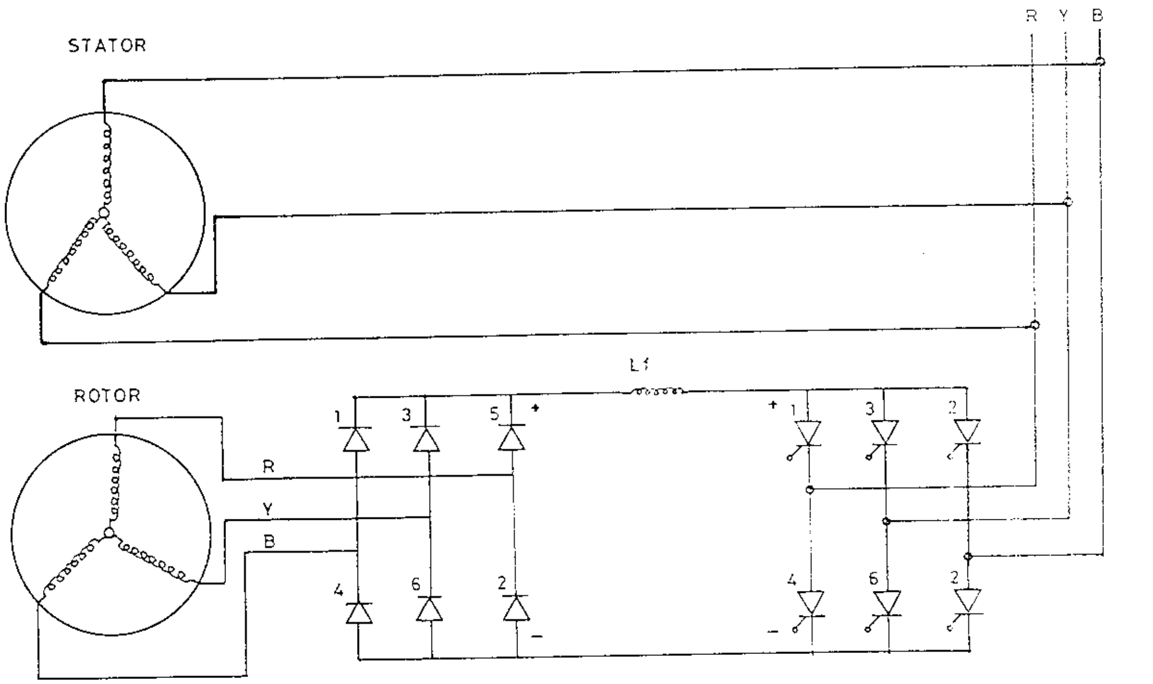
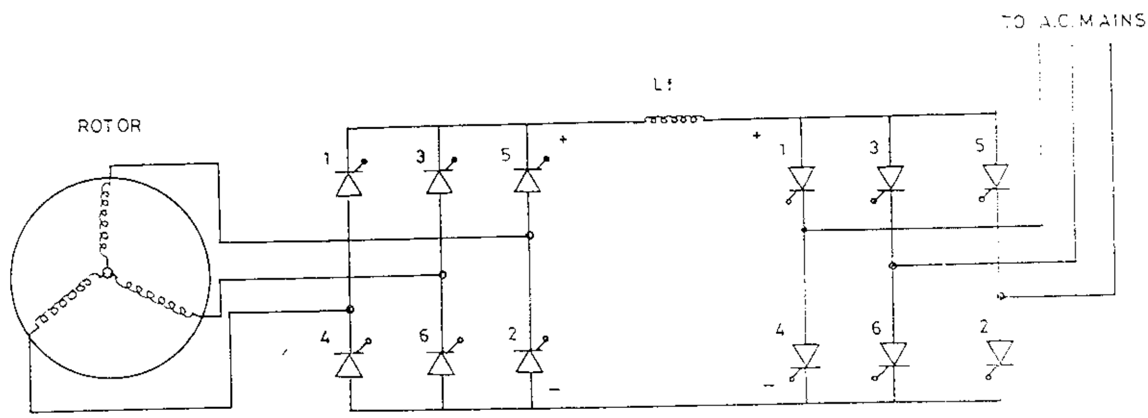
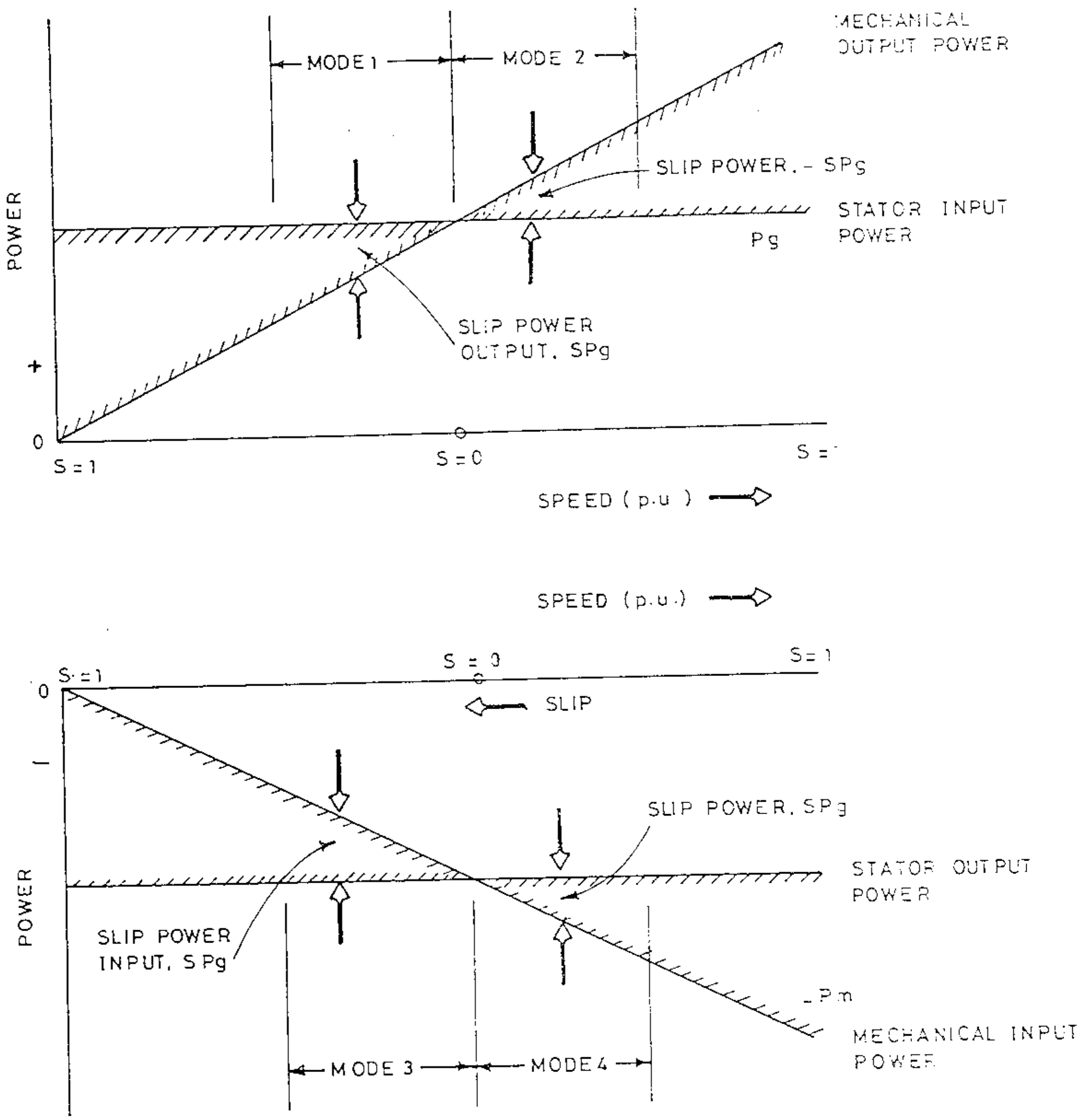


FIG. 6. STATIC SCHERBIUS DRIVE



A WIDE RANGE OF STATIC SCHERBIUS DRIVE



POWER DISTRIBUTION VERSES SLIP IN THE SUPER / SUB SYNCRONOO
SPEED RANGE

(a) MOTORING AT CONSTANT TORQUE (b) GENERATION AT CONSTANT
TORQUE

FIG.7. STATIC SCHERBIUS SYSTEM

3.4 Static Scherbius Drive :

The dual-converter system in a static kramer drive can be replaced by a single phase - controlled line - commutated cycloconverter, shows in Fig.6. The scheme is known as a static scherbius system drive and has found applications in very large horse-power pump and blower-type drives. The cycloconverter permits the slip power to flow in either direction, and therefore the machine speed can be controlled in both Sub-syochronous and Supersynchronous ranges with motoring and regeneration features.

The various modes of operation shown in Fig. 7 can be explained as follows. It is assumed that the machine shaft torque is constant and that losses in the machine and cycloconverter are negligible.

Mode 1 : Subsynchronous Motoring :

This mode is identical to that of the static Kramer System. The stator input or air gap power Pg remains constant and the slip power Spg, which is proportional to the slip. is returned back to the line. Therefore, the line supplies the net mechanical power Pm consumed by the shaft. The slip frequency power in the rotor creates a rotating field in the same direction as in the Stator and the rotor speed corresponds to the difference ($\omega_r = \omega_e - \omega_s$) between these two frequencies. At true synchronous speed (S = 0), the Cycloconverter supplies dc excitation to the rotor and the machine behaves like a synchronous motor.

The use of a Cycloconverter instead of a dual converter system means additional cost and complexity, but the resulting advantages are obvious. The problem of commutation near synchronous speed disappears and the near Sinusoidal current wave in the rotor substantially improve harmonic heating and torque pulsation effects. The line current waveform is improved correspondingly. The Cycloconverter is to be controlled so that the output frequency tracks precisely with the slip frequency. The drive system can be designed to operate within a fractional slip range about the synchronous speed. The Scherbius drive System does not permit speed reversal and requires a reversing contactor in the stator side for this function.

the generalised electric machine theory

CHAPTER - IV

THE GENERALISED ELECTRICAL MACHINE[1]

For the purpose of predetermining its performance under any given conditions an electrical machine may be regarded as an assemblage of windings characterised by parameters which are assumed to be constant. This assumption excludes the non-linear effects of Saturation and commutation, and subsequent corrections have to be made in the light of experience. The basic laws of Faraday, ohm and Kirchoff may then be used to formulate equations expressing the currents, voltages and flux linkages associated with the several windings, thereby leading to the complete determination of the operating characteristics.

4. 1. The Generalized Machine :

This process may be facilitated, especially in connection with automatic control systems, by adopting the concept of a generalised machine which, for the present purposes, is a basic structure comprising a cylindrical rotating member in association with a fixed field system having two salient poles. This depicts a generalisation of the basic commutator machine shown in figure (8), coil F representing the actual field winding and coil D representing the armature winding. The original machine will usually be of multipolar construction but, for the purpose of calculation, may be replaced by an equivalent 2 pole machine having the same winding parameters.

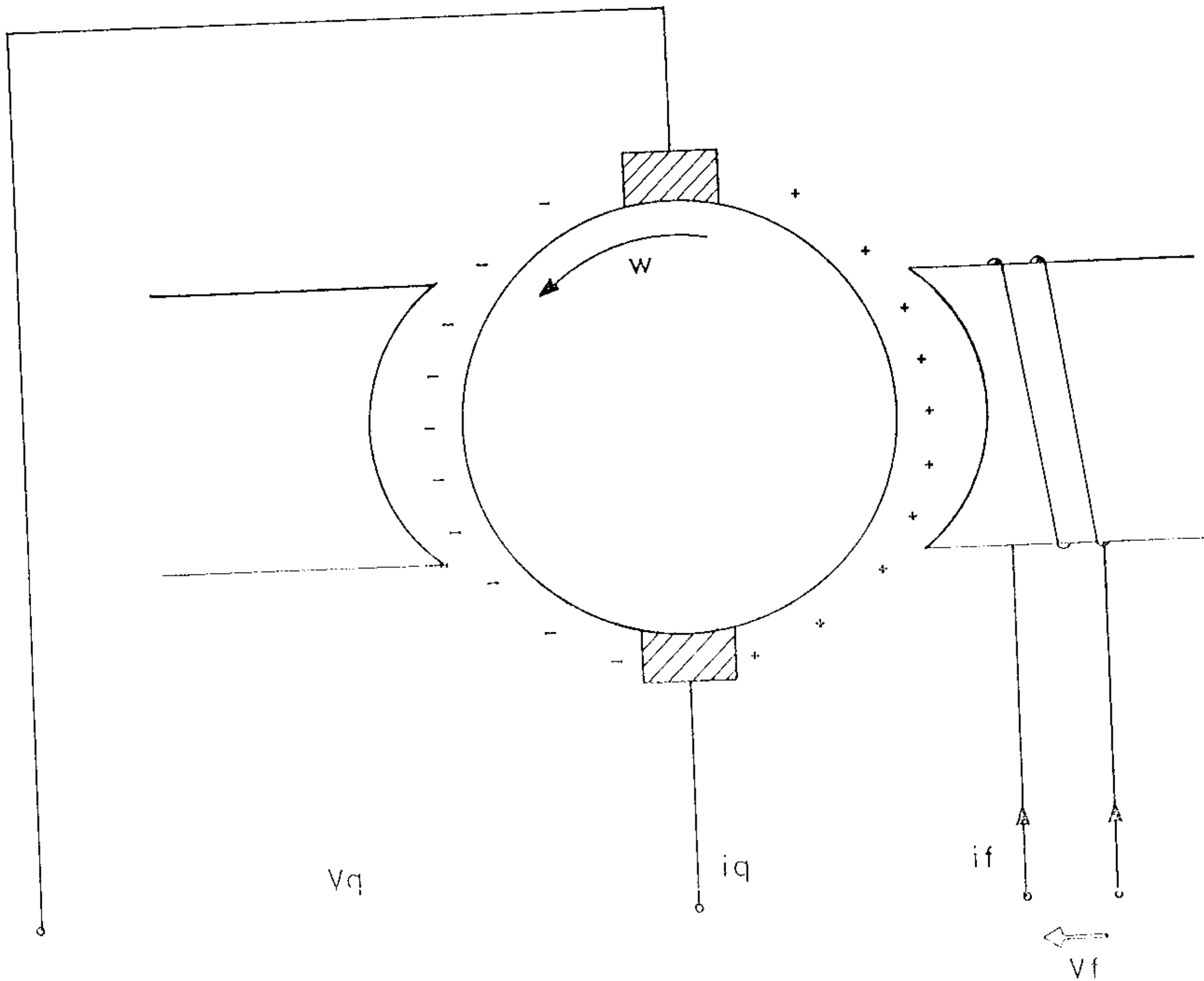


FIG.X

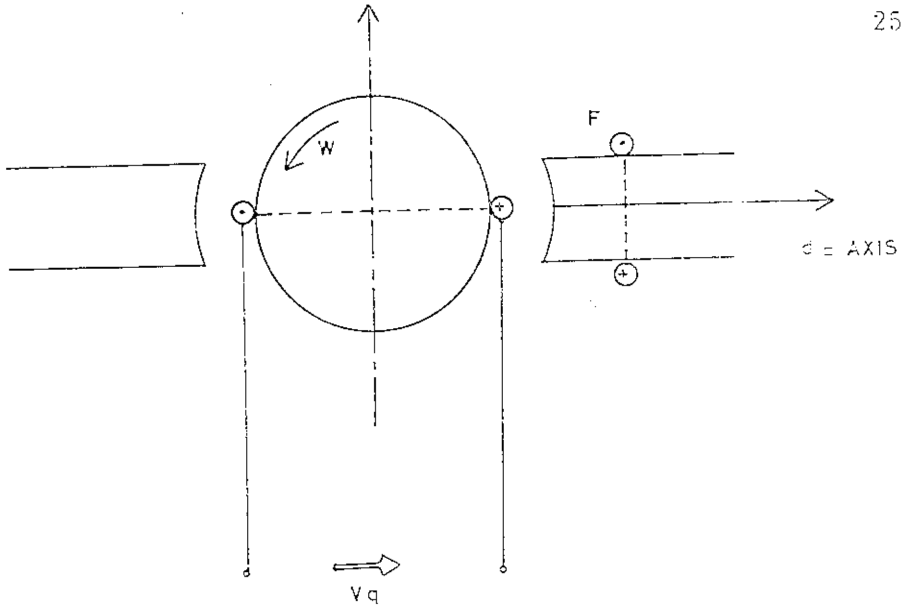


FIG. 9

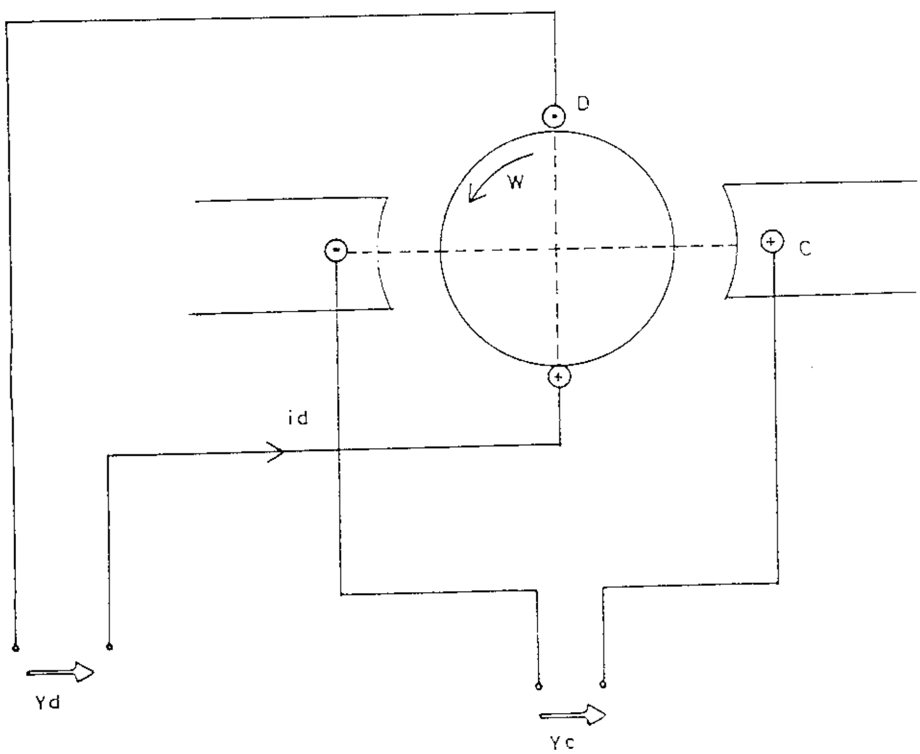


FIG. 10

Each actual machine winding is thus represented in the generalised machine by a single coil, the plane of which is normal to one or other of the two axes, as in figure (9). These are magnetic axes and are : (a) the direct axis or d.axis, which coincides with the axis of the field poles, and (b) the quadrature axis, or q-axis, which is perpendicular to the d.axis and therefore coincides with the brush axis of a commutator machine with brushes in the geometric neutral plane.

The equivalent coils (two only in this simple case) are thus grouped so as to magnetise along either the d axis or the q axis. Positive d axis flux is assumed to be directed horizontally to the right and positive q axis flux directed vertically upwards. The positive directions of current are thus determined, as shown, and assuming the positive direction of rotation to be counter clockwise, the right hand rule shows that the positive direction of the induced e.m.f. is in the direction of the positive current.

An external Voltage applied between any pair of terminals is defined to be positive when it acts in such a direction as to send positive direction current through the winding concerned.

A more general type of machine has a cross magnetizing winding placed in the pole faces, and also a second pair of brushes in the d axis. The corresponding modifications to the generalised machine are indicated in figure (10), in which coil D represents the directly magnetising effect of the armature winding, and coil C, acting on the q axis, represents the cross magnetising winding. The commutating winding exercises only a local effect in the air-gap, and although it increases the inductance ...ture circuit to some extent, it is not taken into account here.

The combined effects of coils D and F gives a flux ϕ_d centred on the d axis and assumed to be sinusoidally distributed round the air gap. Similarly coils Q and C produce a quadrature flux ϕ_2 also assumed to be sinusoidally distributed. All magnetic leakage effects are here ignored, in order to simplify the discussion.

E.M.F. Equations :

In general, when the machine is running and the fluxes are varying, an e.m.f. will be induced in Q by rotation of the flux C_d, and an e.m.f. will also be induced in Q by variations of the flux ϕ_q.

Similarly, these will be in coil D a rotational e.m.f. due to the flux ϕ_q and transformer e.m.f. due to the flux ϕ_d. These emf's may be expressed simply as follow :

If the armature winding has N_a turns, the flux linkage with a sinusoidally distributed field of flux O is

$$\Psi = \text{Average flux. linkage per turn x Turns in series between brushes}$$

$$= 2/\Pi \cdot \phi \cdot N_a/2$$

A rotational e.m.f. may therefore be expressed in the form

$$e_r = \phi \cdot Zn$$

$$= \phi \cdot 2 \cdot Na \cdot \omega/2\pi$$

$$= \omega\Psi \text{ in magnitue}$$

A transformer e.m.f. is given by

$$e_t = d\psi/dt$$

$$= p\psi \text{ in magnitue.}$$

Let the resistances of the four coils be r_d, r_q, r_f and r_c respectively, then, referring to figures (9) and (10), we have

$$v_q - p\,\Psi_a + \omega\Psi_d = r_q\, i_q$$

$$v_d - p\,\Psi_d - \omega\Psi_q = r_d\, i_d$$

The signs are due to the fact that the right hand rule show that a positive rotational e.m.f. acts in conjuction with v_q (as stated previously) but in opposition to v_d.

Now let L_d and L_q denote the self inductances of coils D and Q, and let M_{df}, M_{qc} denote the mutual inductances between coils D and F, and between coils Q and C, respectively. Then the flux linkage of coils D and Q, due to both self and mutual inductance, are given by

$$\Psi_d = L_d \cdot i_d + M_{df} \cdot i_f$$

$$\Psi_q = L_q \cdot i_q + M_{qc} \cdot i_c$$

On substituting these expressions we obtain

$$V_d = r_d \cdot i_d + p.\,L_d.i_d + p.M_{df}.i_f + \omega(L_q.i_q + M_{qc}.i_c)$$

$$V_q = r_q \cdot i_q + p.L_q.i_q + p.M_{qc}.i_c - \omega(L_d.i_d + M_{df}.i_f)$$

The corresponding equations for the field coils are

$$v_f = r_f \cdot i_f + p.\,L_f \cdot i_f + p.\,M_{df} \cdot i_d$$

$$v_c = v_c \cdot i_c + p \cdot L_c \cdot i_c + p.\,M_{qc}.i_q$$

On the assumption of linear parameters, the above equations contain nine variable quantities, namely four currents, four voltages and the speed. Taking speed as an independent variable, it is thus necessary

The currents i_d and i_q flow through external circuits of known impedances Z_d (p) and Z_q (p) respectively, where

$$Z_d(p) = r_d + p \cdot L_d$$

$$Z_q(p) = r_q + p \cdot L_q$$

Therefore

$$V_d = i_d \cdot Z_d (p) + e_d$$

$$V_q = i_q \cdot Z_q (p) + e_q$$

where e_d and e_q are the emf's (if any) in the external circuits, assumed to be in opposition to V_d and V_q respectively.

A further relation is determined by the mode of excitation of the machine. Thus, when coils F and Q are connected in parallel, $V_f = V_q$; when F and Q are in series, $i_f = i_q$. If the machine is separately excited, V_f must be specified. A final relation is provided by the mode of action of the coil C. If the functions as a compensating winding we must have $i_e = -i_q$; if as a damping winding, $V_c = O$.

Calculations may now proceed in order to express unknown quantities in terms of the speed, ω. When all currents and voltages are known the calculations of the machine performance may be completed.

It may be added that it is generally impracticable to calculate the winding parameters of a given machine from first principles, and they are therefore determined by emphirical methods on the basis of test results on similar machines.

Matrix Methods :

The four principal voltage equations deduced above may be displayed consisely in the form of a matrix equation thus

$$\begin{bmatrix} v_f \\ v_d \\ V_q \\ V_c \end{bmatrix} = \begin{bmatrix} r_f + pL_f & pM_{df} & & \\ pM_f & r_d + pL_d & \omega L_q & \omega M_{qc} \\ -\omega M_{df} & -\omega L_d & (r_q + pL_q) & pM_{qc} \\ & & pM_{qc} & (r_c + pL_c) \end{bmatrix} \begin{bmatrix} i_f \\ i_d \\ i_q \\ i_c \end{bmatrix}$$

or, in abbreviated form,

$$[V] = [Z] \quad [i]$$

Where [Z] denotes the impedence matrix and [V] and [i] the column matrics of voltages and currents respectively.

The voltage matrix is thus represented as the product of the impedance matrix by the current matrix, in that order.

Any additional winding, e.g., a second winding on the field poles, may be taken into account by adding a corresponding element to the column matrics of Voltage and current and also a corresponding row and column to the impedance matrix. In like manner, if any winding is absent the corresponding row and cloumn elements are omitted.

The form of the impedance matrix Z is clearly governed by the corresponding equations previously deduced from electromagnetic considerations, and the introduction of the matrix representation may therefore appear to be unnecessary. However, inspection of the impedance matrix reveals that it is characterized by a degree of symmetry enabling it to be written out by a routine process, thus leading automatically to the circuit equations. It is evident that this procedure is simpler than the traditional approach followed previously, and the advantage is more pronounced with more complex assemblages of windings. The symmetrical structure of the impedance matrix will now be considered in more detail, with reference to the four-winding assembly considered above.

A box, in place of the orthodox frame, containing four rows and four columns, as shown in Fig. (11) is first set out and the individual rows and columns are then lettered to correspond to the assumed order of the windings: this will be taken as before.

Examination of the Z matrix shows that -

1. All self impedance terms go into the diagonal compartments, from top left hand to bottom right hand corner.

2. All mutual impedance terms go into off-diagonal compartments and are balanced about the main diagonal. Since mutual coupling cannot exist between coils on difficient axes the corresponding compartments contain no mutual inductance terms.

3. All rotational terms go into the rows associated with the moving coils D and Q, in accordance with the rules that (a) every co-efficient of P in row d reappears in row q in the same column, but with P replaced by - ω, and

   (b) every co-efficient of P in row q reappears in row d in the same column, but with P replaced by ω. The disposition of the negative signs is governed, of course, by the basic assumptions regarding positive directions of flux on the two axis, and of rotation

	f	d	q	c
f	$v_f + pL_f$	pM_{df}		
d	pM_{df}	$r_d + pL_d$	ωL_q	ωM_{qc}
q	$-\omega M_{df}$	$-\omega L_d$	$v_q + pL_q$	pM_{qc}
c			pM_{qc}	$v_c + pL_c$

Fig. (11)

In applying these rules it must be assumed that both coils D and Q are present; if either is absent, the corresponding row and column also omitted from the final impedance matrix.

TORQUE = The total apparent power supplied is given by

$$P = [i]\,[e] = [i]\,[Z]\,[i]$$

The elements of the impedance matrix Z are typified by the expression $(r + L_p + G\,\omega)$, where L may denote either self or mutual inductance, and the value of G is determined by the procedure stated above in reference to the routine construction of the impedance matrix. The total power, therefore, is given by

$$P = [i]\,[r + L_p + G\,\omega]\,[i]$$

$$= [i]\,[r]\,[i] + [i]\,[L_p]\,[i] + [i]\,[G\,\omega]\,[i]$$

The first term represents the total I^2R losses, the second term represents the total power absorbed by the electromagnetic field, and the third term therefore represents the gross mechanical power developed, i.e., $T\omega$, where T denotes the gross torque.

