Computer Aided Polar Plot

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Project Report

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Certificate

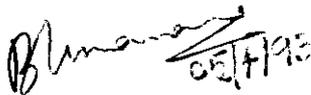This is to Certify that the report entitled

Computer Aided Polar Plot

has been submitted by

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In partial fulfilment for the award of Degree of
Bachelor of Engineering in Electrical and Electronics Engineering
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8927C0006
8927C0032

Internal Examiner   External Examiner

Dedicated to

Our

Beloved Parents

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Synopsis

SYNOPSIS

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In this project, a software is proposed to the developed to draw the polar plots which leads to analysis and design of any control system. This is a graphical plot which involves mapping of points from one plane to another. Out of the various methods of frequency response analysis, polar plots are used as it can be constructed easily and rapidly. Also, the control system can be designed and analysed for stability from this polar plot.

As the accurate plotting of transfer function of higher order systems is generally a tedious process, the aid of computers are used. In the Nyquist stability study, only the general shape of polar plots are required.

Hence, for the complicated systems in order to reduce the time of manual plotting and also to improve the accuracy, this approach will be more useful.

CONTENTS

C H A P T E R - I

INTRODUCTION

1.1 CONTROL SYSTEM:

In the process of development,man produced machine and man himself was only necessary to control in detail the various operations that are required to complete any process. Slowly, the machines and the process become more complicated. Also quick and accurate results were derived. In most of the process,man became unable to perfectly control his own machine. It was in this content that it was tried to replace the human controller by same form of automatic controller which would precisely and speedily do what the man wants. The use of analog and computers has revolutionised the automatic control system.

1.2 MATHEMATICAL MODEL:

Any dynamic system may be characterised by differential equations.The response of the dynamic system to an input may be obtained if these differential equations are solved. Describing dynamic charateristics by mathematical is called as **mathematical model**.It is the important step in analysing and design of any control system. Models may assume many differnt forms.Depending on the particular system and the circumstances one mathematical

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representation may be better suited than other representation.For the transient response analysis or frequency analysis of **siso** system, the transfer function representation is more convienient than other.

1.3 **TRANSFER FUNCTION:**

The transfer function of a linear time invariant system (for definition see appendix A) is defined as laplace transform of the impulse response, with all initial conditions are assumed to be zero. Although the transfer function of a linear system is defined in terms of the impulse reponse, in practice the input-output relation of a linear time invariant system with the continuous data input is often described as the rate of the laplace transform of the output (response function) to the laplace transform of the input (driving function) under the assumption that all initial condtions are zero. Therfore for a linear time invariant system transfer function is

$$G(s) = \frac{Y(s)}{X(s)} = \frac{b_0 s^m + \ldots \ldots + b_m}{a_0 s^n + \ldots \ldots + a_n} \quad (1.1)$$

BY using this concept, one can represent the system dynamics by algebraic equation in s. The higher power of s in the denominator of the transfer fuction is equal to the order of the highest derivative term of the input. If the higest power of s is equal to n, the system is called an

n th order system. Some of the properties of the transfer functions are as follows:

1. Transfer function is defined only for a linear time invariant system.. It is meaningless for non linear system.

2. The transfer function is independent of the input of the system.

3. Transfer function is expressed only as a function of the complex variable s. It is not a function of the real variable time or any other variable that is used as the independent variables. When system is subject to discrete time or digital input, it may be more convenient to model the system by difference equations.

4. It includes the units necessary to relate input to the output, however it does not provide any information concerning the physical structure of the system.

Response of the System

C H A P T E R - II

Response of the System

REASONS FOR STUDYING RESPONSE:

Response of the system to several input reflects

1. STABILITY: If system reaches a steady state condition
    as the same form as the input characteristics.

2. INSTABILITY: If it does not reach in time an
    appropriate steady state condition.

3. Possible oscillatory nature.

4. Rapidity of response or its sluggishness of response.

To study the behaviour of the system, the response
of system to several input is investigated.

We can study the response in two ways,

1. By using time domain method.

2. By using frequency domain method.

2.1TIME DOMAIN METHOD

The variation of the system output with time is
known of time response of the system.Most of the control
systems are inherently are time domain system for which time
response becomes an important factor for design & analysis
of a system.In practice the time response of the system for
a   particular input is measured with arbitary   system

parameters and then these parameters are varied to obtain the desired response of the system.The time reponse of the system consist of transient and steady state response for a particular input.In which transient response will go to zero as time becomes very large.And steady state response will present even the time reaches infinity.The steady state response of a control system is also very important, since when compared to the input,it gives an indication of the final accuracy of the system.

When an excitation is applied at the input terminals of the system an output C(t) is produced at the system output terminals, which changes with time.

This variation of system output C(t) with time is known as the time reponse of the system. The system response for a step excitation is sketched in fig 2.1. Various terms used are defined below and are illustrated in fig 2.1.

1. DELAY TIME Td:

The time required for the system output to reach one half of the final value is defined as delay time.

2. RISE TIME Tr:

The time required for the system output to rise from 10% to 90% of the final value is defined as the rise time.

3. SETTLING TIME Ts:

The time required for the system output to settle down and stay within +2% or -2% of the final value is known as settling time.

4. PEAK TIME Tp:

The time required for the system output to reach the first maximium value is known as peak time.

5. DUPLICATING TIME Td:

The time required for the system output to reach the final value for the first time is known as duplicating time.

6. BUILD UP TIME Td:

The time required for the step excited system output increase from zero to final value when the increase is at a constant rate and is equal to the maximium value of the actual increase rate, is known as build up time.

7. OVERSHOOT:

The ratio of the maximum value of the step excited system output to the final output is known as overshoot in the system.

2.2 FREQUENCY RESPONSE:

Analysis of system response through steady state response due to sinusoidal input is known as steady state requency response analysis or simply as frequency response. For defining the frequency response of a system, without loss of generality, we choose a simple transfer function as follows:

$$M(s) = \frac{1}{s+1} \qquad (2.1)$$

If a sinewave is applied to the system, the corresponding response fuction C(s), should be

$$C(s) = \frac{1}{s+1} \cdot \frac{w}{s+w^2} \qquad (2.2)$$

Inverse laplace transform yields,

$$C(t) = \frac{w}{1+w^2} e^{-t} + \frac{1}{(1+w^2)^{.5}} \sin wt - (\tan^{-1}(w/1)) \quad . \quad (2.3)$$

The second term of the right hand side is usually called the steady-state part of the response, and the first term is called the transient part. This is bacause the former has value s as t ---> infinity ,whereas the latter dies out as t ---> infinity. If only the steady-state part is considered, we have

$$C(t)_{\text{steady-state}} = \frac{1}{(1+w^2)^{.5}} \sin\left\{wt - (\tan^{-1}w/1)\right\} \quad (2.4)$$

Equation 4 is a sinewave, with an amplitude $A = 1+w^2$ and a phase angle $= -(\tan^{-1}w/1)$. Both A and $\dot{\phi}$ are functions of w. Equation 2.4 can be written into many forms; for example, the following are some well known expressions:

1. Trignometric form: $A \sin(wt + \phi)$
2. Polar form : $A\underline{/\phi}$ with the angular velocity w.
3. Rectangular form: $A(\cos\phi + j\sin 0)$, or $u+jv$ with w
4. Exponential form: $A e^{(j\phi)}$ with w

and

$$U(w) = Re(A e^{(j\phi)}) = A \cos(\phi)$$
$$V(w) = Im(A e^{(j\phi)}) = A \sin(\dot{\phi})$$

In deed, once the amplitude and the phase angle of a sine wave in a certain frequency w are determined, the sinewave is uniquely defined. The form in which it is expressed is only a matter of convenience.

By considering w as a variable, we determine a set of corresponding A and ϕ. In other words once w is given, the A's and ϕ's of a linear system are uniquely defined. The procedure mentioned above can be considered as a definition of the frequency response of a system as well as a procedure to obtain the frequency response of a system in practice.