The matrix $G\omega$ contains only those elements in the matrix Z involving ω.

$$T = [i]\,[G]\,[i]$$

$$= [i_f \; i_d \; i_q \; i_c] \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & L_q & M_{qc} \\ -M_{df} & -L_d & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_f \\ i_c \\ i_q \\ i_c \end{bmatrix}$$

$$= M_{qe} \cdot i_c \cdot i_d - M_{df} \cdot i_f \cdot i_f \cdot i_q - (L_d - L_q)\, i_d \cdot i_q$$

after routine matrix multiplication and rearrangement. This result may be confirmed directly as follows :

$$T. = IT_a \cdot \Phi / \P = I\psi$$

in the same terminology as used previously.

Assuming two brush axes, the component of the torque due to the joint effect of the direct-axis component of Ψ and the quadrature axis component of I is

$$T_1 = i_q \; \Psi_d = i_q\,(L_d \cdot i_d + M_{df} \cdot i_f)$$

The torque component due to the quadrature axis component of Ψ and the direct axis component of I is,

$$T_2. = i_d \cdot \Psi_q = i_d\,(L_q \cdot i_q + M_{qc} \cdot i_c)$$

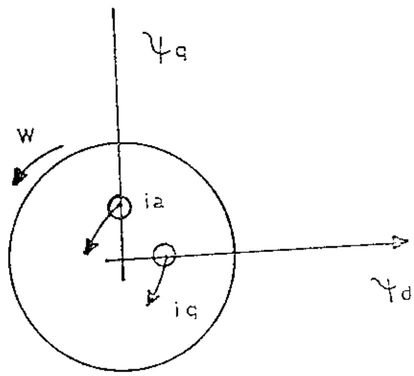


FIG.12

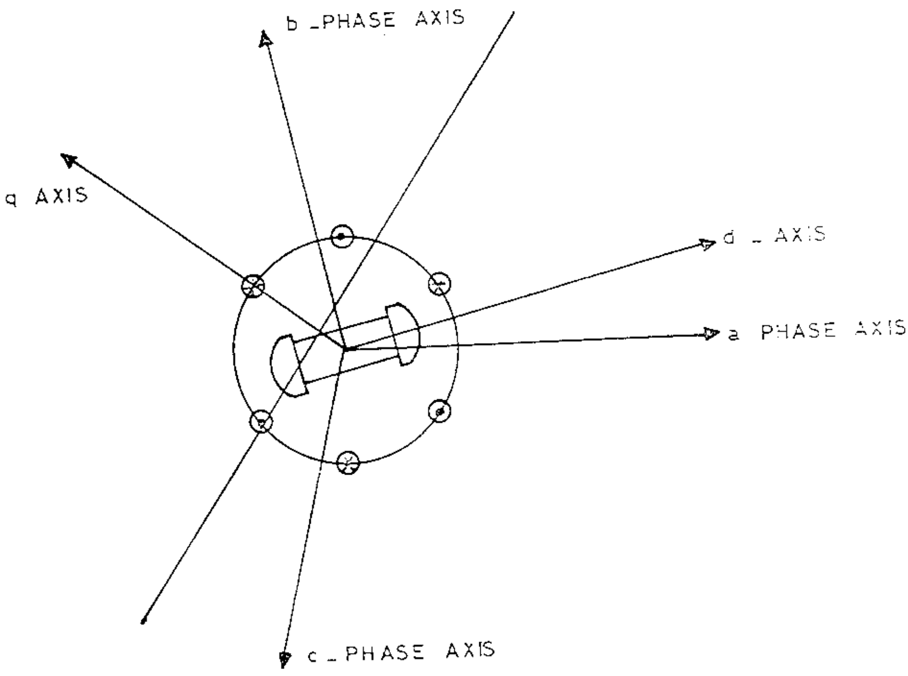


FIG.13. SYNRONOUS MACHINE

The conditions are shown in figure (12) and by the left hand rule it is seen that T_1 opposes and T_2 assists, rotation. The resultant gross torque is therefore.

$$T = T_2 - T_1$$

$$= i_d (L_q.i_q + M_{qe}.i_e) - i_q (L_d.i_d + M_{df}.i_f)$$

which is in agreement with the former result.

Synchronous Machine

A basic difficulty arises in this case due to the fact that the angle between the magnetic axis of any one phase and the direct (field) axis varies with time. This will be evident from figure (13) which is a diagrammatic representation of a 2 pole rotating field machine.

The d, and q-axes are therefore also in rotation, and conse - quently the air-gap permeance with respect to each individual phase varies cyclically. As a result of this the self-inductance of each phase and also its mutual inductances with respect to each of the other phases and to the field winding also vary cyclically. These periodic variation can be expressed by means of Fourier series, but it has been found sufficiently accurate to retain only the lowest harmonic in each case.

For example, the self-inductances, L_{aa}, of phase A must be a maximum when $\Theta = 0$, and again when $\Theta = 180°$, and a minimum when $\Theta = 90°$ or $270°$. It is accordingly represented by

$$L_a = L + L_r \text{ Cos } 2\Theta$$

Similarly for phases B and C,

$$L_b = L + L_r \cos 2\,(\Theta - 120°) = L + L \cos(2\Theta + 120°)$$

$$L_c = L + L_r \cos 2\,(\Theta - 240) = L + L_r . \cos(2\Theta - 120°)$$

It is evident from figure (13) that the mutual inductance between phases A and B is a maximum when the q. axis lies along the axis of phase C, i.e., when $\Theta = -30°$ or $150°$; Thus,

$$M_{ab} = -M - M_r . \cos.2\,(\Theta + 30°)$$

When $\Theta = -30°$ or $150°$, $m_{qb} = -(M + M_r)$, the negative sign arising from the fact that the mutual flux is necessarily directed in the negative sense, as will be clear from figure (13).

The second-harmonic terms in both self and mutual inductance are due to the variation in the permeance of the air gap, and for this reason it is usual to assume that $L_2 = M_2$. Thus

$$M_{ab} = -M + L_r \cos(2\Theta - 120°)$$

Similarly,

$$M_{bc} = -M + L_r \cos 2\Theta$$

and $$M_{ca} = -M + L_r \cos(2\Theta + 120°)$$

Positive flux. linkage in any phase is assumed to be in the sense corresponding to positive current in that phase, and all currents in figure (13) are positive.

The mutual inductance between the field winding and phase A may be represented by

$$M_{af} = M_{af} . \cos \Theta$$

Similarly,

$$M_{bf} = M_{af}.\ \text{Cos}\ (\Theta - 120°)$$

and $$M_{cf} = M_{af}.\ \text{Cos}\ (\Theta + 120°)$$

For an approximate treatment, all magnetic leakage effects are again neglected, and likewise damping windings.

The total flux-linkage with phase A is thus

$$\Psi_a = L_a.\ i_a + m_{ab}.\ i_b + m_{ac}.\ i_c + m_{af}.\ i_f$$

Similarly,

$$\Psi_b = L_b.\ i_f + m_{bc}.\ i_c + m_{ba}.\ i_a + m_{bf} + m_{bf}.\ i_f$$

and $$\Psi_c = L_c.\ i_c + m_{ca}.\ i_a + m_{cb}.\ i_b + m_{cf}.\ i_f$$

and for the field winding.

$$\Psi_f = L_f.\ i_f + M_{af}\ [i_a.\ \text{Cos}\,\Theta + i_b.\ \text{Cos}\ (\Theta - 120°) + i_c(\text{Cos}\ \Theta + 120°)]$$

Where $L_f = M_{af}$ if leakage is neglected.

Complications arise because the various inductance co-efficients are functions of Θ. Further analysis is greatly facilitated by aid of a device, whereby all phase quantities, e.g., current, voltage and flux-linkage are re-expressed in terms of new variables associated with fictitious coils placed on the d- and q-axis, to-gether with a zero sequence component in the phase windings. This procedure, the d-q-o transformation, is analogous to the p-n-o transformation to give symmetrical components in unbalanced 3 phase circuits, and similar ultimate simplifications

Assuming the fictitious, d- and q-axis coils to have each a number of terms equal to the effective number of turns per phase, the phase currents may be related to the d- and q-axis components by equating the m.m.f. along the d and q-axesc in turn. It is found convenient to change the unit of current in the axis coils to one and a half times the unit for the phase coils, we thus obtain.

$$i_d = 2/3\ [i_a.\ \mathrm{Cos}\ \Theta + i_b.\ \mathrm{Cos}\ (\Theta + 240°) + i_c.\ \mathrm{Cos}\ (\Theta + 120°)]$$

$$i_q = -2/3\ [i_a.\ \mathrm{Sin}\ \Theta + i_b.\ \mathrm{Sin}\ (\Theta + 240°) + i_c.\ \mathrm{Sin}\ (\Theta + 120°)]$$

Induction Motor

The analysis is again facilitated by the phase-to-axis transformation. It will be assumed that the d- and q-axis are rotating synchronously as before; and that the primary winding is on the stator. Stator and Rotor quantities will be distinguished by small and the capital letter subscripts respectively, and the d-axis will be assumed to coincide with the axis of stator phase 'a' at the instant $t = 0$.

On substituting $\Theta = wt$, in the expressions given the synchronously machines ; in -

$$i_{qs} = 2/3\ [i_a.\ \mathrm{Cos}\ \Theta + i_b.\ \mathrm{Cos}\ (\Theta + 240°) + i_c.\ \mathrm{Cos}\ (\Theta + 120°)]$$

$$i_{qr} = 2/3\ [i_a.\ \mathrm{Sin}\ \Theta + i_b.\ \mathrm{Sin}\ (\Theta + 240°) + i_c.\ \mathrm{Sin}\ (\Theta - 120°)]$$

we have -

$$i_{ds} = 2/3\ [i_a.\ \mathrm{Cos}\ \omega t + i_b.\ \mathrm{Cos}\ (\omega t - 120°) + i_c.\ \mathrm{Cos}\ (\omega t - 120°)]$$

$$i_{qs} = -2/3\ [i_a.\ \mathrm{Sin}\ \omega t + i_b.\ \mathrm{Sin}\ (\omega t - 120°) + i_c.\ \mathrm{Sin}\ (\omega t + 120°)]$$

or, in matrix form -

$(i_{ps}) = 2/3\ [A]\ [i_s.\ ph]$, say

Hence -

$i_a = i_{ds}.\ \text{Cos}\ \omega t - i_{qs}.\ \text{Sin}\ \omega t$

$i_b = i_{ds}.\ \text{Cos}\ (\omega t - 120°) - i_{qs}.\ \text{Sin}\ (\omega t - 120°)$

$i_c = i_{ds}.\ \text{Cos}\ (\omega t - 120°) - i_{qs}.\ \text{Sin}\ (\omega t + 120°)$

or $(i_s.\ ph) = (A^T)\ [i_{ps}]$

Also, if Θ_s now denotes the instantaneous angular displacements of d-axis ahead of the axis of Rotor phase A;. The axis current components equivalent to the Rotor currents are -

$i_{dr} = 2/3\ [i_A.\ \text{Cos}\ \Theta_s + i_B.\ \text{Cos}\ (\Theta - 120°) + i_c.\ \text{Cos}(\Theta_s + 120°)]$

$i_{qr} = -2/3\ [i_A.\ \text{Sin}\ \Theta_s + i_B.\ \text{Sin}\ (\Theta_s - 120°) + i_c.\ \text{Sin}\ (\Theta + 120°)]$

or $[i_r.\ ph] = [B^J]\ [ipr]$

If it is assumed also that $\Theta_s = 0$ at the instant when $t = 0$; then $\Theta_s = S\,\omega t$ and the angular displacement of the axis of Rotor phase A, ahead of the axis of stator phase 'a' is given by

$\Theta_z = \Theta - \Theta_s = (1-S)\,\omega t.$

where S is the fractional slip. Also,

$\Theta r = (1-S)\ \omega$

It is now necessary to express the flux linkage with the stator and Rotor winding in terms of phase currents. As before self-inductances will be denoted by repeated suffixes; thus for the stator.

$$L_{aa} = L_{bb} = L_{cc}$$

and the Rotor,

$$L_{AA} = L_{BB} = L_{CC}$$

Mutual inductances will be denoted by distinct suffixes; thus for stator

Lab = Lbc = Lca

and the Rotor,

LAB = LBC = LCA

It is evident that -

LaA = LaA. Cos. θr.

LaB = LaB. Cos (θ_2 + 120°)

LaC = LcB = LaC

LbA = LcC = LaA

LbB = LcC = LaA

Lbc = LcA = LaB

also,

LAa = LBb = LCc = LaA

LAb = LBc = LCa = LaC

LAC = LBa = LCb = LaB

Let Laa - Lab = Ls, the apparent 3-phase stator self inductance

LAA - LAB = Lr the apparent 3-phase Rotor self inductance

3/2. LaA = L_{12}, the apparent 3-phase mutual inductance

$$\Psi_a = Laa.\ i_a + Lab.\ i_b + Lac.\ i_c + LaA.\ i_A + LaB.\ i_B. + Lac.\ i_c.$$

Since $\Sigma\,[i_a] = \Sigma\,[i_A] = 0$ in the absence of a Neutral connexion, assuming star - connected primary and secondary windings this may be written as -

$$\Psi_a = L_s.\ i_b. + LaA\ [i_A.\ \text{Cos}\ \Theta_2 + i_B.\ \text{Cos}\ (\Theta_2 + 120°) + i_c\ (\text{Cos}\ \Theta_2 - 120°)]$$

Similarly,

$$\Psi_b = L_s.\ i_b + LA\ [i_A.\ \text{Cos}\ (\Theta_2 - 120°) + 2B.\ \text{Cos}\ \Theta_2 + i_c.\ \text{Cos}\ (\Theta_2 + 120°)]$$

and

$$\Psi_c = L_s.\ i_c + LaA\ [i_A.\ \text{Cos}\ (\Theta_2 + 120°) + i_B.\ \text{Cos}\ (\Theta_2 - 120°) + i_c.\ \text{Cos}\ \Theta_2)]$$

In Matrix form -

$$(\Psi_s ph) = L_s[i_s.ph] + LaA\ [C]\ [i_r.ph]$$

For Rotor winding we obtain similarly -

$$\Psi_a = Lr.\ i_A + LaA.\ [i_a.\ \text{Cos}\ \Theta_2 + i_b.\ \text{Cos}\ (\Theta_2 - 120°) + i_c.\ \text{Cos}\ (\Theta_2 + 120°)]$$

$$\Psi_b = Lr.\ i_B. + LaA.\ [i_a.\ \text{Cos}\ (\Theta_2 + 120°) - i_b.\ \text{Cos}\ (\Theta_2 - 120°)] + ib\ \cos\Theta_2$$

$$\Psi_c = Lr_2.\ i_c.\ LaA\ [i_a.\ \text{Cos}\ (\Theta_2 - 120°)\ i_b.\ \text{Cos}\ (\Theta_2 + 120°) + i_c.\ \text{Cos}\ \Theta_2]$$

or $$(\Psi_r.\ ph) = Lr\ [i_{r.ph}] + LaA\ [c^t]\ (i_{s.ph})$$

The axis flux linkage equivalent to the actual stator and Rotor linkages given above may now be written down, following the same pattern, as for the current transformation.

In Matrix form

$$\begin{bmatrix} \Psi d_s \\ \Psi q_s \end{bmatrix} = 2/3 \begin{bmatrix} \text{Cos}\ \omega t & \text{Cos}(\omega t-\alpha) & \text{Cos}\ (\omega t+\alpha) \\ -\text{Sin}\ \omega t & -\text{Sin}(\omega t-\alpha) & -\text{Sin}\ (\omega t+\alpha) \end{bmatrix} \begin{bmatrix} \Psi a \\ \Psi b \\ \Psi c \end{bmatrix}$$

or

$$\begin{aligned} [\Psi_{Ps}] &= 2/3\ [A]\ [\Psi_s.ph] \\ &= 2/3\ [A]\ \left[L_s\ [i_s.ph] + L_{aA}\ [C]\ .\ [i_2ph]\right] \\ &= 2/3\ [A]\ \left[L_s\ [A^T]\ [i_{Ps}] + LaA.\ [C]\ [B^T]\ [i._{pr}]\right] \end{aligned}$$

$$\begin{aligned} \text{and } [\Psi_{2P}] &= 2/3\ [B]\ .\ [\Psi r.\ ph] \\ &= 2/3\ [B]\ \left[[Lr.\ i_{r.ph}] + LaA\ [C^T]\ [is.ph]\right] \\ &= 2/3\ [B]\ \left[[Lr.\ [B^T]\ [i_{rp}]\ LaA.\ [C^T]\ [A^T]\ [i_{p.s}]\right] \end{aligned}$$

The combined result is -

$$\begin{bmatrix} \Psi_{Ps} \\ \Psi_{Pr} \end{bmatrix} = 2/3 \begin{bmatrix} L_s.\ [A]\ [A^t] & LaA\ [A]\ [C]\ .\ B^t \\ LaA\ [B]\ [C^t]\ [A^t] & Lr\ [B]\ .\ [B^t] \end{bmatrix} \begin{bmatrix} i._{ps} \\ i._{pr} \end{bmatrix}$$

It will be seen that (A) and (B) are identical in the form and that $[B]\ [C^t]\ [A^t]$ is the transpose of $[A]\ [C]\ [B^t]$. On expanding the matrix product transforming the trignomentric products into sums, and simplifying, we find that -

$$[A]\ [A^t] = \begin{bmatrix} 3/2 & 0 \\ 0 & 3/2 \end{bmatrix}$$

$$\text{and } [A]\ [C]\ [B^t] = \begin{bmatrix} 9/4 & 0 \\ 0 & 9/4 \end{bmatrix}$$

Hence -

$$\begin{bmatrix} \Psi_{PS} \\ \Psi_{Pr} \end{bmatrix} = 2/3 \begin{bmatrix} L_s \begin{bmatrix} 3/2 & 0 \\ 0 & 3/2 \end{bmatrix} & L_{aA} \begin{bmatrix} 9/4 & 0 \\ 0 & 9/4 \end{bmatrix} \\ L_{aA} \begin{bmatrix} 9/4 & 0 \\ 0 & 9/4 \end{bmatrix} & L_r \begin{bmatrix} 3/2 & 0 \\ 0 & 3/2 \end{bmatrix} \end{bmatrix} \begin{bmatrix} i.\ ps \\ i.\ pr \end{bmatrix}$$

$$= \begin{bmatrix} L_s & L_{12} \\ L_{12} & L_r \end{bmatrix} \begin{bmatrix} i_{ps} \\ i_{pr} \end{bmatrix}$$

i.e.,

$$\begin{bmatrix} \Psi_{dr} \\ \Psi_{qr} \end{bmatrix} = Lr \begin{bmatrix} i_{dr} \\ i_{qr} \end{bmatrix} + L_{12} \begin{bmatrix} i_{ds} \\ i_{qs} \end{bmatrix}$$

$$\begin{bmatrix} \Psi_{ds} \\ \Psi_{qs} \end{bmatrix} = L_s \begin{bmatrix} i_{ds} \\ i_{qs} \end{bmatrix} + L_{12} \begin{bmatrix} i_{dr} \\ i_{qr} \end{bmatrix}$$

The phase - to - axis transformation for the voltagesis expressed by -

$$(\ V.\ PS) = 2/3\ [A]\ [V_s.ph] = 2/3\ [A]\ \left[r_s\ [i_{s.ph}] + P\ [\Psi_s.ph]\right]$$

$$2/3\ [A]\ r_s.\ [i_s.ph] = 2/3\ .r_s\ [A]\ [A^t]\ [i\ PS] = r_s [i_{PS}]$$

$$2/3\ [A]\ p\ [\Psi ph]$$

$$= 2/3\ [A]\ p.\ \left[[A^t]\ [\Psi_p]\right]$$

$$= 2/3\ [A] \left[P \begin{bmatrix} \text{Cos}\ \omega t & -\text{Sin}\ \omega t \\ \text{Cos}\ (\omega t - \alpha) & -\text{Sin}\ (\omega t - \alpha) \\ \text{Cos}\ (\omega t + \alpha) & -\text{Sin}\ (\omega t + \alpha) \end{bmatrix} \right] [\Psi\ PS]$$

$$+ 2/3\ [A]\ [A^t]\ \ P\ \ [\Psi_{ps}]$$

$$= 2/3\ [A]\ .\ \omega \begin{bmatrix} -\ \mathrm{Sin}\,\omega t & -\ \mathrm{Cos}\ \ \omega t \\ -\ \mathrm{Sin}\ (\omega t - \alpha) & -\ \mathrm{Cos}\ (\omega t - \alpha) \\ -\ \mathrm{Sin}\ (\omega t + \alpha) & -\ \mathrm{Cos}\ (\omega t - \alpha) \end{bmatrix} [\Psi\ PS]$$

$$+ 2/3\ [A]\ [A^t]\ .\ P\ [\Psi\ PS]$$

$$= 2/3\ \omega \begin{bmatrix} 0 & -3/2 \\ 3/2 & 0 \end{bmatrix} [\Psi_{Ps}] + 2/3 \begin{bmatrix} 3/2 & 0 \\ 0 & 3/2 \end{bmatrix} P\ [\omega_{Ps}]$$

Therefore -

$$V_{Ps} = r_s\ \ [i_{Ps}] + P \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} [\Psi_{Ps}] + \omega \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} [\Psi_{Ps}]$$

i.e.,

$$V_{ds} = r_s \cdot i_{ds} + P.\Psi_{ds} - \omega \cdot \Psi_{qs}$$

$$V_{qs} = r_s \cdot i_{qs} + P.\ \Psi_{qs} + \omega \cdot \Psi_{ds}$$

These equations are seen to be identical with those derived for the synchronous machine. This is due to the fact that in the case of the induction motor the d - and q - axes have been arbitrarily assumed to be rotating at Synchronous speed; an essential conditions.

With the salient-pole machine. The equations for the axis voltages could therefore have been assumed to be the same as those deduced for the synchronous machine, but the alternative method of derivation has features of interest.

The corresponding equations for the rotor are obtained on re-

$$V_{dr} = V_r \cdot i_{dr} + P \cdot \Psi_{dr} \quad S.\omega.\Psi_{qr}$$

$$V_{qr} = V_r \cdot i_{qr} + P \cdot \Psi_{qr} - S.\omega\,\Psi_{dr}$$

On substituting the values of the axis flux linkages, we have

$$V_{ds} = V_s.i_{ds} + P(L_s.i_{ds} + L_{12}.i_{dr}) - \omega(L_s i_{qs} + L_{12}\, i_{qr})$$

$$V_{qs} = V_s.i_{qs} + P(L_s.i_{qs} + L_{12}.i_{qr}) + \omega(L_s.i_{ds} + L_{12}.i_{dr})$$

$$V_{dr} = V_r.i_{dr} + P(L_r.i_{dr} + L_{12}.i_{ds}) - S\omega(L_r.i_{qr} + L_{12}.i_{q})$$

$$V_{qr} = V_r.i_{qr} + P(L_r.i_{qr} + L_{12}.i_{qs}) + S\omega(L_r.i_{dr} + L_{12}.i_{ds})$$

or in the matrix form

$$\begin{bmatrix} V_{ds} \\ V_{qs} \\ V_{dr} \\ V_{qr} \end{bmatrix} = \begin{bmatrix} (R_s + PL_s) & -\omega L_s & PL_{12} & -\omega L_{12} \\ +\omega L_s & (R_s + PL_s) & +\omega L_{12} & PL_{12} \\ PL_{12} & -S\omega L_{12} & (R_r + PL_r) & -S\omega L_r \\ S\omega L_{12} & PL_{12} & +S\omega L_r & (R_r + PL_r) \end{bmatrix} \begin{bmatrix} i_{ds} \\ i_{qs} \\ i_{dr} \\ i_{qr} \end{bmatrix}$$

The axis voltages are given by

$$(V_{ps}) = 2/3\ [A]\ [V_s.ph] \text{ and } [V_{pr}] = 2/3.\ [B]\ [V_r.ph]$$

The components of (V_s.ph) are known and in the usual case where the Rotor windings, are short circuited the components of (V_r.ph) are zero.

The current matrices (i_{ps}) and (i_{pr}) may therefore be determined and hence

$$(i_s.ph) = [A^T]\ [ips] \text{ and } [i_r.ph] = [B^T]\ [i_{pr}]$$

Hence the relation between the axis voltages and currents may be written in the form

$$\begin{bmatrix} V_{ds} \\ V_{qs} \\ V_{dr} \\ V_{qr} \end{bmatrix} = \begin{bmatrix} (R_s + pL_s) & -\omega L_s & pL_{12} & -\omega L_{12} \\ +\omega L_s & (R_s + pL_s) & +\omega L_{12} & pL_{12} \\ pL_{12} & -S\,\omega L_{12} & (R_r + pL_r) & -S\,\omega L_r \\ S\,\omega L_{12} & pL_{12} & +S\,\omega L_r & (R_r + pL_r) \end{bmatrix} \begin{bmatrix} i_{ds} \\ i_{qs} \\ i_{dr} \\ i_{qr} \end{bmatrix}$$

TORQUE

The total Instantaneous power supplied to the stator winding is

$$P = [i_{ph}] \cdot [V_{ph}]$$

$$= [A^T] \cdot [i_p] [A^T] [v_p]$$

$$= 3/2 \, [i_p^T] \cdot [v_p]$$

On expanding the matrix product $[A] [A^T]$

The torque is obtained by dividing the mechanical power components of P by the mechanical angular velocity. We have

$$[i_p^T] [v_p] = [i_d \cdot i_q] \begin{bmatrix} v_d \\ v_q \end{bmatrix}$$

$$P = 3/2 \; id \, [p. \, \Psi_a - \omega \Psi_q + R.id) + i_q \, (\omega \Psi_d + p. \, \Psi_q + R_{iq})$$

$$T_e = P/\omega$$

$$= 3/2\ (i_{qs}\Psi_{ds} - i_{ds}\ \Psi_{qs})$$

$$= 3/2\ [i_{qs}(L_s\ i_{ds} + L_{12}\ i_{ds}) - i_{ds}\ (L_s\ i_{qs} + L_{12}.i_{qr})]$$

$$Te = L_{12}\ (i_{qs}\ i_{qr} - i_{ds}\ i_{qr}) \quad \ldots (2)$$

4.2 Modeling of the System :

We developed the equations describing the dynamics of a 3 Φ slip Ring Induction Motor in synchronously Rotating Reference Frame as in (1).

$$\begin{bmatrix} V_{ds} \\ V_{qs} \\ V_{dr} \\ V_{qr} \end{bmatrix} = \begin{bmatrix} (R_s + pL_s) & -\omega L_s & pL_{12} & -\omega L_{12} \\ \omega L_s & (R_s + pL_s) & +\omega L_{12} & pL_{12} \\ pL_{12} & -S\ \omega L_{12} & (R_r + pL_r) & -S\ \omega L_r \\ S\ \omega L_{12} & pL_{12} & S\ \omega L_r & (R_r + pL_r) \end{bmatrix} \begin{bmatrix} i_{ds} \\ i_{qs} \\ i_{dr} \\ i_{qr} \end{bmatrix}$$

If the angular relationship between q axis and magnetic axis of the stator and rotor phases are so selected that these axis coincide at time zero and the a.c voltages at the rectifier are transformed to a synchronously rotating reference frame then the q axis voltage is equal to the V_{rm} while the d axis voltage is maintained at zero. Thus

$$V_{dr} = 0 \quad \ldots (3)$$

$$V_{qr} = V_{rm} \quad \ldots (4)$$

But the output voltage of a 3 phase rectifier is V_R.

$$V_R = 3\sqrt{3}/\Pi\ .\ V_{rm}$$

Therefore, $$V_{qr} = \Pi/3\sqrt{3}\ .\ V_R \quad \ldots (5)$$

Since V_{dr} is always zero, the instantaneous power balance equation can be written as $V_R \cdot I_R = -3/2 \cdot V_{qr} \cdot i_{qr}$

$$IR = [-3/2 \cdot V_{qr} \cdot i_{qr}]/V_R \cdot \quad \ldots (6)$$

Substitute (5) in (6) ;

$$IR = [-3/2 \cdot \Pi/3.\sqrt{3} \cdot V_R \cdot i_{qr}]/V_R$$

$$I_R = -\Pi/2.\sqrt{3} \cdot i_{qr} \quad \ldots (7)$$

$$= -0.906\ i_{qr}$$

Voltage equation for filter circuit is given by

$$V_R = I_R\,(R_f + p.L_f) + V_I$$

$$V_R = V_I + I_R\,(R_f + P/\omega.\ X_f) \quad \ldots (8)$$

Assuming infinitely smoothed direct voltage and neglecting AC source impedance, the opposing emf presented by the inverter which is approximately equal to the direct input voltage can be expressed as

$$V_I = 3.\sqrt{3}/\Pi \cdot V_{sm} \cdot \text{Cos}\,(\Pi - \alpha) \quad \ldots (9)$$

where V_{sm} - Stator line to peak voltage

Since at time t = 0, the q axis of the synchronously rotating reference frame coincides with the magnetic axis of the reference frame. The quadrature components of the supply voltage,

$V_{qs} = V_{sm}$, and direct component $V_{ds} = 0$

The direct component $V_{ds} = 0$ . . . (10)

i.e., $V_I = 3.\sqrt{3}/\Pi \cdot V_{qs} \cdot \text{Cos}\,(\Pi - \alpha)$ . . . (11)

$= -3.\sqrt{3}/\Pi \cdot V_{qs} \cdot \text{Cos}\,\alpha$ . . . (12)

From (4)

$$V_R = -3.\sqrt{3}/\Pi \;.\; V_{qs} \;.\; Cos\,\alpha + i_R\,(R_f + P/\omega \;.\; X_f)$$

$$V_R = -3.\sqrt{3}/\Pi \;.\; V_{qs} \;.\; Cos\,\alpha - 0.906\; i_{qr}\,(R_f + P/\omega . X_f) \qquad \ldots (13)$$

$$3.\sqrt{3}/\Pi .\; V_{rm} = 3.\sqrt{3}/\Pi .\; V_{qr} = -3.\sqrt{3}/\Pi\; V_{qs}.Cos\,\alpha - 0.906\; i_{qr}\,(R_f + P/\omega \;.\; X_f)$$

$$V_{qr} = -V_{qs}\,Cos\,\alpha - 0.55\; i_{qr}\,(R_f + P/\omega\; X_f) \qquad \ldots (14)$$

Incorporating the above equations (3), (10) and (14) in (1) we can modify, under steady state condition the equation describing the dynamics of the 3 phase slip ring induction motor in synchronously rotating reference frame as

$$\begin{bmatrix} V_{ds} \\ V_{qs} \\ V_{dr} \\ V_{qr} \end{bmatrix} = \begin{bmatrix} 0 \\ V_{qs} \\ 0 \\ -V_{qs} \;.\; Cos\,\alpha \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ V_{qs} \\ 0 \\ -V_{qs}.Cos\,\alpha \end{bmatrix} = \begin{bmatrix} R_s+pL_s & -\omega L_s & pL_{12} & -\omega L_{12} \\ \omega L_s & R_s+PL_s & \omega L_{12} & pL_{12} \\ pL_{12} & -S\,\omega L_{12} & (R_r+pL_r) & -S\,\omega L_r \\ S\,\omega L_{12} & pL_{12} & S\,\omega L_r & R_r-0.55\,R_f + p/\omega(R_r-0.55\,X_f) \end{bmatrix} \begin{bmatrix} i \\ i \\ i \\ i \end{bmatrix}$$

$$Te = T_L + 2Hp\,\omega_r + D\,\omega_r$$

The voltage equation is generally represented as

$$V = [R + (P/\omega)X]\ i.$$

$$[R+(P/\omega).X. = \begin{bmatrix} R_s & -X_s & 0 & -X_{12} \\ X_s & R_s & X_{12} & 0 \\ 0 & -SX_{12} & R_r & -SX_r \\ SX_{12} & 0 & SX_r & R_r+0.55\ X_f \end{bmatrix}$$

$$+\ P/\omega \begin{bmatrix} X_s & 0 & X_{12} & 0 \\ 0 & X_s & 0 & X_{12} \\ X_{12} & 0 & X_r & 0 \\ 0 & X_{12} & 0 & X_r+0.55\ R_f \end{bmatrix}$$

$$[V] = [R]\ [i] + [X]\ [P/\omega\ ]\ [i]$$

$$P/\omega\ [i] = [X]^{-1}\ [V] - [X]^{-1}\ [R]\ [i] \qquad \ldots.\ I$$

$$x^{-1} = \left[\begin{array}{cc|cc} x_s & 0 & x_{12} & 0 \\ 0 & x_s & 0 & x_{12} \\ \hline x_{12} & 0 & x_r & 0 \\ 0 & x_{12} & 0 & x_r+0.55x_f \end{array}\right]^{-1} = \begin{bmatrix} B_1 & B_2 \\ B_3 & B_4 \end{bmatrix}$$

$$B_1 = [A_1 - A_2\ A_4^{-1}\ A_3]^{-1}$$

$$B_2 = -\ B_1\ A_2\ A_4^{-1}$$

$$B_4 = A_4^{-1} - A_4^{-1} A_3 B_2$$

put

$$A = X_s \cdot X_r - X_{12}^2$$

$$B = X_s X_r - X_{12}^2 + 0.55 X_s X_f$$

$$A_4^{-1} = \begin{bmatrix} 1/X_r & 0 \\ 0 & 1/X_r + 0.55 X_f \end{bmatrix}$$

$$A_2 A_4^{-1} = \begin{bmatrix} X_{12} & 0 \\ 0 & X_{12} \end{bmatrix} \begin{bmatrix} 1/X_r & 0 \\ 0 & 1/X_r - 0.55 X_f \end{bmatrix}$$

$$A_2 A_4^{-1} = \begin{bmatrix} X_{12} & 0 \\ 0 & X_{12}/(X_r + 0.55 X_f) \end{bmatrix}$$

$$A_2 . A_4^{-1} . A_3 = \begin{bmatrix} X_{12}/X_r & 0 \\ 0 & X_{12}/(X_r + 0.55 X_f) \end{bmatrix} \begin{bmatrix} X_{12} & 0 \\ 0 & X_{12} \end{bmatrix}$$

$$= \begin{bmatrix} X_{12}^2 / X_r & 0 \\ 0 & X_{12}^2/(X_r + 0.55 X_f) \end{bmatrix}$$

$$A_1 - A_2 . A_4^{-1} . A_3 = \begin{bmatrix} X_s & 0 \\ 0 & X_s \end{bmatrix} - \begin{bmatrix} X_{12}^2 & 0 \\ 0 & X_{12}^2/(X_r + 0.55 X_f) \end{bmatrix}$$

$$B_1 = \begin{bmatrix} \dfrac{X_s . X_r - X_{12}^2}{X_r} & 0 \\ 0 & \dfrac{X_s . X_r + 0.55 X_f . X_s - X_{12}^2}{(X_r + 0.55 X_f)} \end{bmatrix}$$

$$= \begin{bmatrix} A/X_r & 0 \\ 0 & B/(X_r+0.55\ X_f) \end{bmatrix}^{-1}$$

$$B_1 = \begin{bmatrix} X_r/A & 0 \\ 0 & (X_r+0.55\ X_f)/B \end{bmatrix}$$

$$B_2 = -\ B_1 \ .\ A_2 \ .\ A_4^{-1}$$

$$= -\begin{bmatrix} X_r/A & 0 \\ 0 & (X_r+0.55\ X_f)/B \end{bmatrix} \begin{bmatrix} X_{12}/X_r & 0 \\ 0 & X_{12}/(X_r+0.55\ X_f) \end{bmatrix}$$

$$= \begin{bmatrix} -[\ (X_r.X_{12})/(A.X_r] & 0 \\ 0 & \dfrac{X_{12}(X_r+0.55\ X_f)}{B(X_r+0.55\ X_f)} \end{bmatrix}$$

$$B_2 = \begin{bmatrix} -X_{12}/A & 0 \\ 0 & X_{12}/B \end{bmatrix}$$

$$B_3 = -\ A_4^{-1} \ .\ A_3 \ .\ B_1$$

$$= -\begin{bmatrix} 1/X_r & 0 \\ 0 & 1/(X_r+0.55X_f) \end{bmatrix} \begin{bmatrix} X_{12} & 0 \\ 0 & X_{12} \end{bmatrix} \begin{bmatrix} X_r/A & 0 \\ 0 & \dfrac{(X_r+0.55\ X_f)}{B} \end{bmatrix}$$

$$B_3 = -\begin{bmatrix} X_{12} & 0 \\ 0 & X_{12}/B \end{bmatrix}$$

$$B_4 = A_4^{-1} - A_4^{-1} . A_3 . B_2$$

$$A_4^{-1} . A_3 = \begin{bmatrix} 1/X_r & 0 \\ 0 & 1/(X_r+0.55\ X_f) \end{bmatrix} \begin{bmatrix} X_{12} & 0 \\ 0 & X_{12} \end{bmatrix}$$

$$A_4^{-1}.A_3.A_2 = \begin{bmatrix} \dfrac{X_{12}}{X_r} & 0 \\ 0 & \dfrac{X_{12}}{(X_r+0.55X_f)} \end{bmatrix} \begin{bmatrix} \dfrac{-X_{12}}{A} & 0 \\ 0 & -X_{12}/B \end{bmatrix}$$

$$= \begin{bmatrix} -X_{12}^2/X_r.A & 0 \\ 0 & -X_{12}^2/(X_r+0.55\ X_f)B \end{bmatrix}$$

$$B_4 = A_4^{-1} - A_4^{-1} . A_3 . B_2$$

$$= \begin{bmatrix} 1/X_r & 0 \\ 0 & 1/(X_r+0.55\ X_f) \end{bmatrix} - \begin{bmatrix} X_{12}^2/X_r.A & 0 \\ 0 & -X^2_{12}/(X_r+0.55X_f) \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{A+X_{12}^2}{X_r.A} & 0 \\ 0 & B+X_{12}^2/(X_r+0.55X_f)^2.B \end{bmatrix}$$

Substituting for A and B

$$B_4 = \begin{bmatrix} X_s/A & 0 \\ 0 & X_s/B \end{bmatrix}$$

$$X^{-1} = \begin{bmatrix} X_r/A & 0 & -X_{12} & 0 \\ 0 & (X_r+0.55\ X_f)/B & 0 & -X_{12}/B \\ -X_{12}/A & 0 & X_s/A & 0 \\ 0 & -X_{12}/A & 0 & X_s/B \end{bmatrix}$$

Substituting X^{-1} in (I).

$$P/\omega[i] = [X]^{-1}[V] - [X]^{-1}[R][i]$$

where $[i] = \begin{bmatrix} I_{ds} \\ I_{qs} \\ i_{dr} \\ i_{qr} \end{bmatrix}$

$$[X]^{-1} = \begin{bmatrix} X_r/A & 0 & -X_{12}/A & 0 \\ 0 & (X_r+0.55\ X_f)/B & 0 & -X_{12}/B \\ -X_{12}/A & 0 & X_s/A & 0 \\ 0 & -X_{12}/B & 0 & X_s/B \end{bmatrix}$$

Here

$$V = \begin{bmatrix} V_{ds} \\ V_{qs} \\ V_{dr} \\ V_{qr} \end{bmatrix} = \begin{bmatrix} 0 \\ V_{qs} \\ 0 \\ -V_{qs}\cdot \\ \text{Cos}\ \alpha \end{bmatrix}$$

where

$$A = X_s \cdot X_r - X_{12}^2$$

$$B = X_s \cdot X_r - X_{12}^2 + 0.55\ X_s \cdot X_f$$

$$[R] = \begin{bmatrix} R_s & -X_s & 0 & -X_{12} \\ X_s & R_s & X_{12} & 0 \\ 0 & -SX_{12} & R_r & -S.X_r \\ SX_{12} & 0 & SX_r & R_r+0.55\ X_f \end{bmatrix}$$

Where $S = [1 - \omega_r]$ in per unit

$$\begin{bmatrix} [(X_r/A).R_s] & [(X_r).(-X_s)+(-X_{12}/A).(-S.X_{12})] & [(r+R_r).(-X_{12}/A)+0] & [(X_r/A).(-X_{12})+(X_{12}.S.X_r)/A] \\ {}_s(X_r+0.55X_f/B)+S.X_{12}(-X_{12}/B)] & [R_s.(X_r+0.55X_f)/B] & [X_{12}(X_r+0.55X_f)/B+ S.X_r(-X_{12}/B)] & [-X_{12}/B.(R_r+0.55X_f)] \\ [-X_{12}/A.\ R_s] & [(-X_{12}/A)(-X_s)+(X_s/A)(-S.X_{12})] & [(X_s/A).(R_r)] & [(X^2_{12}/A)+(X_s/A.S.X_r)] \\ \dfrac{[X_s(-X_{12})+(X_s/B.SX_{12})]}{B} & [(R_s.(-X_{12}/B)] & [(-X_{12}/B)+(S.X_r/B)X_s] & [X_s/B\ (R_r+0.55X_f)] \end{bmatrix}$$

$$
\begin{bmatrix}
[X_r.R_s/A] & [(-X_s.X_r/A)+(S.X^2_{12}/A)] & [-R_r.X_{12}/A] & [(-X_r.X_{12}/A)+(X_{12}.X_r.S/A)] \\
[X_s.(X_r+0.55.X_f/B)-(S.X^2_{12}/B)] & [R_s.(X_r+0.55X_f/B)] & [X_{12}(X_r+0.55X_f/B).(-S.X_r.X_{12}/B] & [(-X_{12}/B)(R_r+0.55X_f)] \\
[-X_{12}.R_s/A] & [(X_s.X_{12}/A)-(S.X_s.X_{12}/A)] & [X_s/A.R_r] & [(-X^2_{12}/A)+(S.X_s.X_r/A)] \\
[(-X_{12}.X_s/B)+(X_s/B).(S.X_{12})] & [-X_{12}.R_s/B] & [(-X^2_{12}/B)+(S.X_r.X_s/B)] & [X_s/B(R_r+0.55X_f)]
\end{bmatrix}
$$

$$\ldots R_s/A)] + [i_{qs}.(-X_s.X_r+X^2_{12}(1-\omega_r)/A] + [i_{dr}.(i_{dr}(R_r.X_{12})/A] + [i_{qr}.(-X_r.X_{12}+X_{12}.X_r(1-\omega_r)/A]$$

$$\ldots(X_r+0.55X_f) - (1-\omega_r)X^2_{12}) + [i_{qs}(R_s(X_r+0.55X_f/B))] + [i_{dr}(X_{12}(X_r+0.55.X_f-(1-\omega_r).(X_r.X_{12}/B))] + [i_{qr}(-X_{12}/B.(R_r.0.55X_f)]$$

$$\ldots_2.R_s/A)] + [i_{qs}(X_s.X_{12}-(1-\omega_r).(X_s.X_{12})/A)] + [i_{dr}(X_s/A).R_r] + [i_{qr}(-X^2_{12} + (1-\omega_r)X_s.X_r/A)]$$

$$\frac{\ldots_2,X_s+X_s(1-\omega_r).X_{12})]}{B} + [i_{qs}(-X_{12}.R_s/B)] + [i_{dr}(-X^2_{12}+(1-\omega_r)X_r.X_s)/B))] + [i_{qr}(X_s/B).(X_s/B(R_r+0.55.X_f))]$$

$$[X]^{-1}[V] = \begin{bmatrix} [0] \\ [V_{qs}(X_r+0.55X_f/B) + (-X_{12}/B)(-V_{qs}.\cos\alpha))] \\ [0] \\ [(-V_{qs}.X_{12}/B) - (V_{qs}.\cos\alpha.X_s/B))] \end{bmatrix}$$

$$= [X]^{-1}[V] - [X]^{-1}[R].[I]$$

$$= [0 - (i_{ds}.X_r.R_s/A)] + [i_{qs}(-X_s.X_r + X_{12}^2(1-\omega_r)/A] + [-i_{dr}.R_r.X_{12}/A] + [i_{qr}\frac{(-X_r.X_{12} + X_{12}.X_r(1-\omega_r)]}{A}$$

$$= [V_{qs}(X_r + 0.55X_f/B) + (X_{12}/B)\ V_{qs}.\cos\alpha)] - [i_{ds}(X_s.(X_r + 0.55X_f) - (1-\omega_r).X_{12}^2)/B))] + [i_{qs}.R_s.(X_r + 0.55X_f/B)] +$$
$$[i_{dr}(X_{12}(X_r + 0.55X_f - (1-\omega_r)(X_r - X_{12})/B))] + [i_{qr}(-X_{12})/B.(R_r + 0.55X_f)]$$

$$= [[o] - (i_{ds}(-X_{12}).R_s/B))] + [i_{qs}(X_s.X_{12} - (1-\omega_r).X_s.X_{12})/A] + [i_{dr}.X_s.R_r/A] + [i_{qr}(-X_{12}^2 + (1-\omega_r)X_s.X_r)/A]$$

$$= [(-V_{qs}.X_{12})/B - (V_{qs}.\cos\alpha\, X_s)/B] - [i_{ds}(-X_{12}X_s + X_s.X_{12}(1-\omega_r)/B] + [i_{qs}.R_s(-X_{12})/B] +$$
$$[i_{dr}(-X_{12}^2 + (1-\omega_r)X_r.X_s.)/B] + [i_{qr}(X_s/B.(R_r + 0.55.X_f))]$$

Torque Equation

$$T_e = T_L + D.\omega_r + 2H.P.\omega_r; \quad \text{Where} \quad P = \frac{d}{dt}$$

$$p.\omega_r = (T_e - T_L - D.\omega_r)/2.H.$$

Substituting for $T_e = [X_{12}(i_{qs}.i_{dr} - i_{ds}.i_{qr})]$

$$p.\omega_r = [X_{12}.(i_{qs}.i_{dr} - i_{ds}.i_{qr}) - T_L - D.\omega_r]/2H$$

implifying :

$$s = [-(i_{ds}.X_r.\ R_s.)/A + [i_{qs}(1 + X_{12}^2 \cdot \omega_r)/A] + [i_{dr}.\ R_r.\ X_{12}]/A + [i_{qr}.\ X_r.\ X_{12}.\ \omega_r]/A \quad \ldots (1)$$

$$s = [V_{qs}/B \cdot (X_r + 0.55\ X_f + X_{12}.\ \mathrm{Cos}\ \alpha)] - [i_{ds}(1 + \omega_r.\ X_{12}^2)/B] - [i_{qs}.\ R_s.\ (X_r + 0.55\ X_f)/B]$$

$$- [i_{dr}(0.55.\ X_f.\ X_{12} + X_r.\ X_{12}.\ \omega_r)] + [i_{qr}(X_{12}/B.\ (R_r + 0.55\ X_f)] \quad \ldots (2)$$

$$i_{dr} = [i_{ds}.\ X_{12}.\ R_s]/A - [i_{qs}/A\ .(X_s.\ X_{12}.\ \omega_r)] - [i_{dr}/A\ .\ X_s.\ R_r] + [i_{qr}(1 - X_s.\ X_r.\ \omega_r)/A] \quad \ldots (3)$$

$$i_{qr} = \frac{-V_{qs}}{B} \cdot (X_{12} + X_s.\ \mathrm{Cos}\ \alpha) + [i_{ds}(X_s.\ X_{12}.\ \omega_r)/B] + [i_{qs}./B\ R_s.\ X_{12}] - [i_{dr}/B\ (A - \omega_r.\ X_s.\ X_r)]$$

$$- [i_{qr}(X_s/B\ .\ (R_r + 0.55\ X_f)] \quad \ldots (4)$$

$$r = [((X_{12}(i_{qs}.\ i_{dr} - i_{ds}.\ i_{qr}) - T_L - D.\ \omega_r)/2H] \quad \ldots (5)$$

These five equations are used for the analysis of static scherbius drive under transient conditions, using Runge - Kutta 4^{th} order method. A flow chart for the above method is shown in flow chart 1 & 2.

4.3. TRANSIENT ANALYSIS

(Sub Synchronous Mode)

During steady-state operation a balanced induction motor fed from a balanced supply develops a constant torque, while the stator currents pulsate sinusoidally at supply frequency. However following a switching operation or some other changes inoperating circumstances, the motor experiences a transient condition during which the instantaneous currents and torque may reach several times their steady-state values. Although this condition may only persist for a very short time, the transient torque may impose undue strain on the mechanical components, and shaft failures have been attributed to this cause.

The basic differential equation describing the transient condition has been developed in the modelling section. A useful Analytical Solution is only possible when the equations are linearised by assuming that the speed remains unchanged during the transient period.

Of the total computing time used in solution i.e.. expended in solving the algebric equations, the interconnected networks at each step often forms the major parts. Any method of digital computation which promote an increase in the length of step interval, without increasing the errors in the solution are therefore valuable, since they reduce the total computing time. However increasing the step length of an essentially approximate mathematical procedure reduces its accuracy and while the necessary network solution at each step usually consumes most of the total computing time, the overall accuracy is largely controlled by the integration routine.

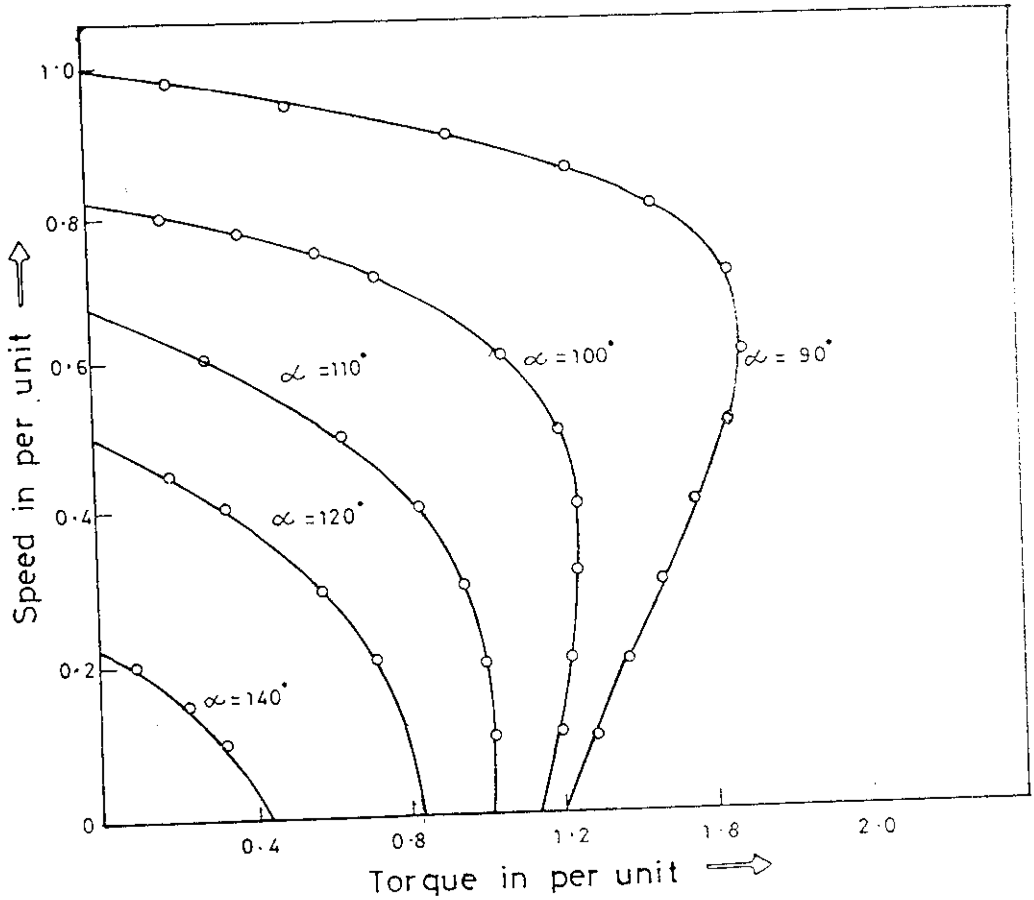
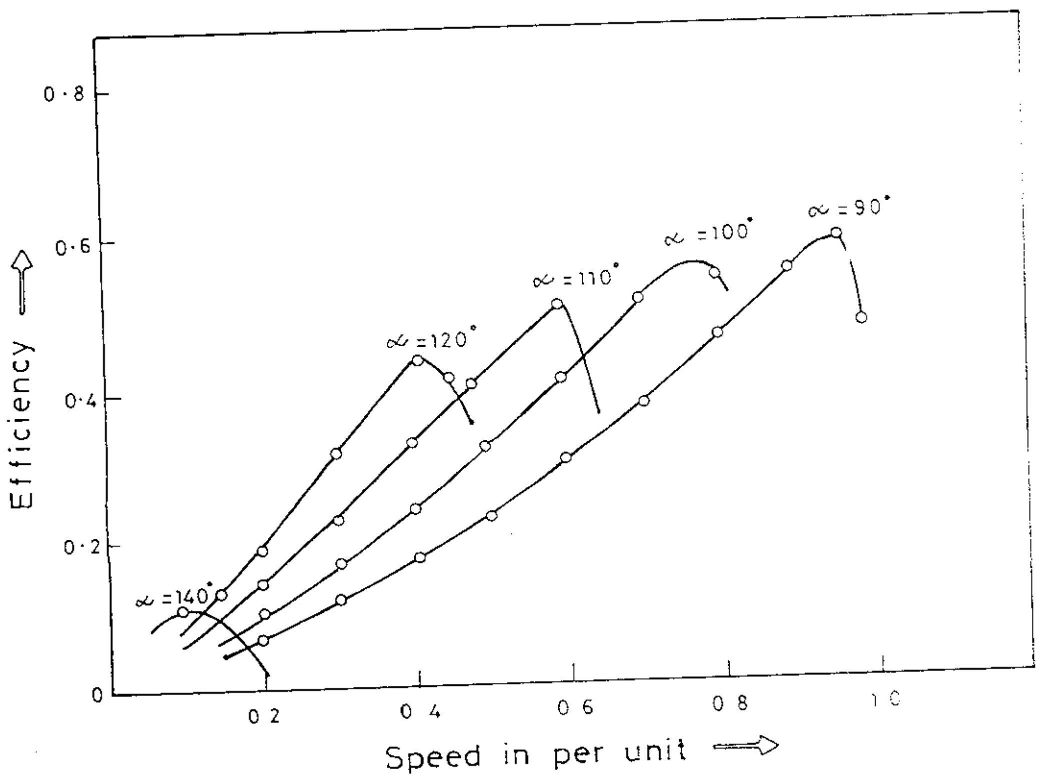