Performance specifications of control system with regard to frequency domain are known as frequency domain specifications. Frequency demain specification for a system are expressed in following terms,

1. BAND - WIDTH (BW):

The frequency at which the magnitude of $M(jw)$ has dropped to 70.7% percent of its zero frequency level or 3db below fronm the zero frequency is known as band width of the system as shown in the figure 2.2.

2. PEAK RESONANCE (M(W)p:

The maximium value of the magnitude of closed loop transfer function $(M(jw))$ is defined as the peak resonance Mp or $M(w)p$ of the system.

3. RESONANT FREQUENCY(wp):

The frequency at which the magnitude of $M(w)$ is maximium is known as the resonant frequency of the system.

4. CUTT OFF RATE:

The rate of cutt off for the frequency response characteristics at higher frequencies is known as the cut off rate of the system. This indicates the system ability to distinguish betweeen the signal and the noise present in the system.

5. GAIN MARGIN :

This is defined as the magnitude of the reciprocal of the open loop transfer function evaluated at the phase cross over frequency for which the phase of open loop

transfer function is -180 degrees. Hence the gain margin(G.M.) of the open loop system transfer function is equal to 20 log (1/(G(jw)) at the phase cross over frequency.

6. PHASE MARGIN:

This is defined as 180 degrees plus the phase $\dot{\phi}(w)$, of the transfer function at which the magnitude is unity. This indicates the relative stability of the system.

The magnitude and phase angle of function G(jw) for various frequencies are represented by various graphical plots in different co_ordinates which give better insight for analysis and design of control systems.

The graphical plots generally used are:

1. POLAR PLOT:

This is the plot of the magnitude M(w) versus phase angle $\phi(w)$. In polar co_ordiantes for various values of frequencies.

2. BODE PLOT:

This is the plot of magnitude M(w) in decibels versus log w and phase angle $\phi(w)$ versus log w in rectangular co_ordinates.

3. MAGNITUDE VERSUS PHASE ANGLE PLOT :

This is the plot of magnitude M(w) in decibels versus phase angle $\phi(w)$ in rectangular co_ordinates with frequency as varying parameter. These are also known as gain phase plot of the system.

2.3 **ADVANTAGES AND DISADVANTAGES:**

ADVANTAGES:

Frequency response method is preferred for investigation system performance due to the following reasons.

1. As the frequency response method and the pole zero method are two different ways of applying the same principles of analysis and design, hence one method may be used to check the accuracy of other.

2. Experimental data for control systems are often presented in terms of frequency response measurements can be made with any degree of accuracy and convenience as compared to other methods.

3. The design specifications for control system are generally given in terms of system frequency response characteristics. This is particularly true when the input signals are random functions of time having only satistical properties.

4. In control system having multiple loops the frequency response method gives the design and analysis specifications more quickly than are obtained by pole-zero method.

5. Some of the procedure used for analysis of non - linear system are based on the frequency response method hence it is preferred as compared to other methods.

DISADVANTAGE:

The main disadvantage of the frequency response method for analysis and design of control system is the indirect link between the frequency and time domain.

2.4 POLAR PLOT

The curve that gives the information regarding the gain and phase shift of the frequency function is known as the frequency response curve of the system.

The polar plot of a sinusoidal transfer function G(s) is a plot of the magnitude of G(jw) versus the phase angle of G(jw) on polar co_ordinates as w varied from zero to infinity. Thus the polar plot is the locus of vectors :G(jw) as w varied from zero to infinity.

In polar plots a positive phase angle is measured counter clockwise, from positive real axis (we say it as phase lead transfer function). And for negative phase angle, it is measured clockwise frim positive real axis (phase lag). For plotting the polar plots we are assuming the feedback is unity (H(s) = 1), so that open loop transfer function (G(s)H(s)) becomes simply G(s).