FIG.14.1. SPEED_TORQUE CHARACTERISTIC



software and results

CHAPTER - V

DIGITAL COMPUTER SOFTWARE

5.1. PROGRAM DEVELOPMENT

Let

$$A = XS.\ X_r - X_{12}^2$$

$$B = A + 0.55.\ X_f\ .XS$$

$$S_1 = Cos\ (ANG\ (3.1416/180.0)$$

$$P_1 = (-X_r).\ R_s/A.$$

$$R_1 = X_{12}^2\ /A$$

$$S_1 = X_{12}.R_r/A$$

$$T_1 = X_{12}.\ X_r/A$$

$$A_1 = (X_r + 0.55.X_f + X_{12}.D_1).\ SVQ/B$$

$$Q_2 = (-X_{12}).\ X_{12}/B$$

$$R_2 = (-R_s).\ (X_r + 0.55\ X_f)/B$$

$$T_2 = (-X_{12}).\ X_r/B$$

$$B_1 = X_{12}.(R_r + 0.55\ R_f)/B$$

$$P_3 = X_{12}.R_s/A$$

$$Q_3 = (-X_{12}).X_s/A$$

$$R_3 = (-X_s).\ R_r/A$$

$$T_3 = (-X_s).\ X_r/A$$

$$A_2 = (-X_{12}-X_x.D_1).\ SVQ/B$$

$$P_4 = X_{12}.\ X_s/B$$

$$Q_4 = X_{12}.R_s/B$$

$$R_4 = (-A)/B$$

$$S_4 = X_s.X_r/B$$

$$T_4 = (-X_s).\ .\ (R_r + 0.55\ R_f)/B$$

Substituting the above constants in the following equations :

$$\frac{P}{\omega} \cdot L_{ds} = \left[\frac{-(X_r . R_s). i_{ds}}{A} + [(1 + \frac{X_{12}^2 \cdot \omega_r}{A}) \cdot i_{qs}] + [\frac{R_r \cdot X_{12}}{A} \cdot i_{dr} + (\frac{X_r \cdot X_{12} \cdot \omega_r}{A}) \cdot i_{qr} \right] \quad (1)$$

$$\frac{P}{\omega} \cdot i_{qs} = \left[[-\frac{V_{qs}}{B} (X_r + 0.55 X_f + X_{12} \cdot \cos\alpha)] - i_{ds}(-\frac{1 + \omega_r . X_{12}^2}{B}) - [i_{qs}.R_s(\frac{X_r + 0.55 X_f}{B})] - [i_{dr}(0.55 X_f.X_{12} + X_r.X_{12}. \omega_r)] + [i_{qr}(\frac{X_{12}}{B} (R_r + 0.55 X_f))] \right] \quad (2)$$

$$\frac{P}{\omega} \cdot i_{dr} = \left[[\frac{i_{ds} \cdot X_{12} . R_s}{A}] - [\frac{i_{qs}}{A} (X_s \cdot X_{12} \cdot \omega_r)] - [\frac{i_{dr}}{A} . X_s . R_r] + [i_{qe} . (1 - \frac{X_s X_r . \omega_r}{A})] \right] \quad (3)$$

$$\frac{P}{\omega} \cdot i_{qr} = \left[[\frac{-V_{qs}}{B} (X_{12} + X_s . \cos\alpha)] + [\frac{i_{ds}. (X_s . X_{12} . \omega_r)}{B}] + [\frac{i_{qs}}{B} R_s . X_{12}] - [\frac{i_{dr}}{B} (A - \omega_r . X_s . X_r)] - [i_{qr} (\frac{X_s}{B} (R_r + 0.55 X_f)] \right] \quad (4)$$

$$p. \omega_r = -[\frac{3/2 (X_{12} (i_{qs} \cdot i_{dr} - i_{ds} \cdot i_{qr}) - T_2 - D. \omega_r]}{2H}$$

We obtain the required equations used in the program -

$$F(1) = [(P_1 . S(1) + (1.0 + R_1 . \omega_r) . S(2) + S_1 . S(3) + T_1 . \omega_r . S(4)]$$

$$F(2) = [(A_1 + (-1.0 + Q_2 . \omega_r) . S(1) + R_2 . S(2) + (S_2 + T_2 . \omega_r)$$

$$S(3) + B_1 . S(4)]$$

$$F(3) = [(P_3 . S(1) + Q_3 . \omega_r . S(2) + R_3 . S(3) + (1.0 + T_3 \; \omega_r) . S(4)]$$

$$F(4) = [(A_2 + P_4 . \omega_r . S(1) + Q_4 . S(2) + (R_4 + S_4 . \omega_r) . S(3) + T_4 . S(4)]$$

$$F(5) = [(A_3 . (S(2) . S(3) - S(1) . S(4))) + B_3 + G_3 . \omega_r]$$

where

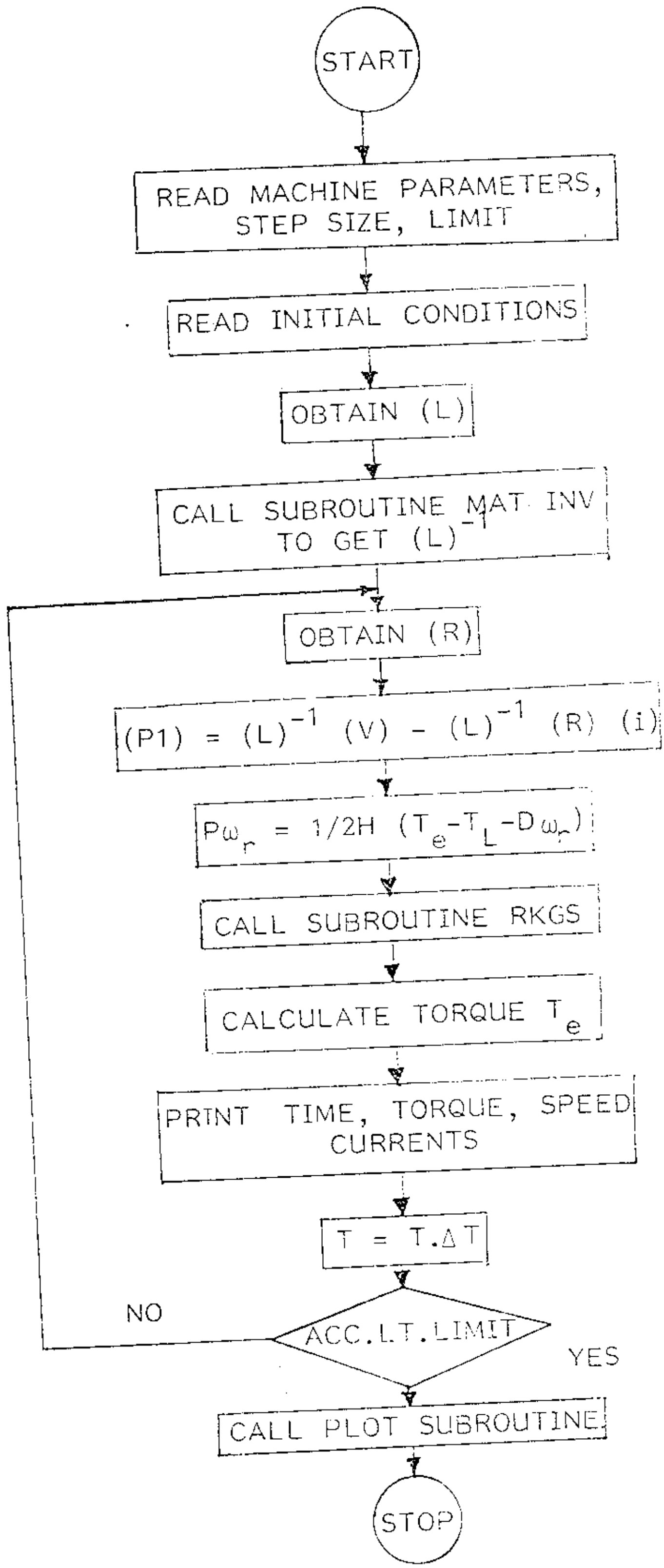
$$F(1) = \frac{d.\ i_{ds}}{dt} = p.\ i_{ds} \; ; \; S(1) = i_{ds}$$

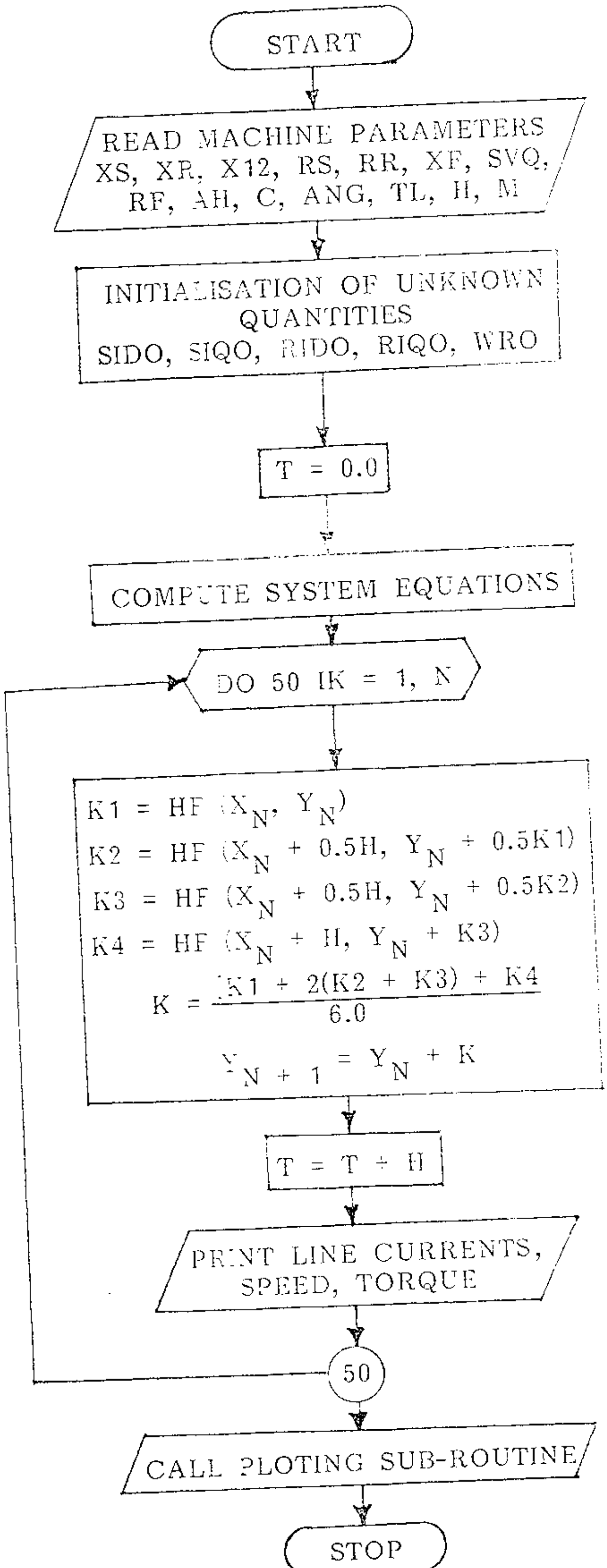
$$F(3) = \frac{d_{idr}}{dt} = p.\ i_{dr} \; ; \; S(3) = i_{dr}$$

$$F(4) = \frac{d_{iqr}}{dt} = p.\ i_{qr} \; ; \; S(4) = i_{qr}$$

$$F(5) = \frac{d.\ \omega_r}{dt} = p.\ \omega_r$$

$$F(2) = \frac{di_{qs}}{dt} = p.i_{qs} \; ; \; S(2) = i_{qs}$$


START
READ MACHINE PARAMETERS, STEP SIZE, LIMIT
READ INITIAL CONDITIONS
OBTAIN (L)
CALL SUBROUTINE MAT INV TO GET $(L)^{-1}$
OBTAIN (R)
$(P1) = (L)^{-1} (V) - (L)^{-1} (R) (i)$
$P\omega_r = 1/2H (T_e - T_L - D\omega_r)$
CALL SUBROUTINE RKGS
CALCULATE TORQUE T_e
PRINT TIME, TORQUE, SPEED CURRENTS
$T = T.\Delta T$
ACC.LT.LIMIT
NO
YES
CALL PLOT SUBROUTINE
STOP


START
READ MACHINE PARAMETERS
XS, XR, X12, RS, RR, XF, SVQ,
RF, AH, C, ANG, TL, H, M
INITIALISATION OF UNKNOWN
QUANTITIES
SIDO, SIQO, RIDO, RIQO, WRO
T = 0.0
COMPUTE SYSTEM EQUATIONS
DO 50 IK = 1, N
K1 = HF (X_N, Y_N)
K2 = HF (X_N + 0.5H, Y_N + 0.5K1)
K3 = HF (X_N + 0.5H, Y_N + 0.5K2)
K4 = HF (X_N + H, Y_N + K3)
K = (K1 + 2(K2 + K3) + K4) / 6.0
Y_N + 1 = Y_N + K
T = T + H
PRINT LINE CURRENTS,
SPEED, TORQUE
50
CALL PLOTING SUB-ROUTINE
STOP

```
C                 5.2  COMPUTER PROGRAM
C       ****************************************************
C       PROGRAM TO CALCULATE TRANSIENT RESPONSE OF STATIC SHERBIUS
C       DRIVE SYSTEM USING RUNGE-KUTTA FOURTH ORDER METHOD
C       ****************************************************
C                                BY
C                         GANESH BABU.R
C                         NACHIAPPAN.K.V
C                         REMESH DHANABAL.V
C                         SENTHIL VEL.R
C                         SURESH.D
C                         VINAYAGAM.M
C                [ FINAL  B.E. {E.E.E} : 1987 - '88 ]
C       ****************************************************
        INTEGER CHAR(15)
        DIMENSION S(4),AK(5),BK(5),CK(5),DK(5),SS(4)
        DIMENSION TIM(101),SPD(101),TOR(101),Z(101,9)
        COMMON A,B,D1,P1,R1,S1,T1,A1,Q2,R2,S2,T2,B1,P3,Q3,
     *  R3,T3,A2,P4,Q4,R4,S4,T4,A3,B3,C3,H
        COMMON IPLOT,IVAR(10)
        DATA CHAR(1),CHAR(2),CHAR(3),CHAR(4),CHAR(5),
     *  CHAR(6),CHAR(7),CHAR(8),CHAR(9),CHAR(10),CHAR(11),
     *  CHAR(12),CHAR(13),CHAR(14),CHAR(15)/' 1',' 2',' 3',' 4',
     *  ' 5',' 6',' 7',' 8',' 9','10',' E',' U',' Y',' R','  '/
        OPEN(1,FILE='PT.DOT', STATUS='OLD')
        READ(1,100)(S(I),I=1,4),WR
  100   FORMAT(5F8.4)
        WRITE(*,401)(S(I),I=1,4),WR
  401   FORMAT(5F8.4)
        READ(1,101)XS,XR,X12,RS,RR,XF,SVQ
  101   FORMAT(7F8.4)
        WRITE(*,402)XS,XR,X12,RS,RR,XF,SVQ
  402   FORMAT(7F8.4)
        READ(1,102)RF,AH,D,ANG,TL,H,M
  102   FORMAT(6F8.4,I3)
        WRITE(*,403)RF,AH,D,ANG,TL,H,M
  403   FORMAT(6F8.4,I3)
        WRITE(*,400)
  400   FORMAT(5X,1HT,6X,3HIDS,5X,3HIQS,5X,3HIDR,5X,3HIQR,5X,3HIAS,
     *  5X,3HIBS,5X,3HICS,6X,2HWR,4X,6HTORQUE)
        T=0.0
        A=XS*XR-X12*X12
        B=A+0.55*XF*XS
        D1=COS(ANG*3.1416/180.0)
        P1=(-XR)*RS/A
```

```
      S1=X12*RR/A
      T1=X12*XR/A
      A1=(XR+0.55*XF+X12*D1)*SVQ/B
      Q2=(-X12)*X12/B
      R2=(-RS)*(XR+0.55*XF)/B
      S2=(-0.55)*XF*X12/B
      T2=(-X12)*XR/B
      B1=X12*(RR+0.55*RF)/B
      P3=X12*RS/A
      Q3=(-X12)*XS/A
      R3=(-XS)*RR/A
      T3=(-XS)*XR/A
      A2=(-X12-XS*D1)*SVQ/B
      P4=X12*XS/B
      Q4=X12*RS/B
      R4=(-A)/B
      S4=XS*XR/B
      T4=(-XS)*(RR+0.55*RF)/B
      A3=X12/(2.0*AH)
      B3=(-TL)/(2.0*AH)
      C3=(-D)/(2.0*AH)
      DO 200 JK = 1,M
      CALL FUN(S,WR,AK)
      T=T+0.5*H
      DO 203 I=1,4
203   SS(I)=0.5*AK(I)+S(I)
      SWR=0.5*AK(5)+WR
      CALL FUN(SS,SWR,BK)
      DO 205 I=1,4
205   SS(I)=0.5*BK(I)+S(I)
      SWR=0.5*BK(5)+WR
      CALL FUN(SS,SWR,CK)
      DO 207 I=1,4
207   SS(I)=CK(I)+S(I)
      SWR=CK(5)+WR
      T=T+0.5*H
      CALL FUN(SS,SWR,DK)
      DO 209 I=1,4
209   S(I)=S(I)+(AK(I)+2.0*(BK(I)+CK(I))+DK(I))/6.0
      WR=WR+(AK(5)+2.0*(BK(5)+CK(5))+DK(5))/6.0
      TORQUE=X12*(S(2)*S(3)-S(1)*S(4))
      SIDF=S(1)*COS(314.16*T)-S(2)*SIN(314.16*T)
      SIQF=S(1)*SIN(314.16*T)+S(2)*COS(314.16*T)
      SIA=SIDF
      SIB=(-0.5)*SIQF-0.866*SIDF
      SIC=(-0.5)*SIQF+0.866*SIDF
      WRITE(*,601)T,S(1),S(2),S(3),S(4),SIA,SIB,SIC,WR,TORQUE
601   FORMAT(1X,10F8.4)
```

```
      TIM(JK)=T
      SPD(JK)=WR
      TOR(JK)=TORQUE
200   CONTINUE
      DO 500 J=1,3
      DO 500 I=1,101
      GOTO(30,40,50),J
30    Z(I,1)=TIM(I)
      GOTO 500
40    Z(I,2)=SPD(I)
      GOTO 500
50    Z(I,3)=TOR(I)
500   CONTINUE
      DO 607 I =1,8
607   IVAR(I)=CHAR(15)
      READ(1,1008)(IVAR(I),I=1,8)
1008  FORMAT(8A2)
      IPLOT=2
      CALL Y8VSX(Z,101,2,10)
      STOP
      END
C     ******************************************************************
      SUBROUTINE FUN(S,WR,F)
      DIMENSION S(4),F(5)
      COMMON A,B,D1,P1,R1,S1,T1,A1,Q2,R2,S2,T2,B1,P3,Q3,
     * R3,T3,A2,P4,Q4,R4,S4,T4,A3,B3,C3,H
      DO 300 J=1,5
      GO TO (310,320,330,340,350),J
310   F(1)=(P1*S(1)+(1.0+R1*WR)*S(2)+S1*S(3)+T1*WR*S(4))*H*314.16
      GO TO 300
320   F(2)=(A1+(-1.0+Q2*WR)*S(1)+R2*S(2)+(S2+T2*WR)*S(3)+
     * B1*S(4))*H*314.16
      GO TO 300
330   F(3)=(P3*S(1)+Q3*WR*S(2)+R3*S(3)+(1.0+T3*WR)*S(4))*H*314.16
      GO TO 300
340   F(4)=(A2+P4*WR*S(1)+Q4*S(2)+(R4+S4*WR)*S(3)+
     * T4*S(4))*H*314.16
      GO TO 300
350   F(5)=(A3*(S(2)*S(3)-S(1)*S(4))+B3+C3*WR)*H
300   CONTINUE
      RETURN
      END
C     ******************************************************************
      SUBROUTINE Y8VSX(A,N,M,NGRID)
C     THIS SUBROUTINE PLOTS UP TO 8 VARIABLES VERSES TIME
C     THIS IS BOTH A X-T AND X-Y PLOT ROUTINE
       INTEGER CHAR(15)
```

```
     *    CHAR(7),CHAR(8),CHAR(9),CHAR(10),CHAR(11),CHAR(12),
     *    CHAR(13),CHAR(14),CHAR(15)/'1','2','3','4','5','6',
     *    '7','8','9','A','E','U','Y','R',' '/
          DATA ISTAR,II,IPER,IDASH,IBLANK/'*','I','.','-',' '/
100       FORMAT(////,9X,11(1PE10.2))
101       FORMAT(1PE13.2,2X,101A1)
102       FORMAT(15X,101A1)
          YMAX=A(1,2)
          YMIN=A(1,2)
          MP1=M+1
          DO 4 J=2,MP1
          DO 4 I=1,N
          IF(YMAX-A(I,J)) 1,2,2
1         YMAX=A(I,J)
2         IF(YMIN-A(I,J)) 4,4,3
3         YMIN=A(I,J)
4         CONTINUE
5         YSHFT=YMIN*100.0/(YMAX-YMIN)
6         NM1=N-1
          DO 8 I=1,NM1
          IP1=I+1
          DO 8 K=IP1,N
          IF(A(K,1)-A(I,1)) 7,8,8
7         DO 88 J=1,MP1
          ATEMP=A(I,J)
          A(I,J)=A(K,J)
          A(K,J)=ATEMP
88        CONTINUE
8         CONTINUE
          XMIN=A(1,1)
          XMAX=A(N,1)
          ABSCA(1)=XMIN
          ABSCA(11)=XMAX
          ORDIN(1)=YMIN
          ORDIN(11)=YMAX
          DO 9 I=2,10
          Q=I-1
          ABSCA(I)=(XMAX-XMIN)*Q/10.0+XMIN
9         ORDIN(I)=(YMAX-YMIN)*Q/10.0+YMIN
          WRITE(*,100) (ORDIN(J),J=1,11)
          STEPX=(XMAX-XMIN)/100.0
          KDELX=1
          KLINE=1
          LINE=1
          DO 26 IND=1,N
          IF(NGRID.EQ.0) GO TO 200
          KSTEP=LINE
          GO TO 201
200       KSTEP=(A(IND,1)-XMIN)/STEPX+1.5
```

```
201     DO 10 J=2,MP1
10      TEMPY(J)=A(IND,J)*100.0/(YMAX-YMIN)-YSHFT
11      IF(KLINE-LINE) 12,12,18
12      DO 13 I=2,100
13      KAXIS(I)=IDASH
        DO 14 I=1,101,10
14      KAXIS(I)=ISTAR
        IF(KSTEP-LINE) 15,15,17
15      DO 16 I=2,MP1
        K=TEMPY(I)+1.5
        MM=IVAR(I-1)
16      KAXIS(K)=CHAR(MM)
17      WRITE(*,101)ABSCA(KDELX),(KAXIS(J),J=1,101)
        IF(NGRID.EQ.0) GO TO 202
        KLINE=KLINE+NGRID
        ABSCA(KDELX)=A(KLINE,1)
        GO TO 24
202     KLINE=KLINE+10
        KDELX=KDELX+1
        GO TO 24
18      DO 19 I=2,100
19      KAXIS(I)=IBLANK
        DO 20 I=1,101,10
20      KAXIS(I)=IPER
        IF(KSTEP-LINE) 21,21,23
21      DO 22 I=2,MP1
        K=TEMPY(I)+1.5
        MM=IVAR(I-1)
22      KAXIS(K)=CHAR(MM)
23      WRITE(*,102) (KAXIS(J),J=1,101)
24      LINE=LINE+1
        IF(LINE-102) 25,25,27
25      IF(KSTEP-LINE) 26,11,11
26      CONTINUE
27      STOP
        END
C       ************************************************************
```

3 : R E S U L T S

(Graphical repersentation of this set of results is shown in plot - 3)

N	M
4	100

H	DA	DELTA	TL	ANG
.1000	.0100	.0050	.0000	110.0000

XS	XR	X12	XF
.0000	3.0000	2.8000	.9000

RS	RR	RF	SVQ
.0580	.0720	.0400	1.1000

IDS	IQS	IDR	IQR	WR	-INITIAL VALUES
.0000	.0000	.0000	.0000	.0000	

T	IDS	IQS	IDR	IQR	IAS	IBS	ICS	WR	TORQUE
.0050	.8898	1.1270	-.5934	-.8546	.8898	.5311	-1.4209	.0000	.2565
.0100	2.0022	.9867	-1.4829	-1.0090	-.9867	2.2272	-1.2406	.0064	1.5597
.0150	2.0826	.4955	-1.8422	-.7474	-2.0826	.6122	1.4704	.0454	1.8023
.0200	1.7764	.5167	-1.7549	-.5264	.5167	-1.7967	1.2800	.0904	.0796
.0250	1.8945	.5816	-1.6655	-.3715	1.8945	-.4435	-1.4510	.0924	-.7419
.0300	1.8330	.3090	-1.3849	-.2671	-.3091	1.7419	-1.4329	.0738	.1726
.0350	1.4017	.2402	-1.0792	-.3992	-1.4017	.4928	.9089	.0781	.8406
.0400	1.2841	.5768	-1.1453	-.6319	.5768	-1.4004	.8236	.0991	.4225
.0450	1.6968	.7740	-1.4865	-.6646	1.6968	-.1781	-1.5187	.1097	-.0641
.0500	1.9382	.5296	-1.5833	-.4807	-.5297	1.9433	-1.4136	.1080	.2607
.0550	1.6514	.3062	-1.3353	-.3883	-1.6514	.5605	1.0909	.1145	.6506
.0600	1.3858	.4433	-1.1810	-.4920	.4433	-1.4217	.9784	.1308	.4430
.0650	1.5411	.6276	-1.3158	-.5730	1.5410	-.2270	-1.3141	.1418	.1604
.0700	1.7381	.5387	-1.4239	-.4990	.5388	1.7746	-1.2358	.1458	.2809
.0750	1.6087	.3873	-1.2983	-.4287	-1.6087	.4688	1.1309	.1528	.5230
.0800	1.4183	.4448	-1.1745	-.4846	.4448	-1.4506	1.0058	.1658	.4619
.0850	1.4874	.5717	1.2477	.5503	1.4873	.2485	1.2388	.1773	.2944
.0900	1.5344	.5549	-1.3442	.5071	.5350	1.6829	-1.1478	.1846	.3073
.0950	1.5745	.4220	-1.2730	-.4391	-1.5745	.4217	1.1528	.1923	.4319
.1000	1.4229	.4306	-1.1554	-.4583	.4307	-1.4475	1.0169	.2030	.4331

1050	1.4239	.5169	-1.1672	-.5130	1.4238	-.2642	-1.1597	.2138	.3558
1100	1.5200	.5192	-1.2382	-.5036	-.5193	1.5759	-1.0567	.2226	.3436
1150	1.5100	.4490	-1.2154	-.4554	-1.5100	.3661	1.1439	.2311	.3976
1200	1.4109	.4329	-1.1312	-.4507	.4330	-1.4383	1.0054	.2410	.4097
.1250	1.3305	.4790	-1.1102	-.4825	1.3805	-.2753	-1.1052	.2512	.3760
.1300	1.4274	.4923	-1.1457	-.4857	-.4924	1.4823	-.9899	.2605	.3619
.1350	1.4318	.4538	-1.1399	-.4564	-1.4317	.3228	1.1089	.2695	.3814
.1400	1.3709	.4326	-1.0838	-.4439	.4328	-1.4035	.9708	.2790	.3909
.1450	1.3326	.4527	-1.0515	-.4586	1.3325	-.2741	-1.0584	.2887	.3783
.1500	1.3463	.4654	-1.0606	-.4645	-.4655	1.3986	-.9331	.2981	.3689
.1550	1.3501	.4461	-1.0576	-.4482	-1.3501	.2886	1.0615	.3072	.3734
.1600	1.3134	.4273	-1.0211	-.4348	.4275	-1.3511	.9236	.3165	.3771
.1650	1.2775	.4319	-.9880	-.4380	1.2775	-.2646	-1.0128	.3258	.3719
.1700	1.2713	.4391	-.9799	-.4413	-.4392	1.3205	-.8813	.3351	.3662
.1750	1.2682	.4294	-.9724	-.4320	-1.2681	.2621	1.0060	.3441	.3651
.1800	1.2435	.4150	-.9462	-.4205	.4151	-1.2844	.8692	.3532	.3646
.1850	1.2129	.4119	-.9162	-.4175	1.2128	-.2496	-.9632	.3622	.3612
.1900	1.1967	.4134	-.8989	-.4171	-.4136	1.2431	-.8295	.3711	.3570
.1950	1.1865	.4073	-.8859	-.4106	-1.1865	.2404	.9461	.3800	.3539
.2000	1.1664	.3963	-.8638	-.4009	.3964	-1.2083	.8119	.3887	.3508
.2050	1.1402	.3898	-.8370	-.3948	1.1402	-.2324	-.9078	.3974	.3469
.2100	1.1199	.3871	-.8155	-.3912	-.3872	1.1634	-.7762	.4060	.3427
.2150	1.1044	.3814	-.7979	-.3850	-1.1044	.2218	.8825	.4144	.3385
.2200	1.0852	.3723	-.7767	-.3764	.3724	-1.1259	.7535	.4228	.3340
.2250	1.0617	.3644	-.7520	-.3688	1.0616	-.2151	-.8465	.4310	.3291
.2300	1.0400	.3568	-.7289	-.3628	-.3589	1.0800	-.7211	.4391	.3241
.2350	1.0213	.3523	-.7084	-.3560	-1.0213	.2054	.8159	.4471	.3190
.2400	1.0017	.3439	-.6869	-.3476	.3440	-1.0394	.6954	.4550	.3133
.2450	.9796	.3354	-.6633	-.3392	.9796	-.1992	-.7804	.4627	.3074
.2500	.9578	.3281	-.6400	-.3316	-.3282	.9935	-.6653	.4703	.3014
.2550	.9376	.3206	-.6181	-.3239	-.9375	.1910	.7465	.4777	.2953
.2600	.9174	.3121	-.5962	-.3153	.3123	-.9506	.6383	.4850	.2888
.2650	.8962	.3033	-.5733	-.3064	.8962	.1853	-.7100	.4921	.2820
.2700	.8748	.2950	.5504	-.2979	.2951	.9051	-.6100	.4990	.2751

750	.8543	.2867	-.5282	-.2893	-.8543	.1787	.6756	.5057	.2681
800	.8342	.2780	-.5064	-.2804	.2781	-.8614	.5833	.5123	.2608
850	.8137	.2690	-.4843	-.2713	.8137	-.1738	-.6399	.5187	.2533
900	.7932	.2602	-.4622	-.2622	-.2603	.8171	-.5568	.5249	.2458
950	.7733	.2515	-.4406	-.2533	-.7732	.1687	.6045	.5309	.2381
.000	.7537	.2426	-.4195	-.2442	.2427	-.7741	.5313	.5367	.2304
3050	.7343	.2336	-.3985	-.2350	.7343	-.1647	-.5696	.5424	.2225
3100	.7151	.2247	-.3777	-.2259	-.2249	.7317	-.5068	.5478	.2146
3150	.6963	.2160	-.3574	-.2169	-.6963	.1610	.5353	.5530	.2067
3200	.6780	.2073	-.3376	-.2079	.2074	-.6908	.4834	.5580	.1988
3250	.6601	.1986	-.3182	-.1990	.6600	-.1579	-.5021	.5629	.1908
3300	.6425	.1901	-.2992	-.1902	-.1902	.6514	-.4613	.5675	.1830
3350	.6253	.1817	-.2807	-.1816	-.6253	.1552	.4701	.5719	.1752
3400	.6088	.1735	-.2628	-.1731	.1736	-.6139	.4404	.5762	.1675
3450	.5927	.1654	-.2453	-.1648	.5926	-.1530	-.4396	.5802	.1599
3500	.5770	.1575	-.2284	-.1567	-.1576	.5785	-.4209	.5841	.1524
3550	.5619	.1498	-.2121	-.1488	-.5619	.1511	.4107	.5877	.1451
3600	.5473	.1423	-.1963	-.1411	.1424	-.5452	.4027	.5912	.1379
3650	.5333	.1351	-.1812	-.1336	.5333	-.1496	-.3837	.5945	.1310
3700	.5198	.1281	-.1666	-.1264	-.1282	.5143	-.3861	.5976	.1242
.3750	.5069	.1214	-.1526	-.1194	-.5069	.1483	.3586	.6006	.1177
.3800	.4945	.1149	-.1392	-.1128	.1150	-.4857	.3707	.6034	.1114
.3850	.4827	.1086	-.1264	-.1063	.4827	-.1472	-.3355	.6060	.1053
.3900	.4714	.1027	-.1142	-.1002	-.1028	.4596	-.3568	.6085	.0994
.3950	.4607	.0970	-.1026	-.0943	-.4606	.1463	.3144	.6108	.0938
.4000	.4504	.0915	-.0915	-.0887	.0916	-.4359	.3442	.6130	.0884
.4050	.4407	.0864	-.0810	-.0834	.4407	-.1455	-.2952	.6151	.0833
.4100	.4316	.0815	-.0711	-.0783	-.0816	.4145	-.3329	.6170	.0784
.4150	.4229	.0768	-.0617	-.0735	-.4229	.1448	.2780	.6188	.0737
.4200	.4147	.0724	-.0528	-.0689	.0725	-.3953	.3228	.6205	.0693
.4250	.4069	.0682	-.0445	-.0646	.4060	-.1443	-.2626	.6221	.0652

.4300	.3996	.0643	-.0366	-.0606	-.0644	.3783	-.3139	.6235	.0612
.4350	.3928	.0606	-.0292	-.0568	-.3928	.1438	.2490	.6249	.0575
.4400	.3864	.0572	-.0222	-.0532	.0572	-.3632	.3060	.6262	.0540
.4450	.3803	.0539	-.0157	-.0498	.3803	-.1434	-.2369	.6274	.0507
.4500	.3747	.0508	-.0095	-.0467	-.0509	.3499	-.2990	.6285	.0476
.4550	.3694	.0480	-.0038	-.0437	-.3693	.1431	.2263	.6295	.0447
.4600	.3644	.0453	.0015	-.0410	.0454	-.3383	.2929	.6305	.0420
.4650	.3598	.0428	.0066	-.0384	.3598	-.1428	-.2170	.6314	.0394
.4700	.3555	.0405	.0112	-.0360	-.0405	.3281	-.2875	.6322	.0371
.4750	.3514	.0383	.0156	-.0337	-.3514	.1425	.2089	.6330	.0348
.4800	.3477	.0362	.0196	-.0316	.0363	-.3192	.2829	.6337	.0328
.4850	.3442	.0344	.0234	-.0297	.3442	-.1423	-.2019	.6344	.0308
.4900	.3409	.0326	.0269	-.0278	-.0327	.3116	-.2789	.6350	.0290
.4950	.3379	.0310	.0302	-.0261	-.3379	.1421	.1958	.6355	.0274
.5000	.3351	.0294	.0332	-.0246	.0295	-.3050	.2754	.6361	.0258

graphic plots

Plot - 1

N M
4 100

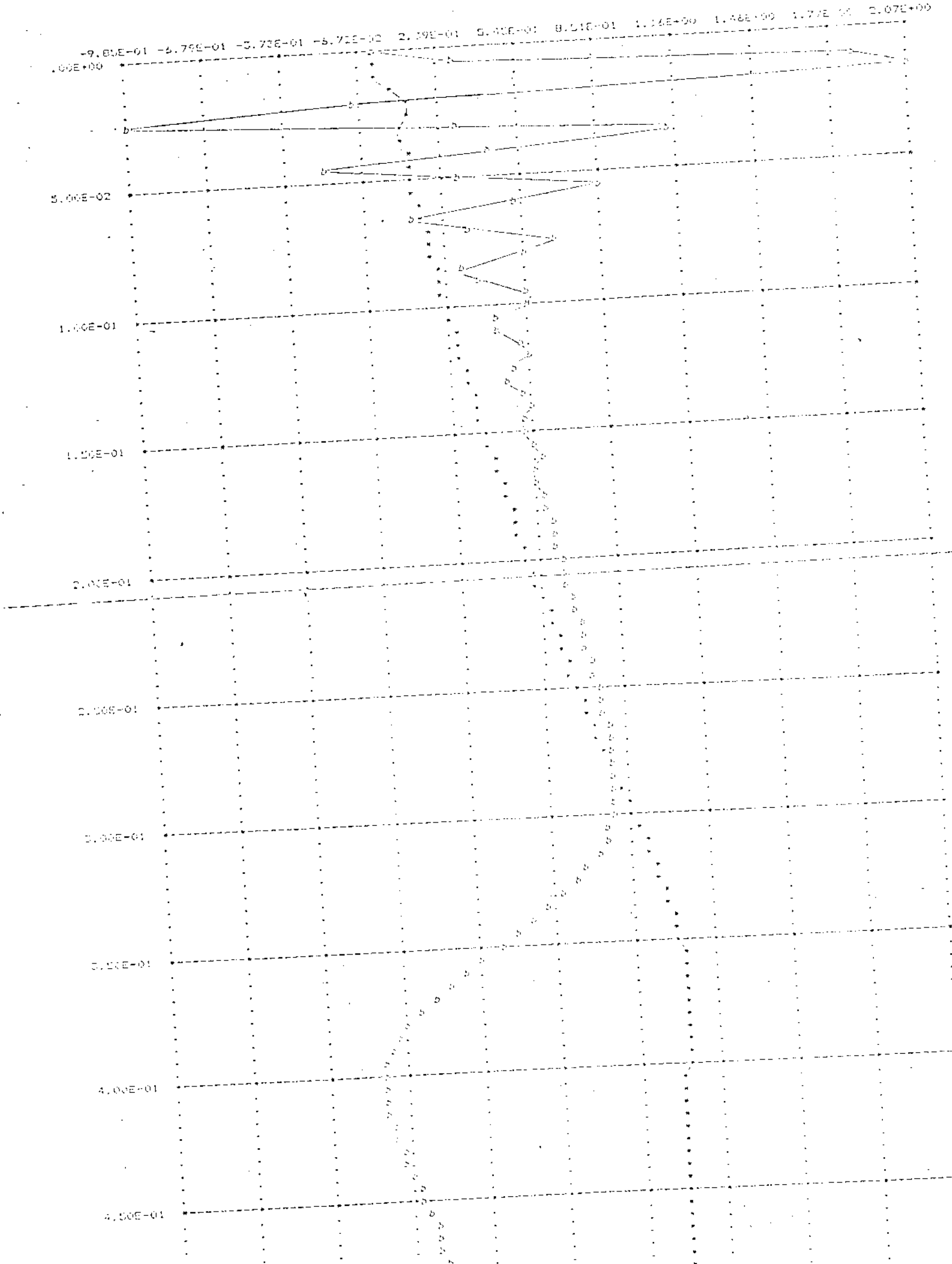
H	DA	DELTA	TL	ANG
.1000	.0100	.0050	.0000	90.0000

XS	XR	X12	XF
3.0000	3.0000	2.8000	.9000

RS	RR	RF	SVQ
.0500	.0720	.0400	1.0000

IQS	IDS	IDR	IQR	WR
.0000	.0000	.0000	.0000	.0000

-INITIAL VALUES

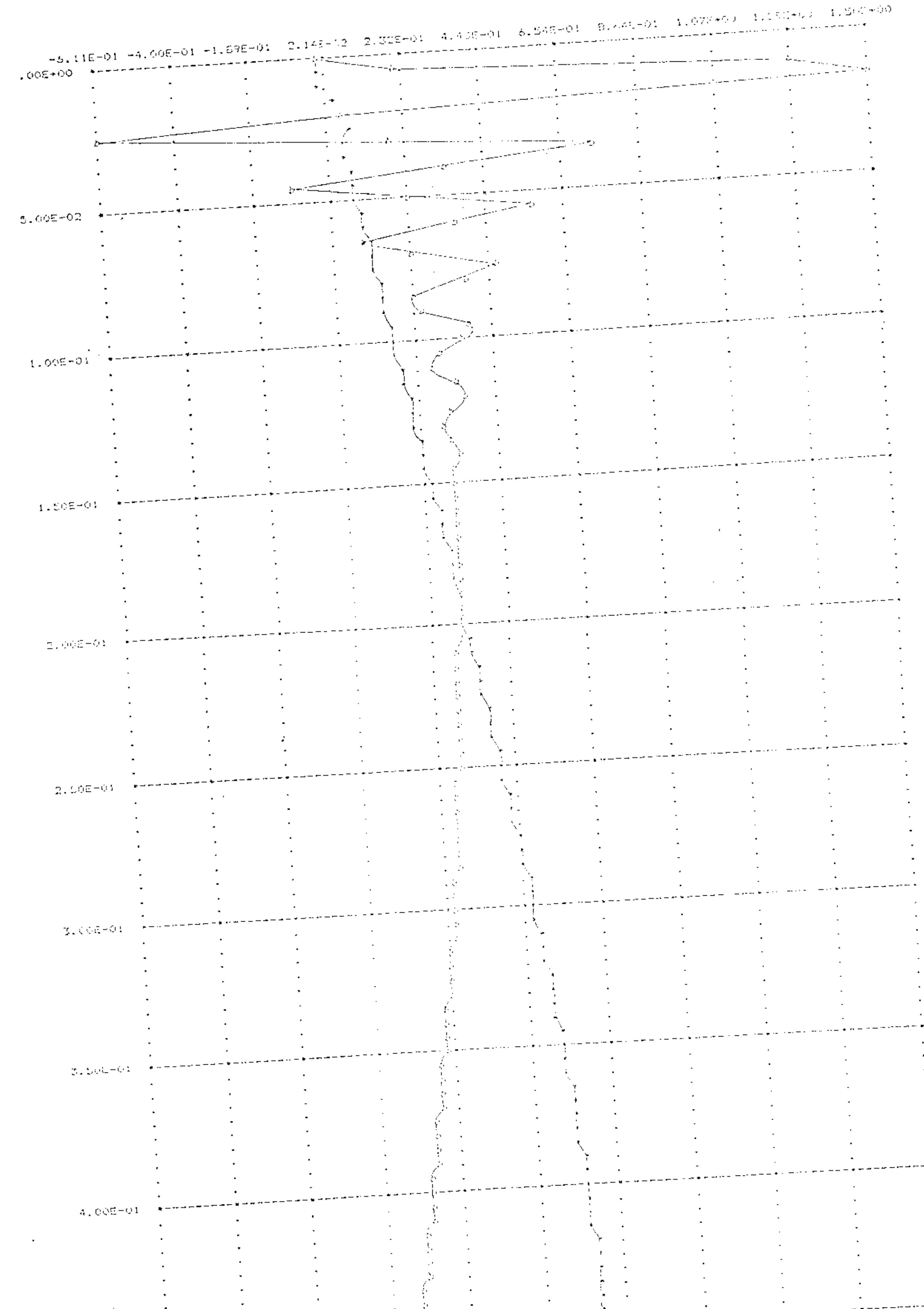


ANG = 110

Plot - 2

XS	XR	X12	XF	
3.0000	3.0000	2.8000	.9000	
RS	RR	RF	SVS	
.0580	.0720	.0400	1.0000	
IDS	IQS	IDR	IQR	WR
.0000	.0000	.0000	.0000	.0000

-INITIAL VALUES



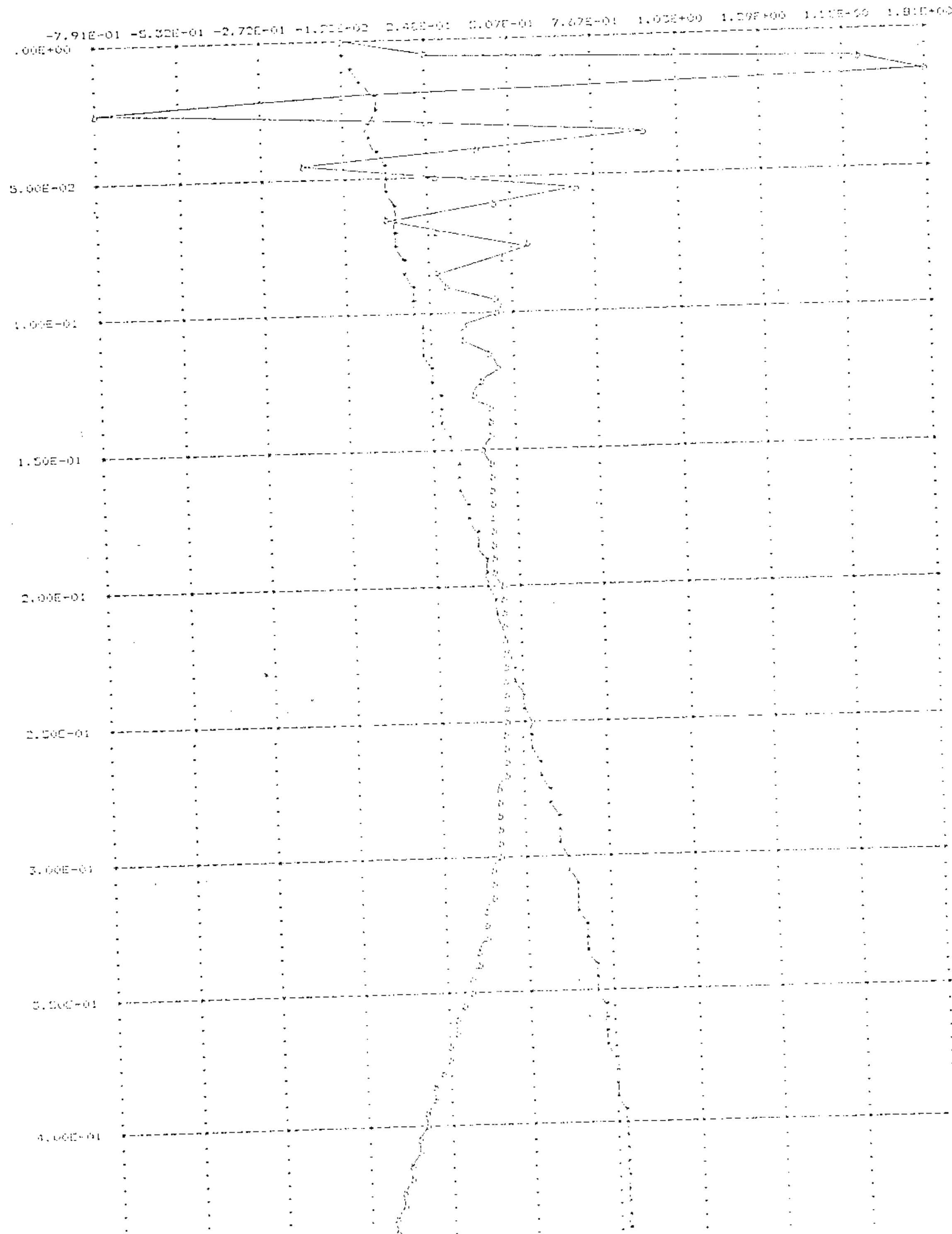
RS	RR	RF	SVD
.0500	.0720	.0400	1.0000

AHG = 110

IDS	IRS	IDR	ICR	WR
.0000	.0000	.0000	.0000	.0000

INITIAL VALUES

Plot - 3



Plot - 4

4 100

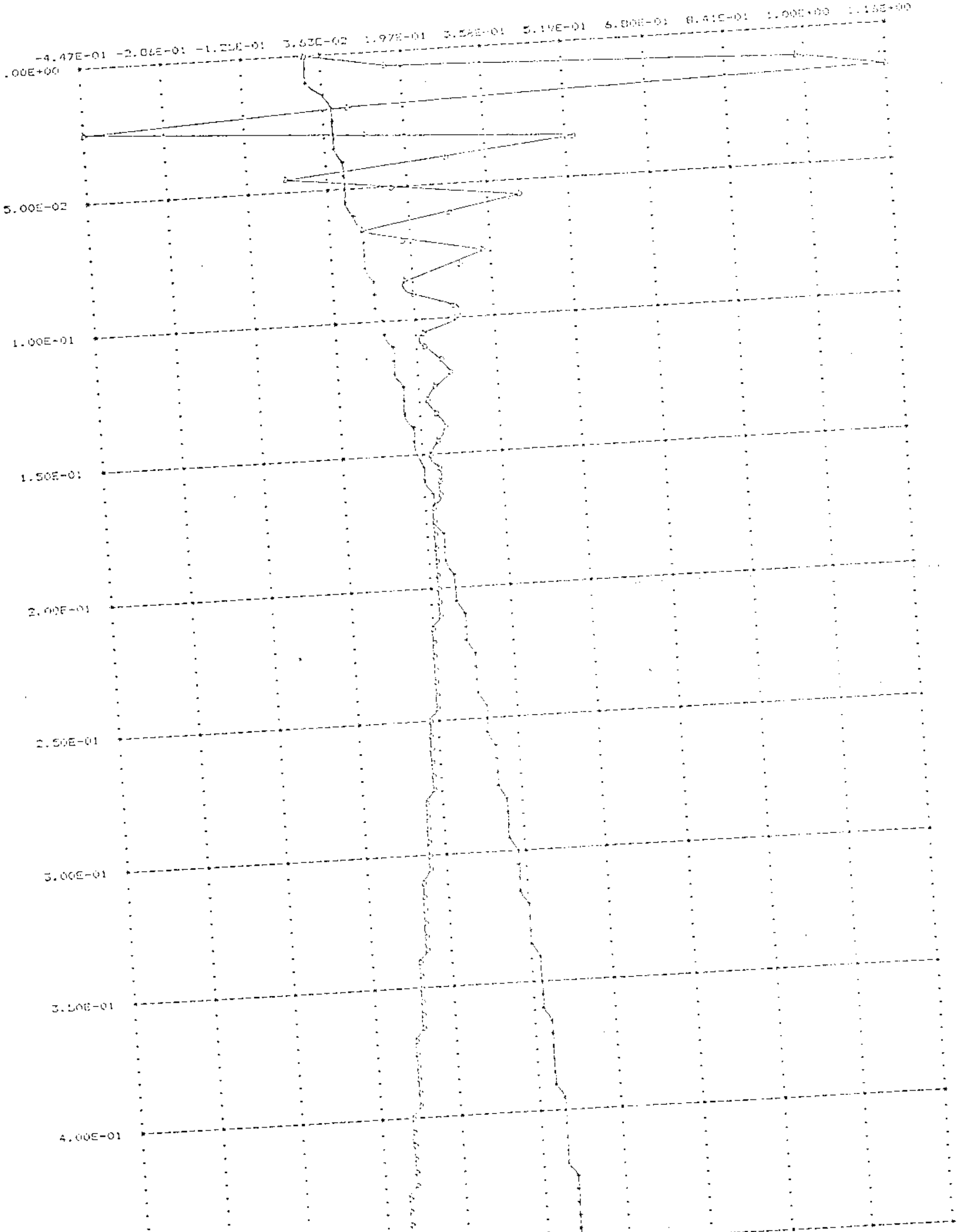
H	DA	DELTA	TL	ANG
.1000	.0100	.0050	.0000	120.0000

XS	XR	X12	XF
3.0000	3.0000	2.8000	.9000

RS	RR	RF	SVQ
.0580	.0720	.0400	1.0000

IDS	IQS	IDR	IQR	WR
.0000	.0000	.0000	.0000	.0000

-INITIAL VALUES



Plot - 5

N	M
4	100

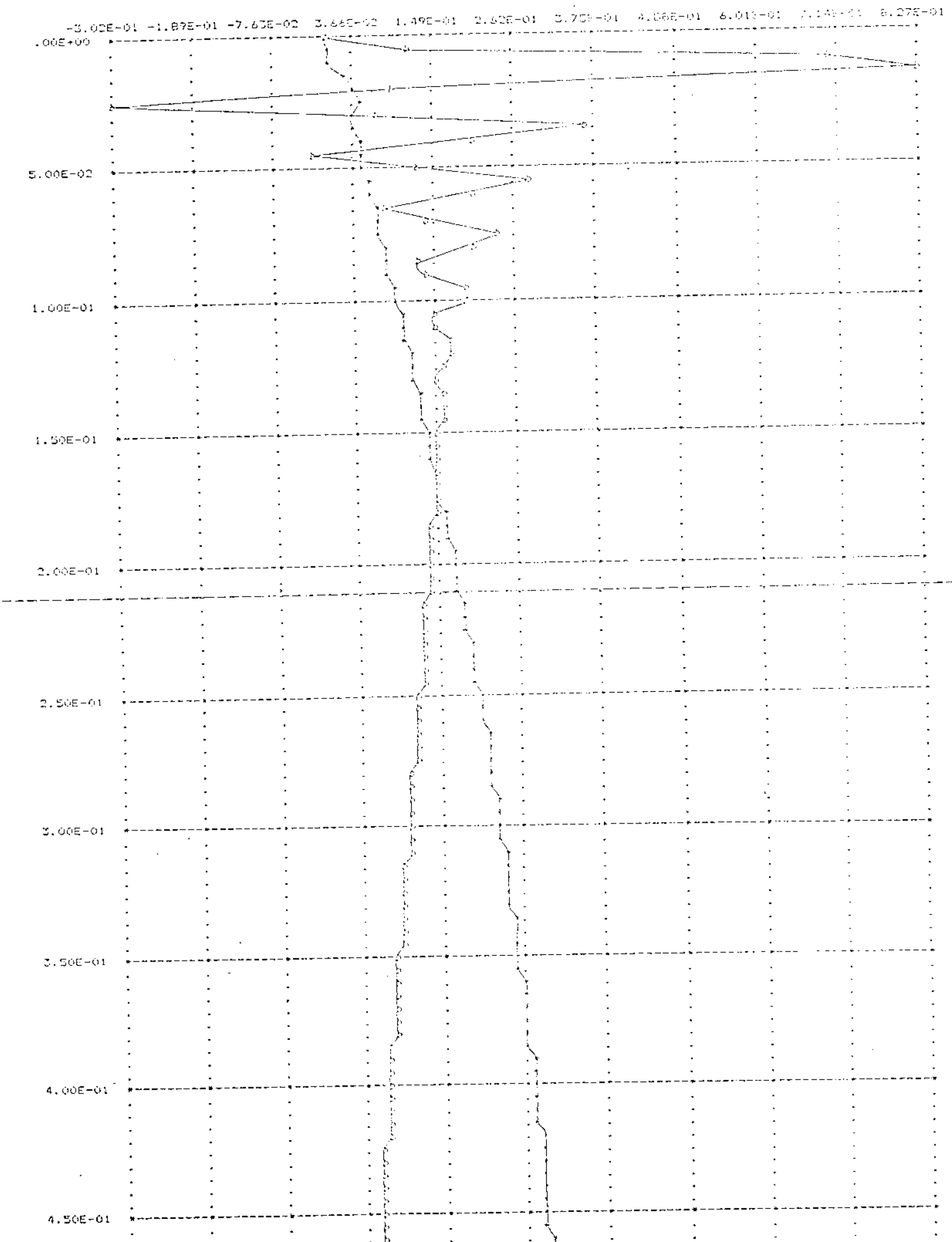
H	DA	DELTA	TL	ANG
.1000	.0100	.0050	.0000	130.0000