2.4.1 PROCEDURE

The procedure for plotting polar plots is given below,

1. Determine the transfer function G(s) of the system.

2. Substitue s = jw in the transer function G(s) and obtain the frequency response function G(jw).

3. Obtain the magnitude of G(jw) at w = 0 and w = infinity by having $\underset{w \to 0}{Lt} |G(jw)|$ and $\underset{w \to \infty}{Lt} |G(jw)|$

4. Calculate the phase angle of G(jw) at w = 0 and w = infinity by having $\underset{w \to 0}{Lt} \underline{/G(jw)}$ and $\underset{w \to \infty}{Lt} \underline{/G(jw)}$

5. Rationalize the comples frequency function G(jw) and seperate real and imaginary parts.

6. Determine the frequencies at which the plot intersects the real axis by equating the imaginary part of G(jw) equal to zero ie Im(G(jw)) = 0. Hence claculate the value of G(jw) at the intersection point by substituting the determined value of frequency in the rationalized expression of G(jw).

7. Determine the frequencies at which the curve intersects the imaginary axis by equating real part of the G(jw) to zero ie Re(G(jw)) = 0. Hence calculate the value of the G(jw) at the intersection point by substituting the determined value of w in the rationalized expression of G(jw).

8. Sketch the complete polar plots of the system which satistifes the above specifications.

2.4.2 CHARACTERISTICS

For sketching the polar plots of an open loop transfer function G(s) the following criteria are used to determine the important position of the complete plot.

1. From the transfer fuction G(s) in general the frequency function G(jw) is obtained by substituting $s = jw$ (i.e)

$$G(s) = \frac{K\ (1+ST_a)(1+ST_b)\ldots\ldots(1+ST_m)}{(S)^T\ (1+ST_1)(1+ST_2)\ldots(1+ST_n)} \qquad (2.5)$$

where the value of T defines the type of the system.

$$G(jw) = \frac{K(1+jwT_a)(1+jwT_b)\ldots(1+jwT_m)}{(jw)^T\ (1+jwT_1).\ (1+jwT_n)} \qquad (2.6)$$

magnitude and phase angle at w ---> 0 is obtained by taking the limit of 1 at w tends to zero.

2. At higher frequencies (i.e) w tends to infinity the magnitude and pjhase angle are obtained by taking the limit of magnitude and phase angle of 1 at w tends towards infinity. Depending upon the type of the system (i.e) the value of T the magnitude can be zero or infinity and the phase angle is (m-n+-T) degrees. In actual linear system the value of the (T+n) for positive values of T will always be greater than m. Hence the curve for w tends towards infinity approaches to the

2.5 EFFECT OF ADDING POLES AND ZEROS

1. Addition of a non zero pole to a transfer function results in further rotation of the polar plot through an angle of -90 as w tends towards infinity.

2. Addition of a pole at the origin to a transfer function rotates the polar plot at zero and infinite frequencies by a further angle of -90.

3. The effect of addition of a zero to a transfer function is to rotate the high frequency portion of the polar plot by 90 degrees in counter-clockwise direction.

Studying the effect of adding pole and zero is useful while we design a system.

2.6 STABILITY

2.6.1 CONCEPT:

Stability in a system implies that small changes in the system input, in initial conditions or in system parameters, do not result in large changes in system output. Stability is a very important characteristic of the transient performance of a system.

Almost every working system is designed to be stable. Within the boundaries of parameter variations permitted by stability considerations, we can then seek to improve the system performance.

linear time - invariant system is stable if the following two notions of system stability are satisfied,

1. When the system is excited by a bounded input the output is bounded.

2. In the absence of the input, the output tends towards zero (the equilibrium state of the system) irrespective of initial conditions.(This stability concept is known as asymptotic stability.

The second notion of stability generally concerns a free system relative to its transient behaviour. For non - linear system, because of the possible existence of multiple equilibrium states and other anomalies, the concept of stability is difficult even to define, so that there is no clearcut correspondence between the two notions of stability defined above. For a free stable non - linear system, there is no guarantee that output will be bounded whenever input is bounded. Also if the output is bounded for a particular bounded input it may not be bounded for other bounded inputs. Many of the important results obtained thus far concern the stability of the non - linear systems in the sense of the second notion above (i.e) when the system has no input.

The possibility of unstable operation is inherent in all feedback control systems because of very nature of the feedback itself. An unstable system, obviously cannot perform the control task required of it. Therefore, while

rigin in the clockwise direction making the curve tangent to the proper axis at the origin.