XS	XR	X12	XF
3.0000	3.0000	2.8000	.9000

RS	RR	RF	SVQ
.0590	.0720	.0400	1.0000

IDS	IQS	IDR	IQR	WR
.0000	.0000	.0000	.0000	.0000

-INITIAL VALUES



Plot - 6

N	M
4	100

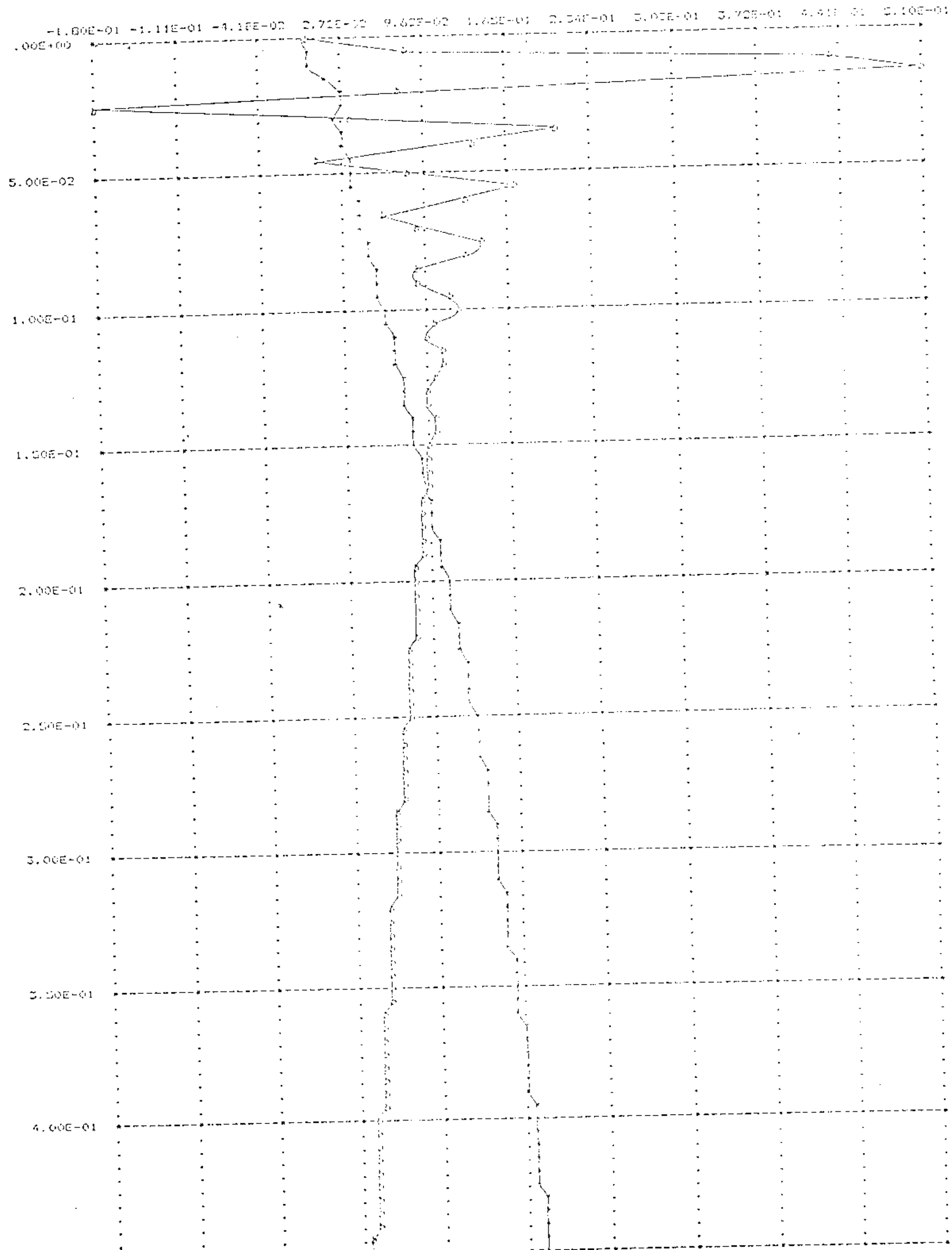
H	DA	DELTA	TL	ANG
.1000	.0100	.0050	.0000	140.0000

XS	XR	X12	XF
3.0000	3.0000	2.8000	.9000

RS	RR	RF	SVQ
.0500	.0720	.0400	1.0000

IDS	IQS	IDR	IQR	WR
.0000	.0000	.0000	.0000	.0000

-INITIAL VALUES



Plot - 7

4 100

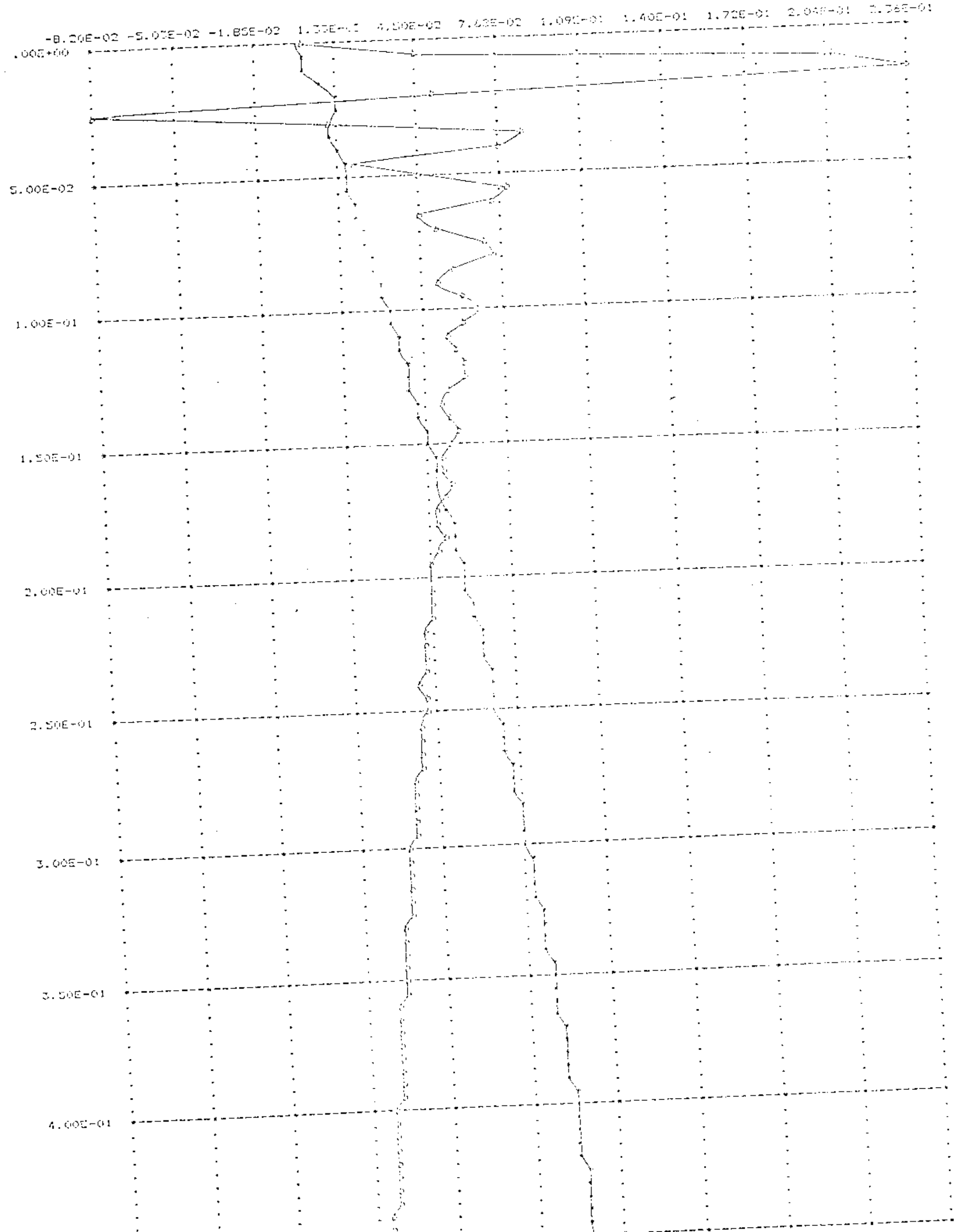
H	DA	DELTA	TL	ANG
.1000	.0100	.0050	.0000	150.0000

XS	XR	X12	XF
3.0000	3.0000	2.8000	.9000

RS	RR	RF	SVQ
.0580	.0720	.0400	1.0000

IDS	IQS	IDR	IQR	WR
.0000	.0000	.0000	.0000	.0000

-INITIAL VALUES



Plot - 8

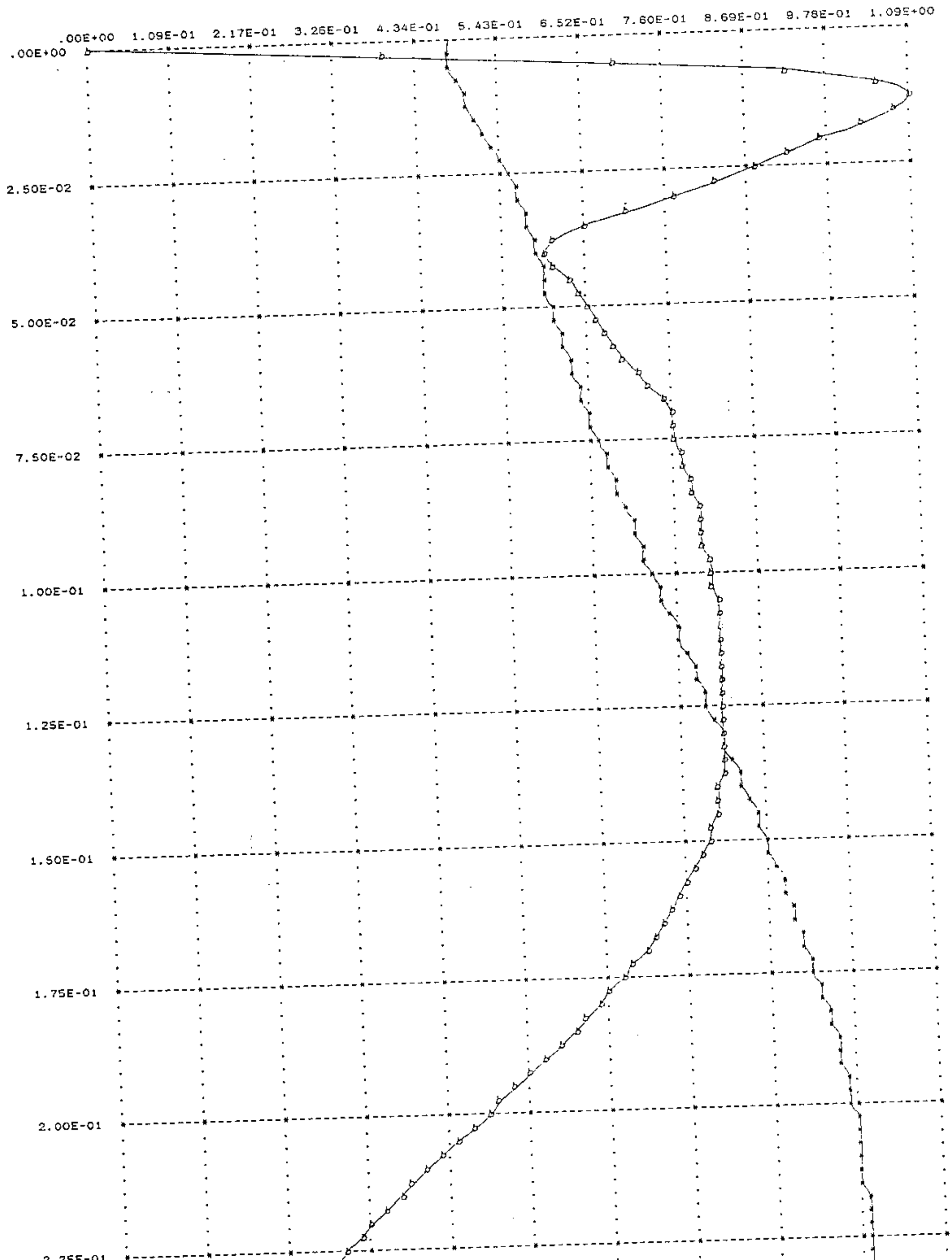
N	M
4	100

H	DA	DELTA	TL	ANG
.1000	.0100	.0025	.2500	90.0000

XS	XR	X12	XF
3.0000	3.0000	2.8000	.9000

RS	RR	RF	SVQ
.0580	.0720	.0400	1.0000

IDS	IQS	IDR	IQR	WR	
.2518	.0085	.0871	-.0039	.4823	-INITIAL VALUES



Plot - 9

N	M
A	100

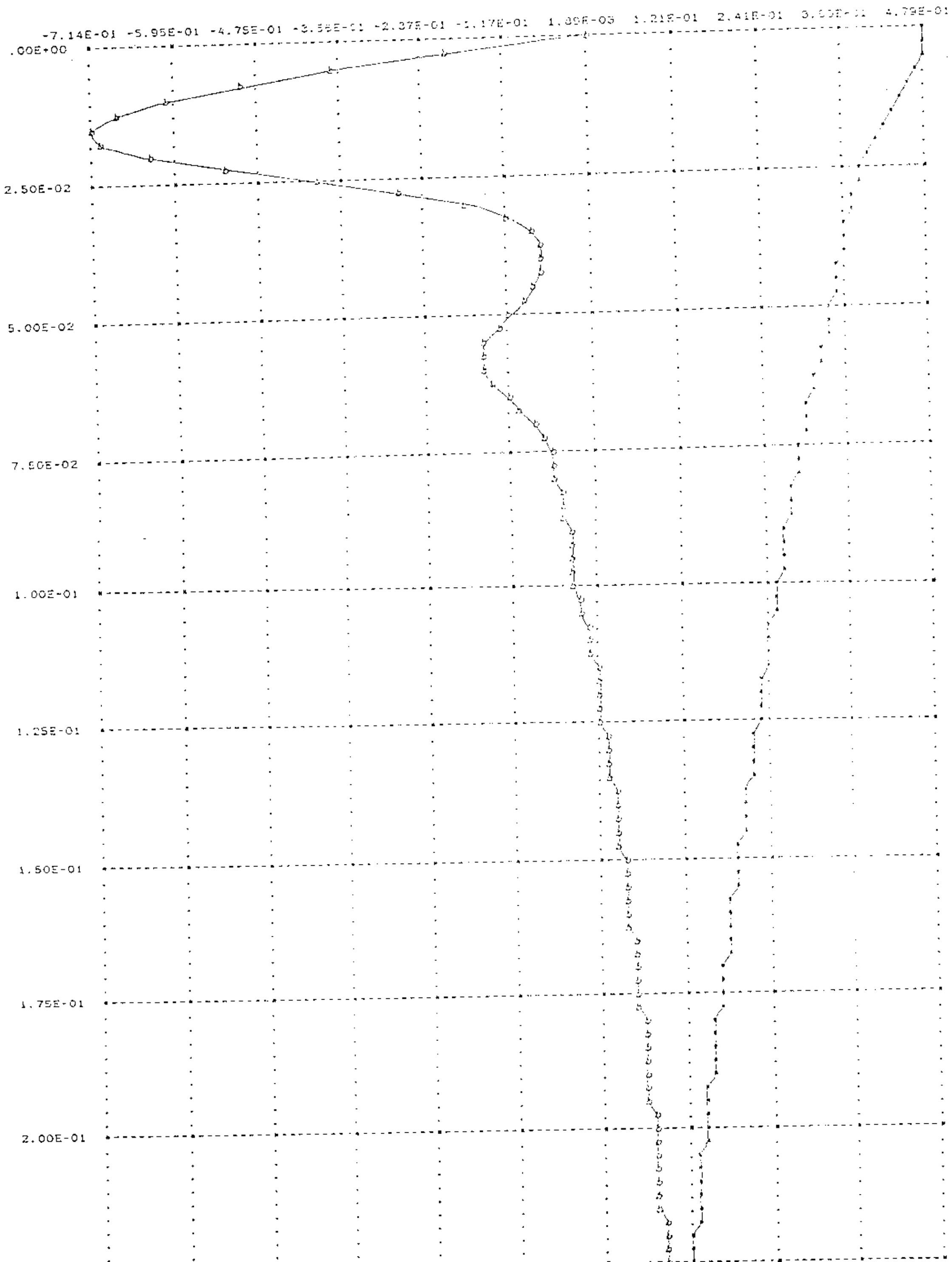
H	DA	DELTA	TL	ANG
.1000	.0100	.0025	.2500	140.0000

XS	XR	X12	XF
3.0000	3.0000	2.8000	.9000

RS	RR	RF	SVQ
.0580	.0720	.0400	1.0000

IDS	IQS	IDR	IQR	WR
.2518	.0085	.0871	-.0039	.4823

-INITIAL VALUES



4 100

H	DA	DELTA	TL	ANG
.1000	.0100	.0025	.2500	120.0000

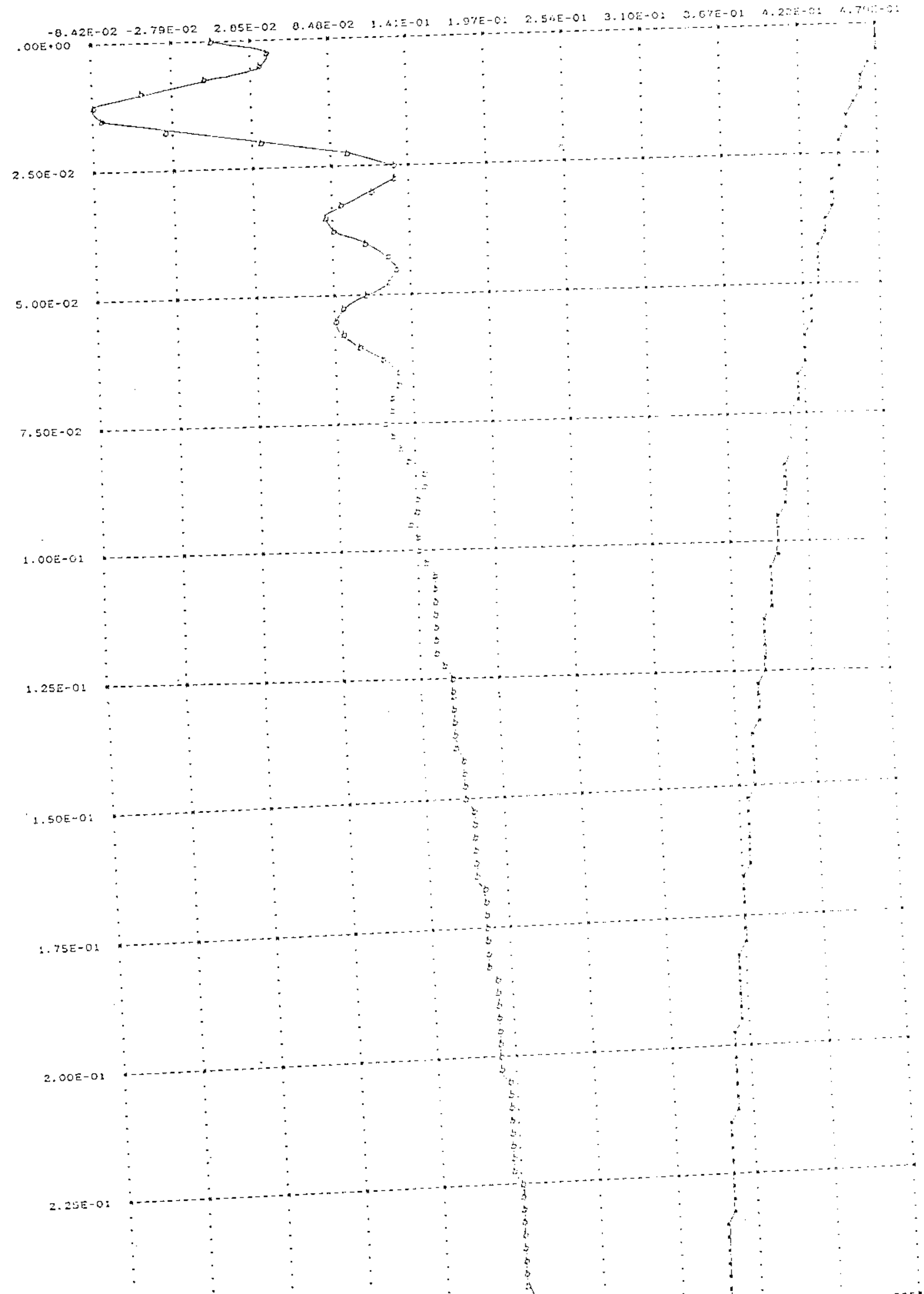
XS	XR	X12	XF
3.0000	3.0000	2.8000	.8000

RS	RR	RF	SVQ
.0500	.0720	.0400	1.1000

IDS	IQS	IDR	IQR	WR
.2518	.0095	.0871	-.0039	.4823

-INITIAL VALUES

Plot - 10



4 100

H	DA	DELTA	TL	ANG
.1000	.0100	.0015	.2500	120.0000

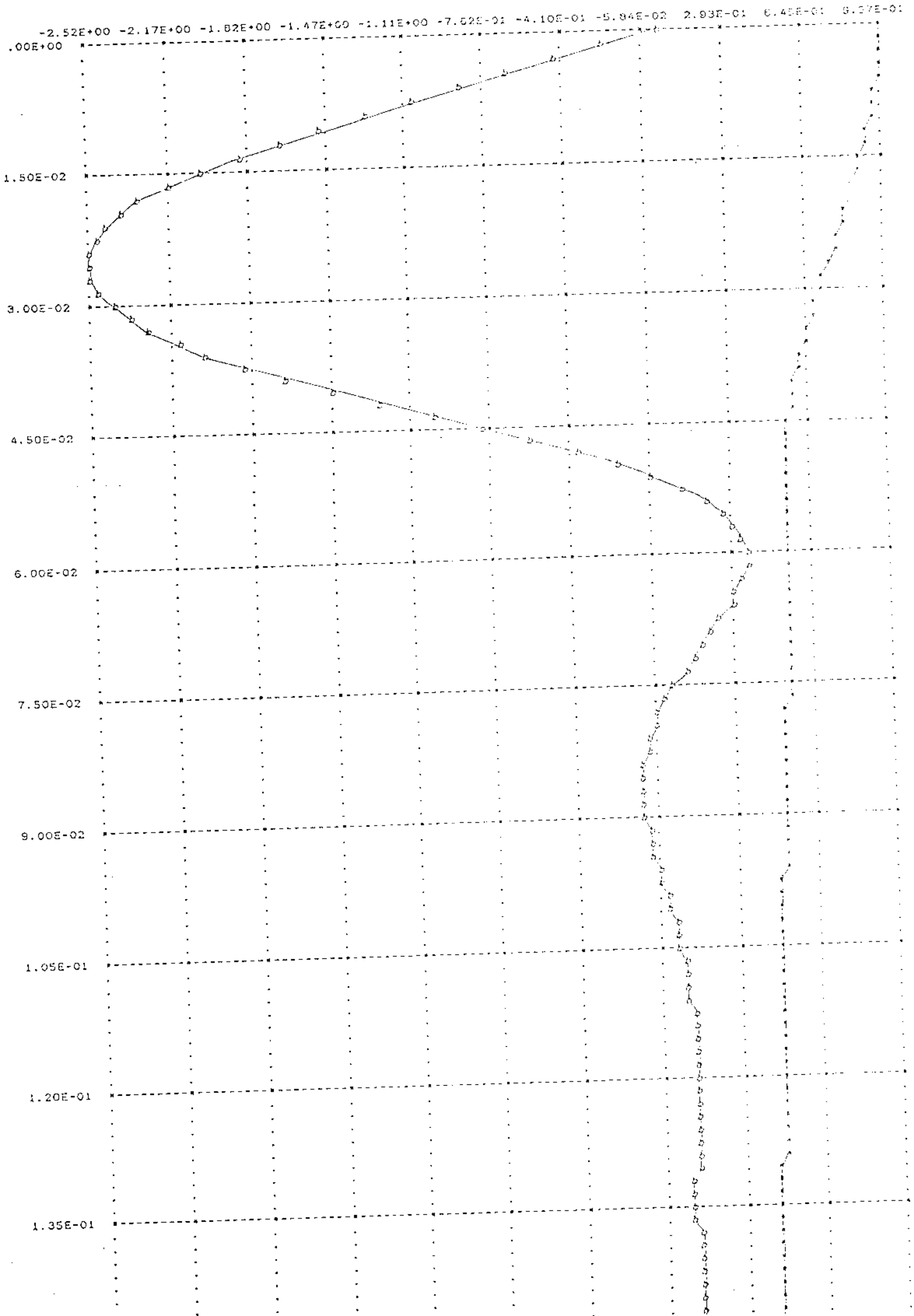
XS	XR	X12	XF
3.0000	3.0000	2.8000	.9000

RS	RR	RF	SVQ
.0500	.0720	.0400	1.0000

IDS	IQS	IDR	IQR	WR
.3328	.0164	.0002	-.0107	.9989

-INITIAL VALUES

Plot - 11



N	M
4	100

H	DA	DELTA	TL	ANG
.1000	.0100	.0025	.2500	120.0000

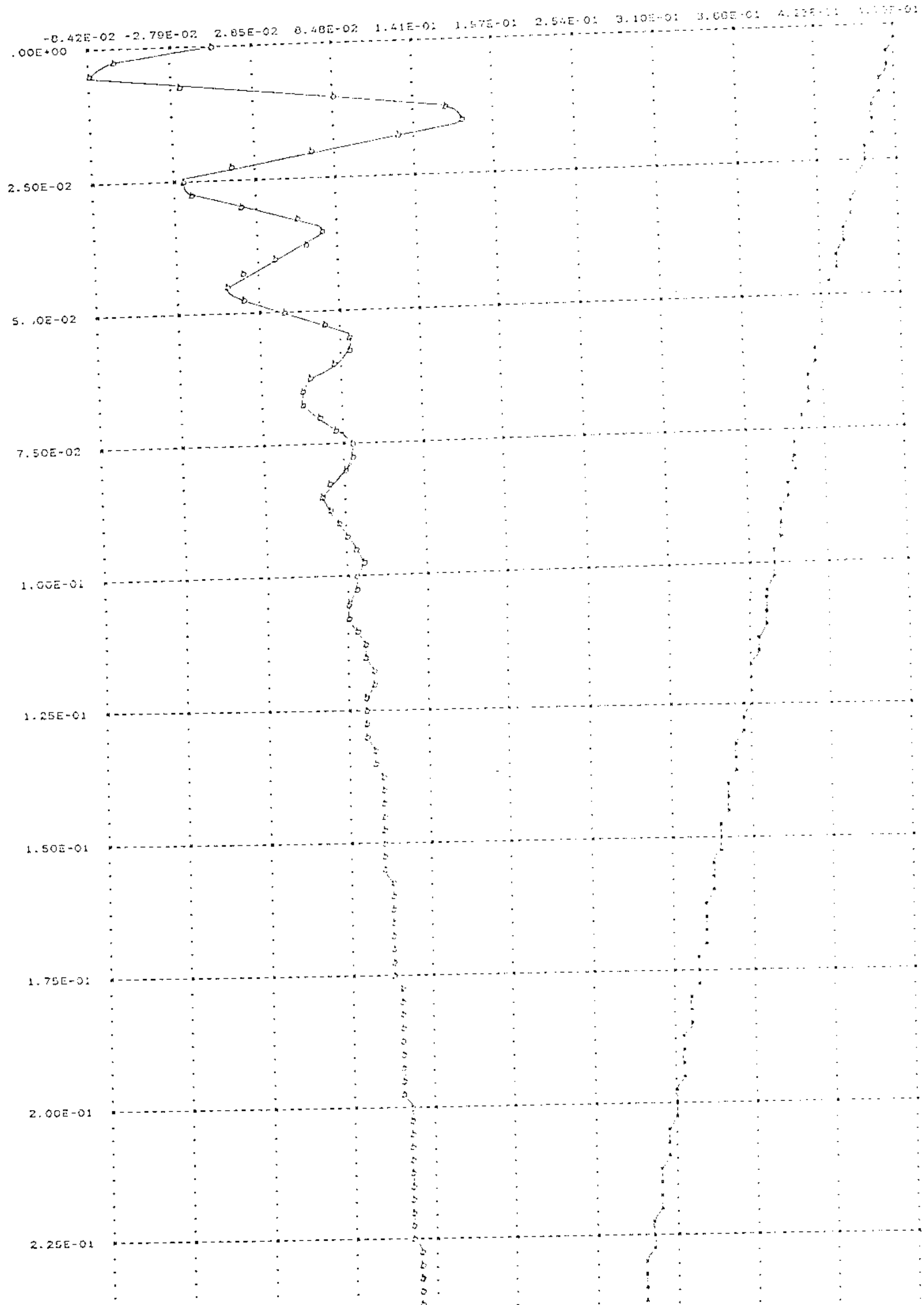
XS	XR	X12	XF
3.0000	3.0000	2.0000	.9000

RS	RR	RF	SVQ
.0580	.0720	.0400	.8000

IDS	IQS	IDR	IQR	VR
.2518	.0085	.0871	-.0039	.4823

-INITIAL VALUES

Plot - 12



N M
4 100

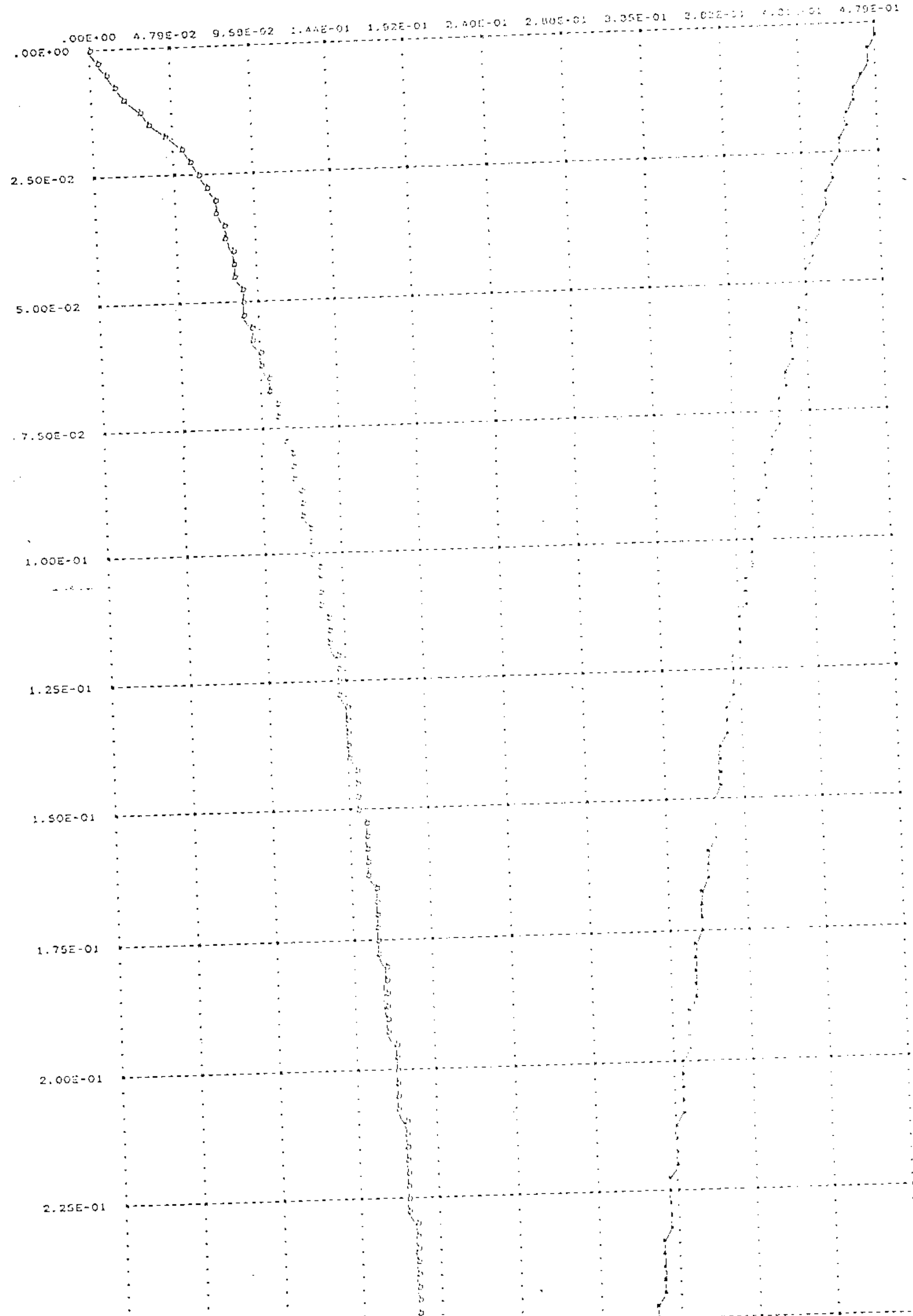
H DA DELTA TL ANG
.1000 .0100 .0025 .2500 120.0000

XS XR X12 XF
3.0000 3.0000 2.8000 .9000

RS RR RF SVQ
.0580 .0720 .0400 1.0000

IDS IQS IDR IQR VR -INITIAL VALUES
.2518 .0085 .0871 -.0039 .4623

Plot - 13



4 100

H	DA	DELTA	TL	ANG
.1000	.0100	.0015	.2500	90.0000

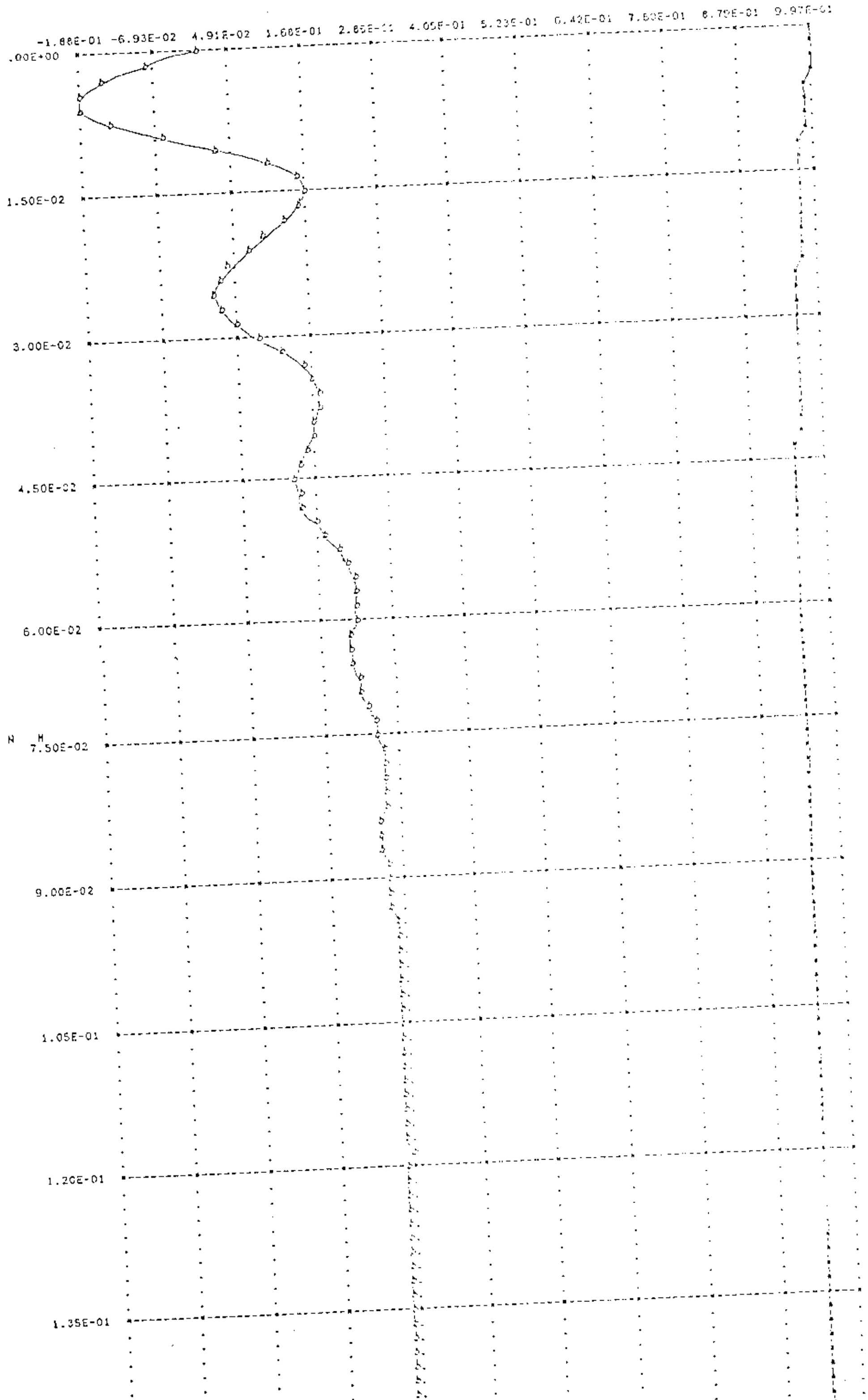
XS	XR	X12	XF
3.0000	3.0000	2.8000	.0000

RS	RR	RF	SVQ
.0580	.0720	.0400	.8000

IDS	IQS	IDR	IQR	VR
.3320	.0164	.0002	-.0107	.9989

-INITIAL VALUES

Plot - 14



N	M
4	100

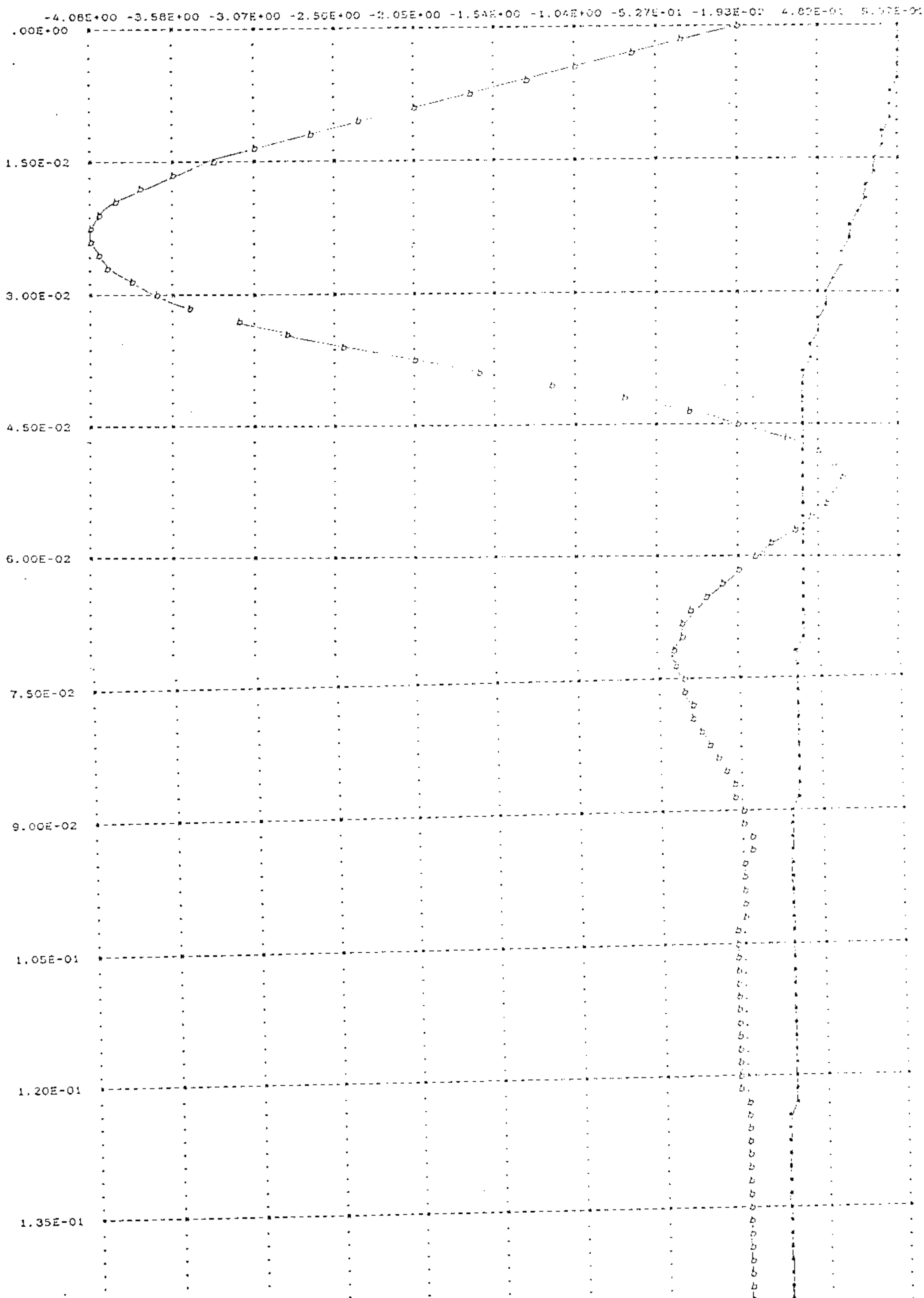
H	DA	DELTA	TL	ANG
.1000	.0100	.0015	.2500	140.0000

XS	XR	X12	XF
3.0000	3.0000	2.8000	.9000

RS	RR	RF	SVQ
.0580	.0720	.0400	1.0000

IDS	IQS	IDR	IQR	WR	
.3328	.0164	.0002	-.0107	.9989	-INITIAL VALUES

Plot - 15



Plot - 16

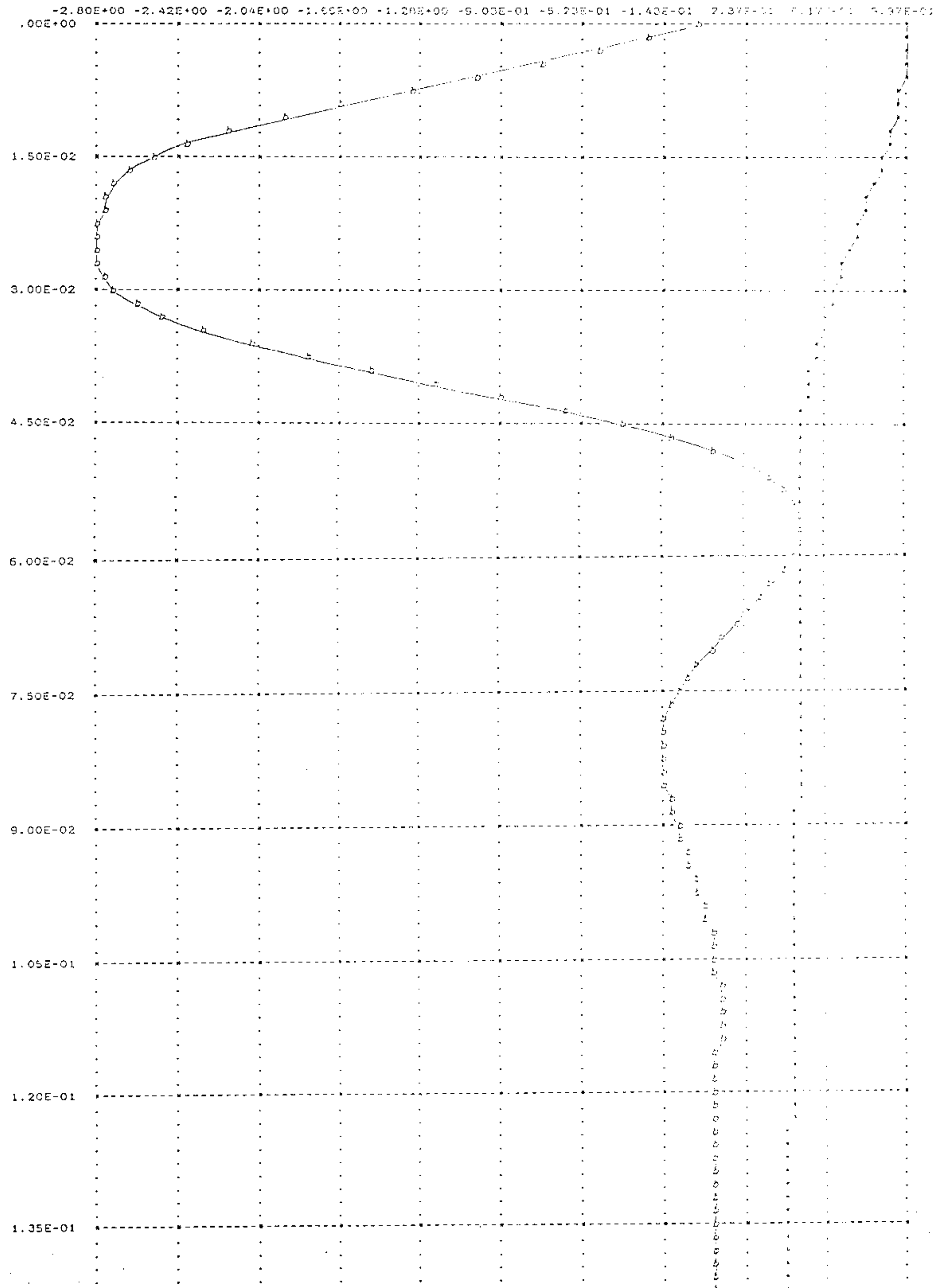
N	M
4	100

H	DA	DELTA	TL	ANG
.1000	.0100	.0015	.2500	120.0000

XS	XR	X12	XF
3.0000	3.0000	2.8000	.9000

RS	RR	RF	SVQ
.0580	.0720	.0400	1.1000

IDS	IQS	IDR	IQR	WR	-INITIAL VALUES
.3328	.0164	.0002	-.0107	.8869	



N	M
4	100

H	DA	DELTA	TL	ANG
.1000	.0100	.0015	.2500	85.0000

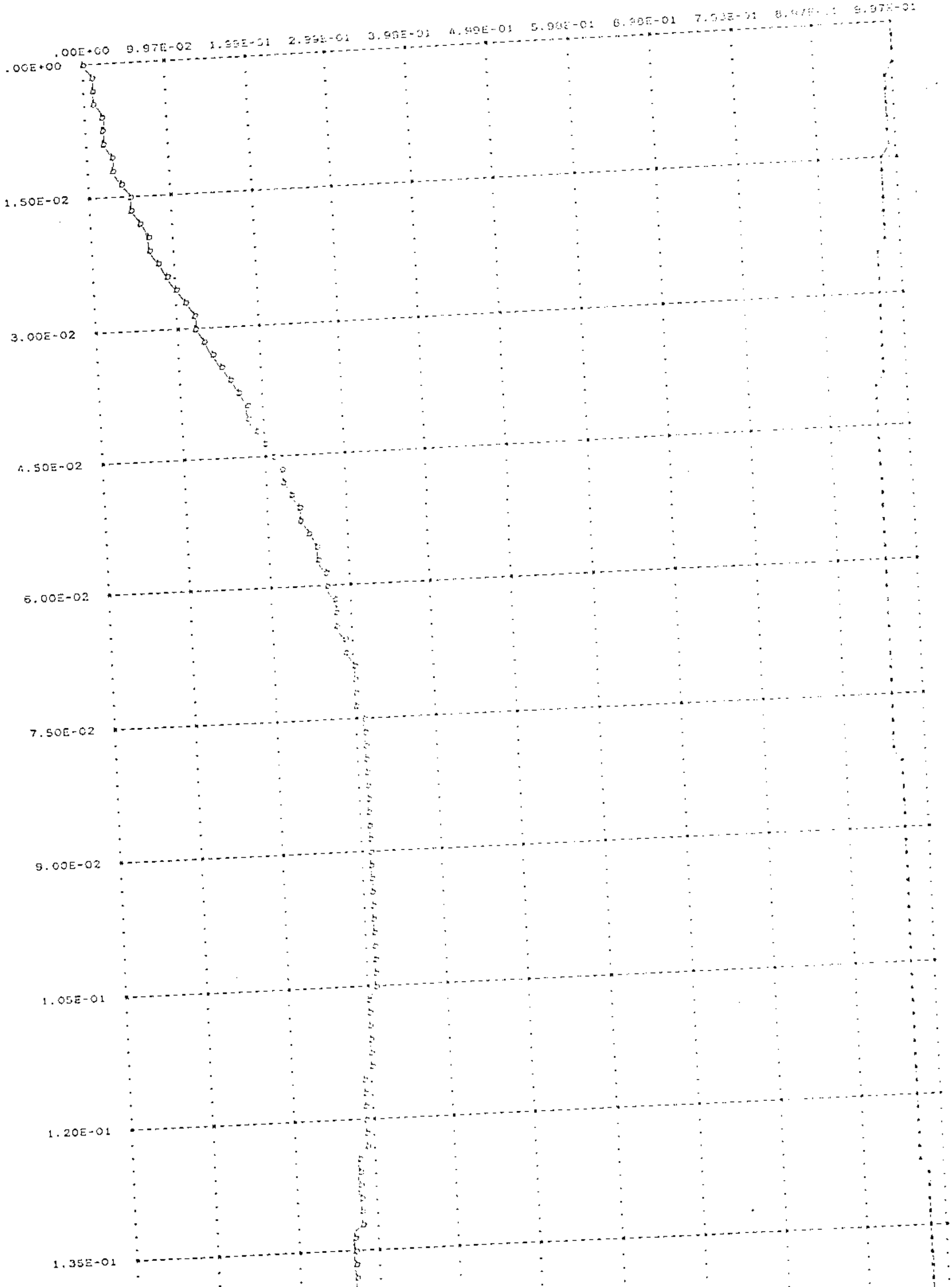
XS	XR	X12	XF
3.0000	3.0000	2.8000	.9000

RS	RR	RF	SVQ
.0580	.0720	.0400	1.0000

IDS	IQS	IDR	IQR	WR
.3328	.0164	.0002	-.0107	.9989

-INITIAL VALUES

Plot - 17



XS	XR	X12	XF
3.0000	3.0000	2.8000	.9000

RS	RR	RF	SVQ
.0580	.0720	.0400	1.1000

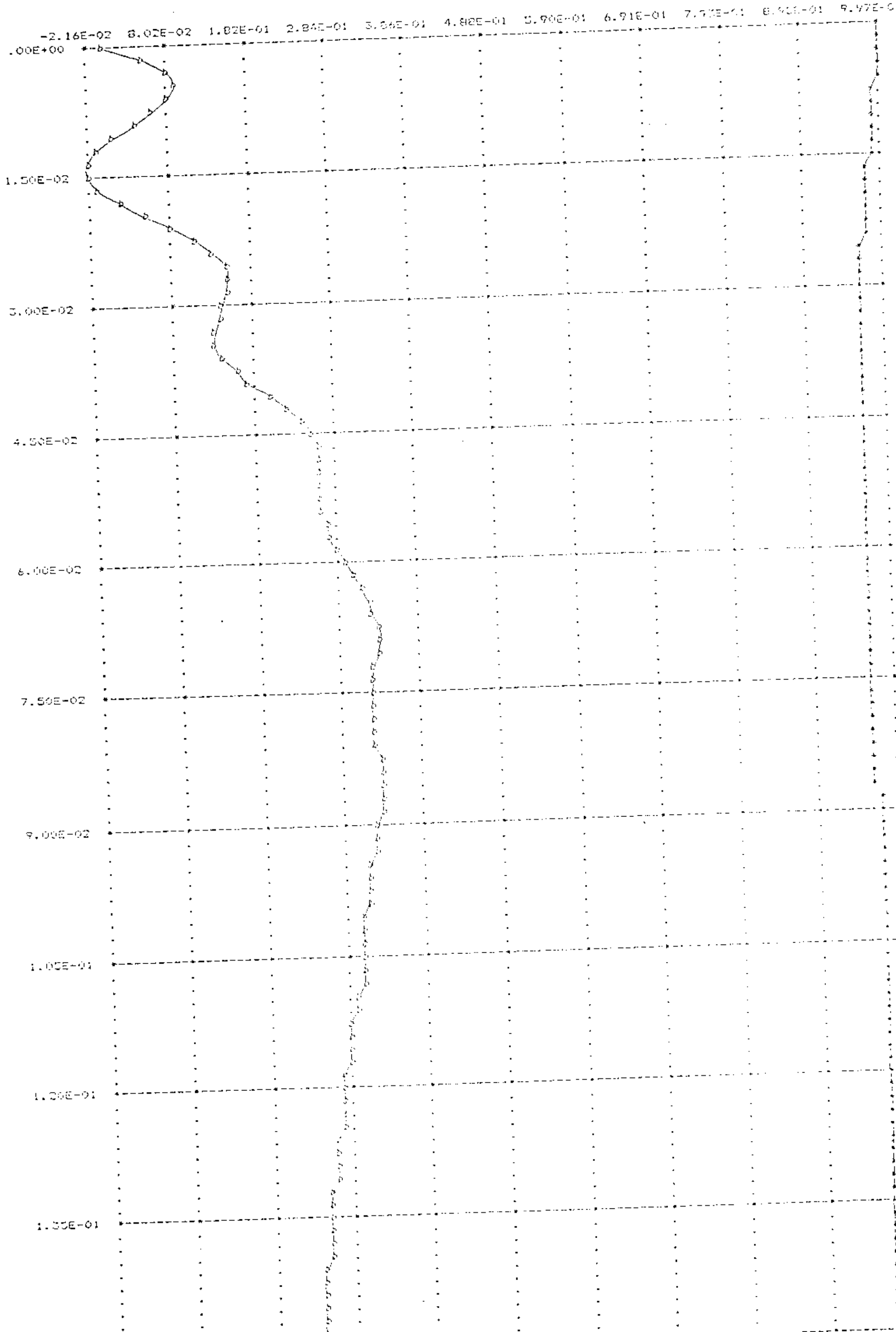
IDS	IQS	IDR	IQR	WR
.3328	.0164	.0002	-.0107	.9759

-INITIAL VALUES

$T_L = 0.25$

ANG = 90

Plot - 18



A 100

H	DA	DELTA	TL	ANG
.1000	.0100	.0015	.8000	[illegible]

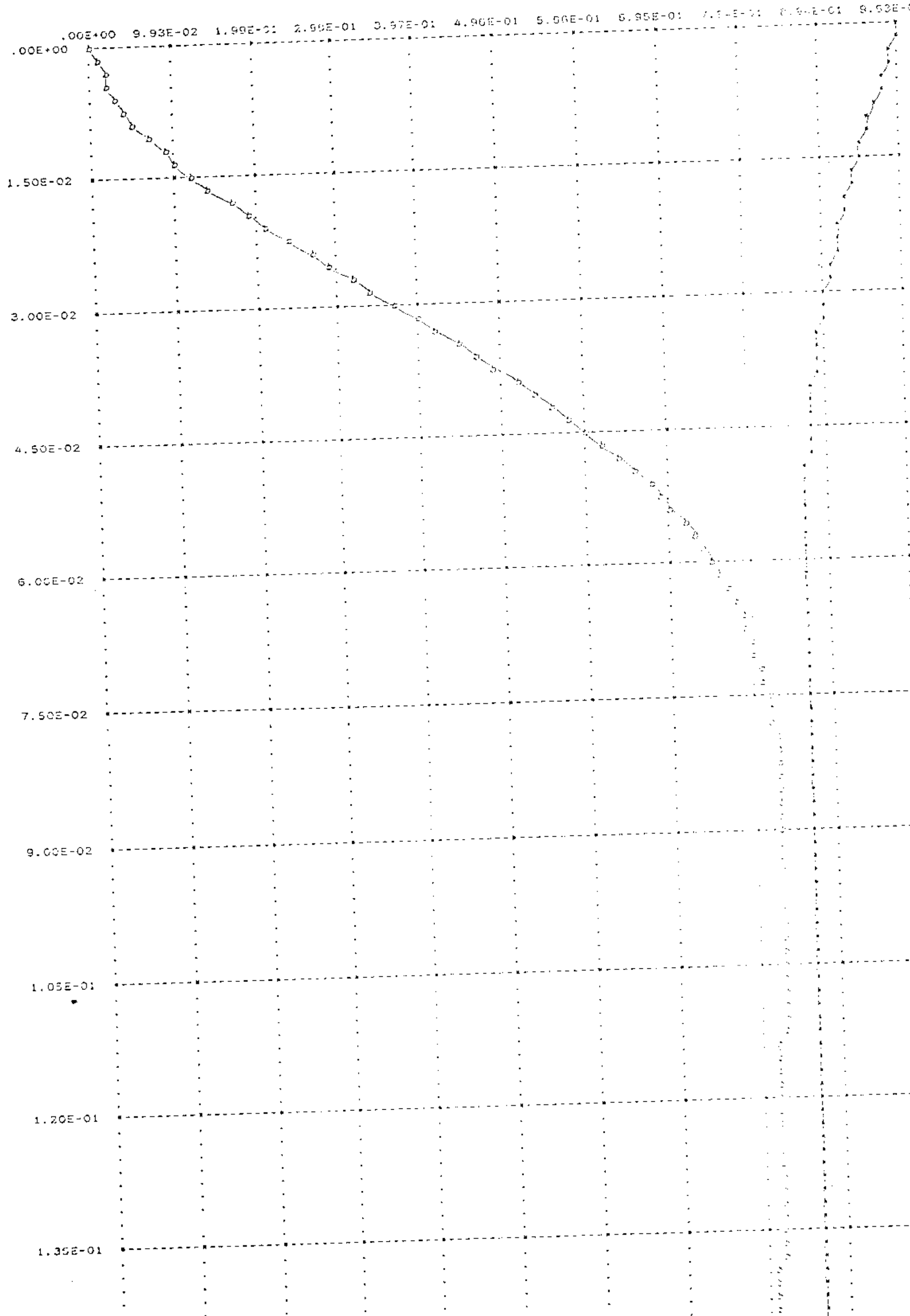
XS	XR	X12	XF
3.0000	3.0000	2.8000	.9000

RS	RR	RF	SVQ
.0580	.0720	.0400	1.0000

IDS	IQS	IDR	IQR	WR
.3328	.0164	.0002	-.0107	.9989

-INITIAL VALUES

Plot - 19



N	M
4	100

H	DA	DELTA	TL	ANG
.1000	.0100	.0015	.8000	90.0000

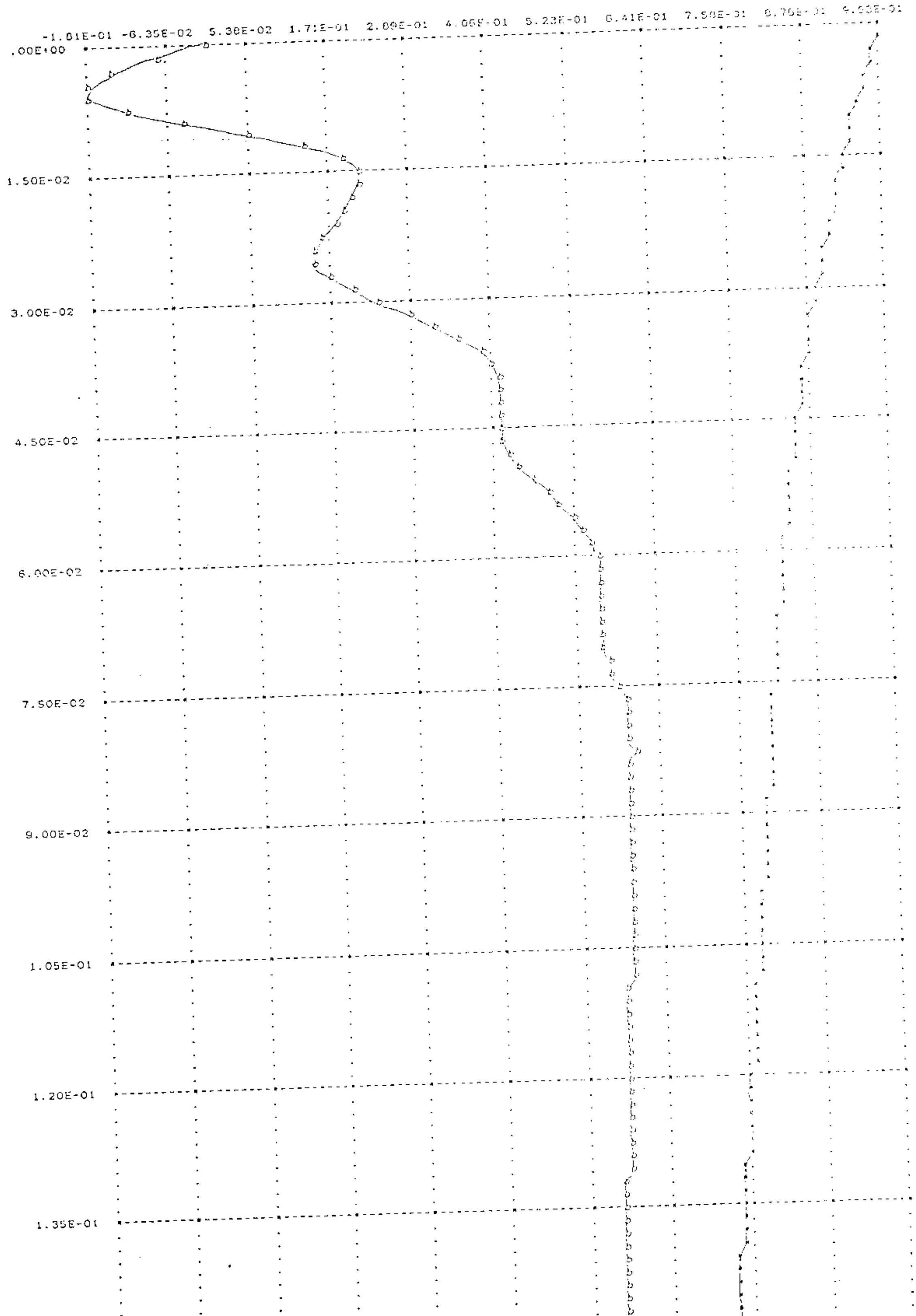
XS	XR	X12	XF
3.0000	3.0000	2.8000	.9000

RS	RR	RF	SVQ
.0580	.0720	.0400	.8000

IDS	IQS	IDR	IQR	WR
.3328	.0164	.0002	-.0107	.9989

-INITIAL VALUES

Plot - 20



4 100

H	DA	DELTA	TL	ANG
.1000	.0100	.0015	.8000	90.0000

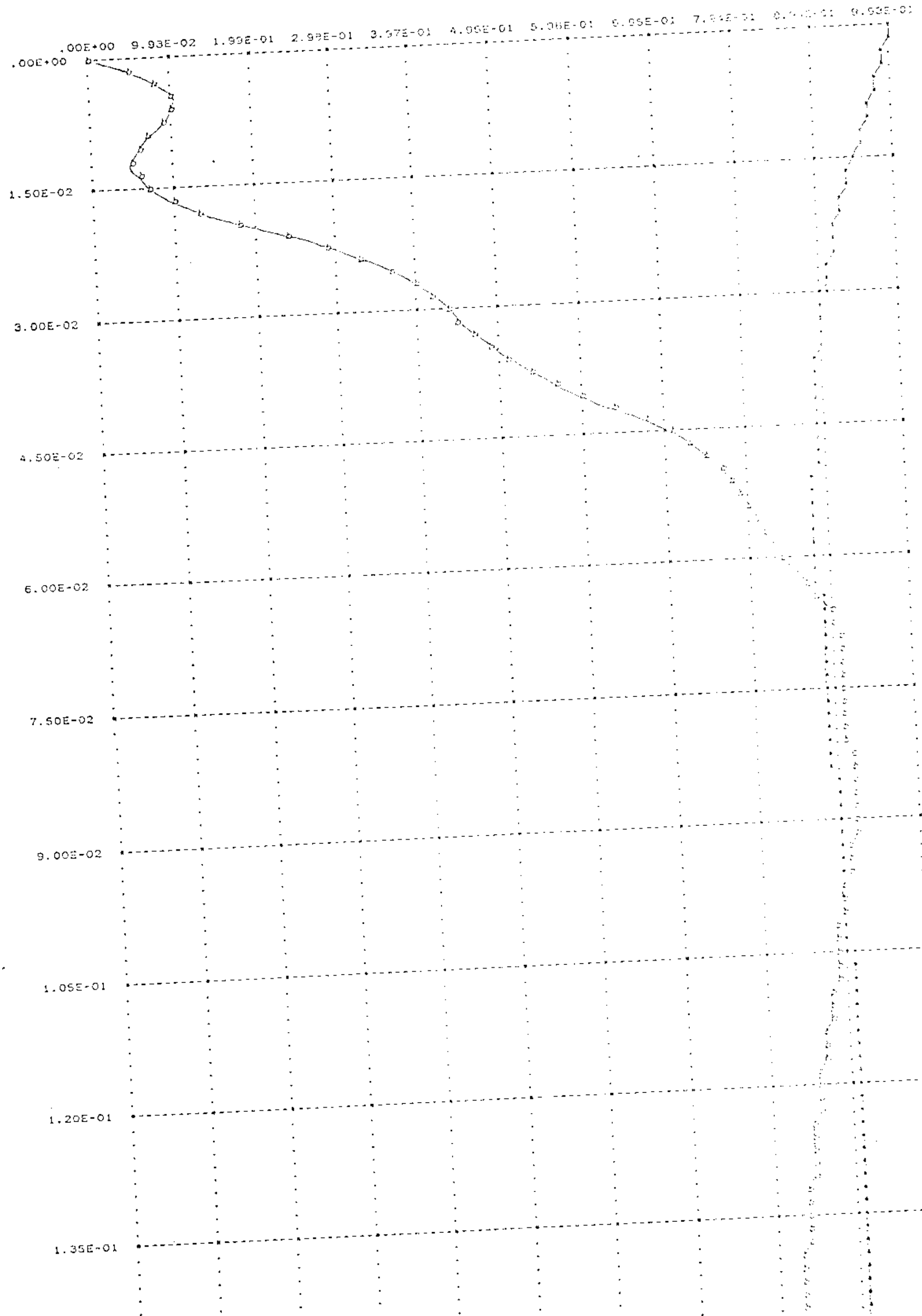
XS	XR	X12	XF
3.0000	3.0000	2.8000	.9000

RS	RR	RF	SVR
.0580	.0720	.0400	1.1000

IDS	IQS	IDR	IQR	VR
.3328	.0164	.0002	-.0107	.9989

-INITIAL VALUES

Plot - 21



A 100

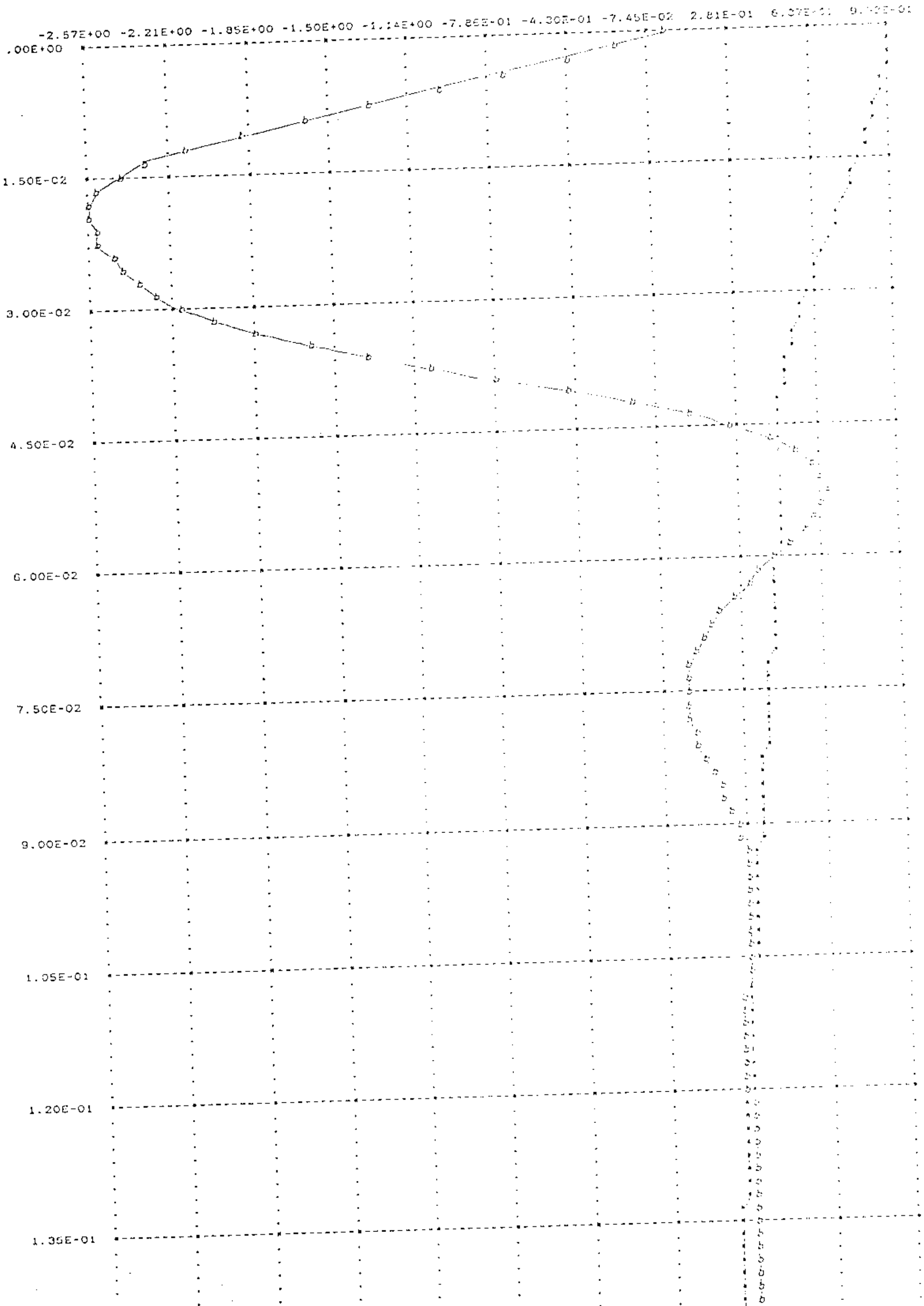
H	DA	DELTA	TL	ANG
.1000	.0100	.0015	.8000	120.0000

XS	XR	X12	XF
3.0000	3.0000	2.8000	.9000

RS	RR	RF	SVQ
.0580	.0720	.0400	1.1000

IDS	IQS	IDR	IQR	WR	
.3328	.0164	.0002	-.0107	.5989	-INITIAL VALUES

Plot - 22



4 100

H	DA	DELTA	TL	ANG
.1000	.0100	.0015	.8000	120.0000

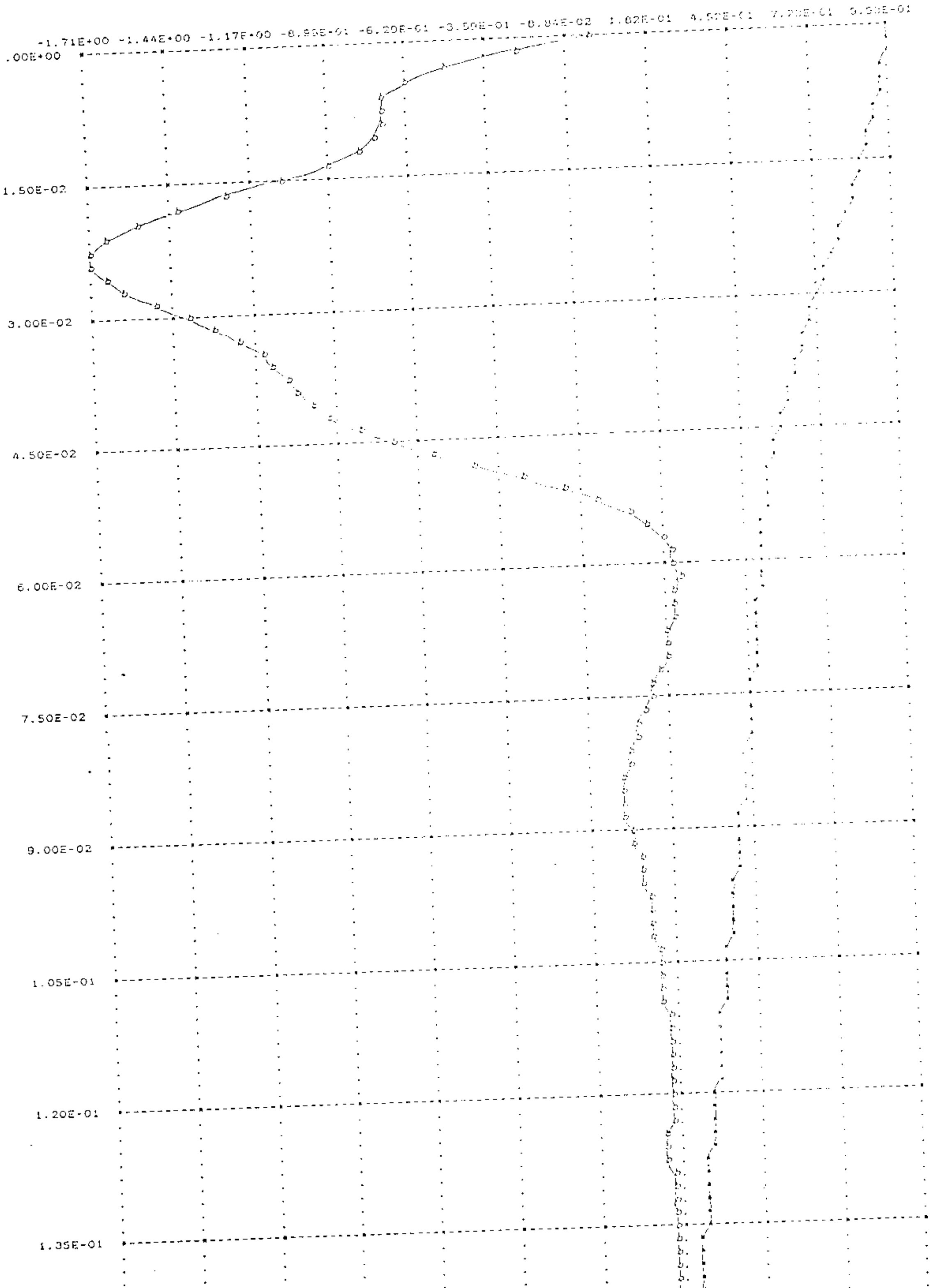
XS	XR	X12	XF
3.0000	3.0000	2.8000	.5000

RS	RR	RF	SVQ
.0580	.0720	.0400	.8000

IDS	IQS	IDR	IQR	WR
.3328	.0164	.0002	-.0107	.9900

-INITIAL VALUES

Plot - 23



Plot - 24

N	M
4	100

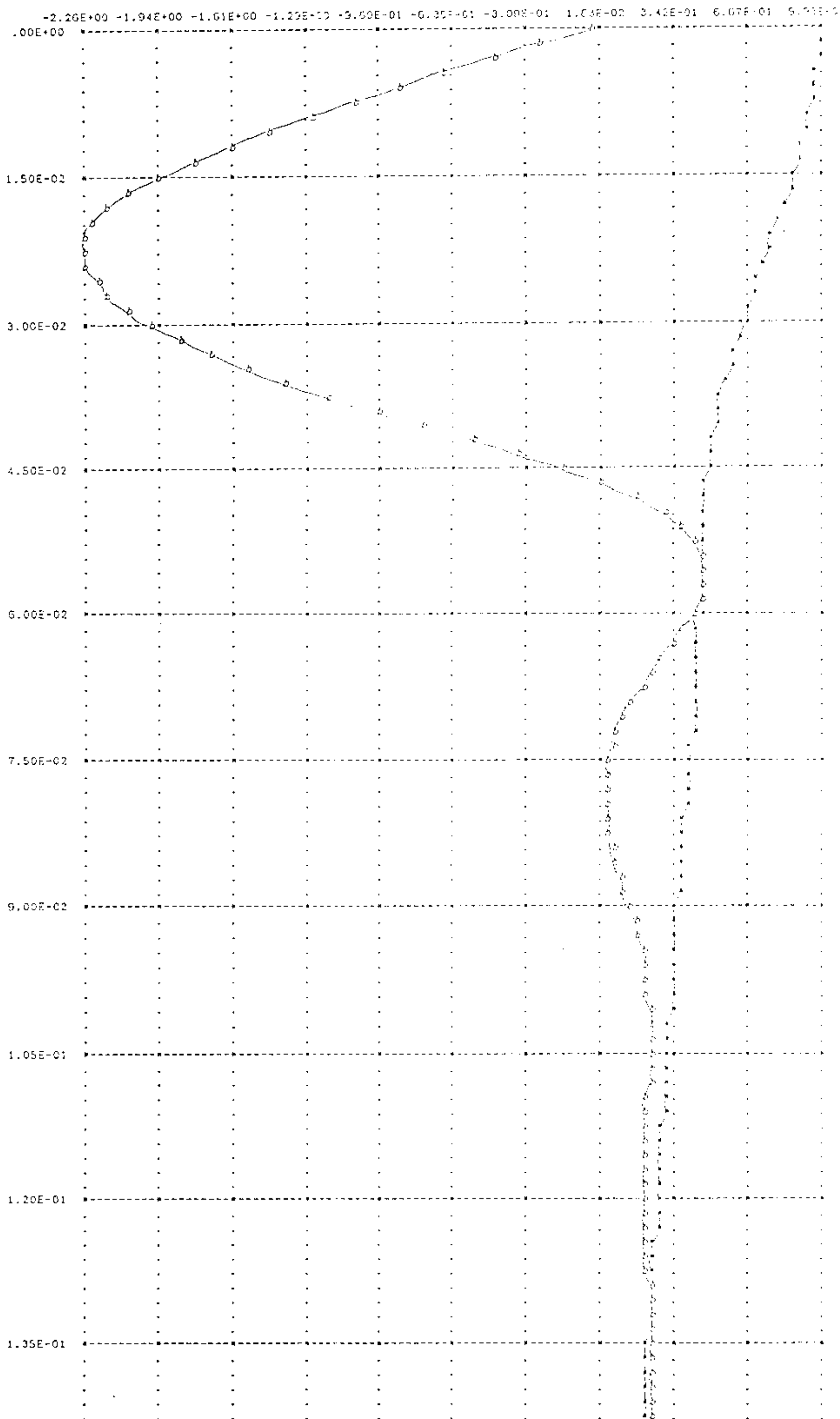
H	DA	DELTA	TL	ANG
.1000	.0100	.0015	.8000	120.0000

XS	XR	X12	XF
3.0000	3.0000	2.8000	.9000

RS	RR	RF	SVQ
.0580	.0720	.0400	1.0000

IDS	IQS	IDR	IQR	WR
.3320	.0164	.0002	-.0107	.9989

-INITIAL VALUES



Plot - 25

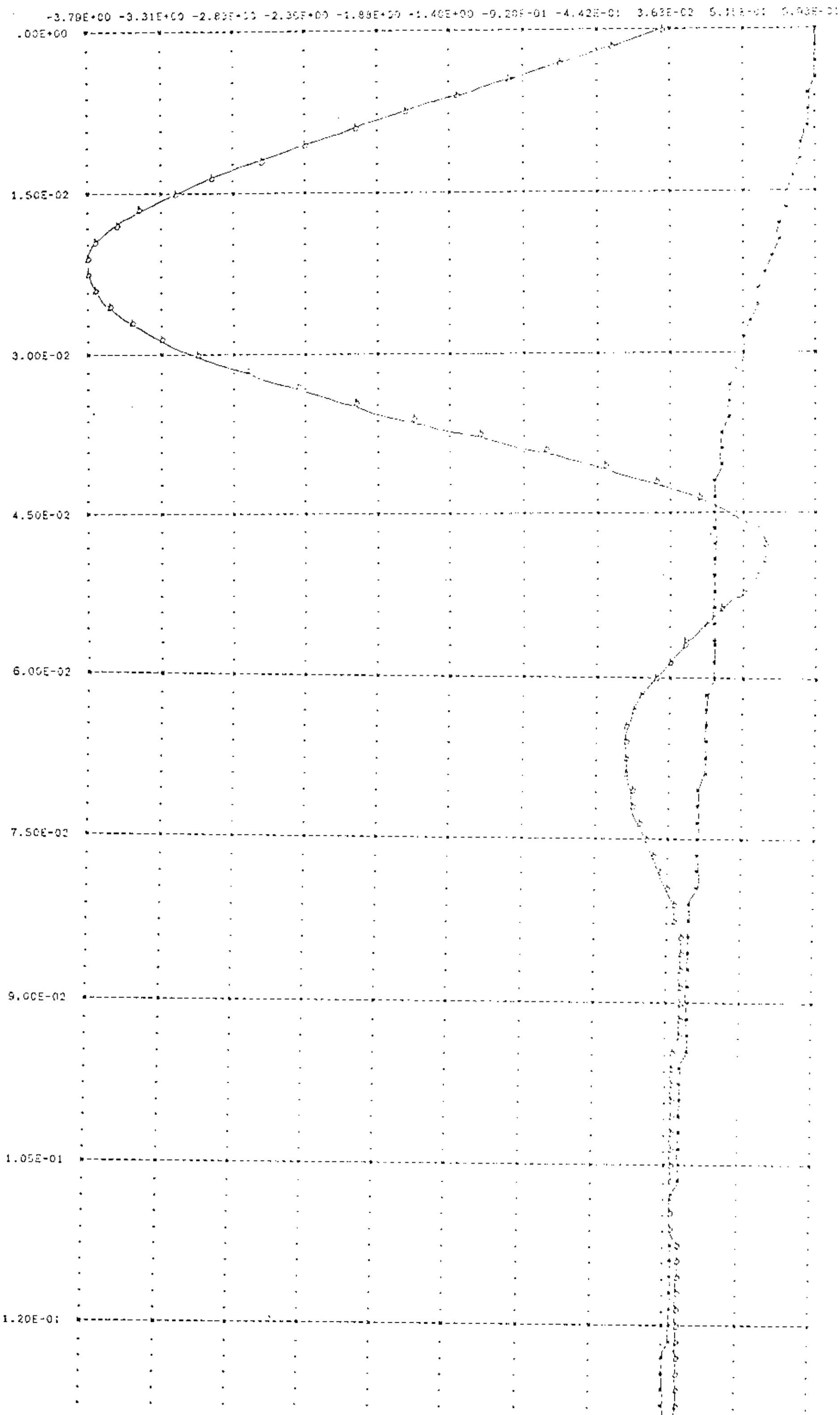
N	K
4	100

H	DA	DELTA	TL	ANG
.1000	.0100	.0015	.8000	14[illegible].0000

XS	XR	X12	XF
3.0000	3.0000	2.8000	.9000

RS	RR	RF	SVQ
.0500	.0720	.0400	1.0000

IDS	IQS	IDR	IQR	VR	-INITIAL VALUES
.3328	.0164	.0002	-.0107	[illegible]	



conclusion

CONCLUSION

To predict the transient stability behaviour of scherbius drive a mathematical model in a synchronously rotating reference frame using generalised machine theory has been developed.

In this Mathematical model analysis the converter and inverter are represented by their instantaneous average values neglecting their uni-directional properties and ripples in the d.c. side voltages and currents. Machine reactance have been taken into account but impedance of a.c. source has been neglected. Instantaneous commutation has to be assumed

Plots : 1-7 illustrates the transient process of the system starting at no load with firing angles set at 90, 110, 130, 140 respectively. Instantaneous torques appear to be oscillatory in all the cases. The frequency of oscillation is distinctly of two orders of values. Initially the torques oscillates at high frequency and finally at low frequency.

When the drive is stated with firing angles set at 90°, the speed of the drive after initial oscillations increases and rises slightly above synchronous speed and then finally settles down at steady state values. The rise in speed above synchronous speed can be explained as follows :

Because of the induction effect, the current flowing in the rotor circuit do not become zero at synchronous speed due to rotor inductance and filter inductance, although the induced voltage become zero. Since the torque depend only on rotor currents and air-gap flux the torque does not reverse at the instant of crossing over synchronous speed.

appendix

APPENDIX

General :

To predict transient performance of the system with the help of Digital Computer it is necessary to determine the parameters of the machine accurately. The slip ring induction motor used for the analytical verification had the following name data.

PHASE : 3, FREQUENCY : 50 Hz CONNECTION : Δ-Y, VOLTS : 420 V

HORSEPOWER : 5.

No. load and Blocked Rotor Test :

No load and blocked rotor test were carried out to determine the machine constants as well as the losses. The magnetising impedance X_m of the motor was first calculated from open circuit test. From blocked rotor test X_{eq} is calculated and assuming $X_1 = X_2$ and using equivalent circuit impedance of the induction motor X_s and X_r are calculated. This stator winding resistance was obtained by measuring the current i passing through a d.c. voltage across the winding. The values thus obtained were multiplied by 1.1 to obtain its effective A.C. resistance.

The rotor resistance has been calculated from the blocked rotor test.

Inertia Constant Determination :

The terminals of the Voltmeter in the armature circuit of the tacho-generator were connected to the eight loop oscillscops for recording the speed changes.

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