3. The frequencies at which the polar plot intersects with the real and imaginary axis are decided by equating the imaginary and real part of G(jw) equal to zero (i.e)

$$Im(G(jw)) = 0$$

$$Re(G(jw)) = 0$$

4. The curve for the frequency function having no time constant terms in the numerator is a smooth one which the values of G(jw) decreases ccntinuously as w is changed from zero to infinity. But when the time constant terms are present in the numerator the phase angle may not change continuously depending upon the values of the time ccnstant and thereby prodeucing ducts in the polar plots.

5. Generally for investigating the system properties the exact slope of the plot near the point (-1+j0) is required hence sufficient points of G(jw) are accurately determined in this area.

6. Correlation exist between the polar plot and steady state output of a feedback control system. The relation is governed by the type the system.

7. For linear time invariant system polar plot will produce a mirror image about central axis for w greater than minus infinity and less the zero.

analyzing a given system, the very first investigation that needs to be made is, whether the system is stable. However, the determination of stability of a system is necessary but not sufficient, for a stable system with low damping is still undesirable. In an analysis problem one must therefore proceed to determine not only the absolute stability but also its relative stability.

2.6.2 RELATIVE STABILITY :

Measure of relative stability of closed loop system which are open loop stable can be conveniently created through Nyquist plot. The stability information of such systems becomes obvious by inspection of the polar plot of the open loop function G(s)H(s) since the stability criterion is merely non encirclement of (-1+j0) point. It can be intuitively imagined that as the polar plot gets closer to (-1+j0) point, the system tends towards instability.

2.6.3 GAIN MARGIN:

It is the factor by which the system gain can be increased to drive it to the verge of instability. The gain margin(GM) can also defined as the reciprocal of the gain at the frequency at which the phase angle becomes 180 degrees. The frequency at which the phase angle is 180 degrees is called phase cross over frequency.

2.6.4 PHASE MARGIN:

The frequency at which $|G(jw)| = 1$ is called the gain cross over frequency. The phase margin is defined as the amount of additional phase - lag at the gain cross - over frequency required to bring the system to the verge of instability. The phase margin is always positive for stable feedback systems.

The value of phase margin for any system can be computed from

phase margin $\phi = \underline{|\ G(jw)H(jw)} \bigm| w = w_1 \quad +180$ degrees
where the angle at w_1 , the gain cross frequency, is measured negatively.

2.7 A FEW COMMENTS ON PHASE AND GAIN MARGIN

The phase and gain margins of a control system are a measure of the closeness of the polar plots to the (-1 + j 0) point. Therefore, these margins may be used as design criteria.

It should be noted that either the gain margin alone or the phase margin alone does not give a sufficient indication of the relative stability. Both should be given in the determination of relative stability.

For a minimium phase system, both the phase and gain margins must be positive for the system to be stable. Negative margins indicate instability.

Proper phase and gain margins ensure us against variations in the system components and are specified for definite values of frequency. The two values bound the behaviour of the closed loop system near the resonant frequency. For satisfactory performance, the phase margin should be between 30 degrees and 60 degrees and the gain margin should be greater than 6 db. With these values, a minimium phase system has guranteed stability, even if the open loop gain and time constants of the components vary to a certain extent. Although the phase and gain margins give only rough estimates of the effective damping ratio of the closed loop system. They do offer a convenient means for

designing control system or adjusting the gain constants of systems.

For minimium phase system, the magnitude and phase characteristics of the open loop transfer fuction are definitly related. The requirement that the phase margin be between 30 degrees and 60 degrees means that in a logarithmic plot the slope of the lg magnitude curve at the gain over frequency be more gradual than -40 db/decade. In most practical cases, a slope of -20 db/decade is desirable at the gain cross over frequency for stability. If it is -40 db/decade, the system could be either stable or unstable. (Even if the system is stable, however the phase margin is small) If the slope at the gain cross over frequency is -60 db/decade or steeper, the system is unstable.

The gain and phase margin concepts are applicable to open - loop transfer functions only.

In the fig 2.3 a typical $G(jw)H(jw)$ locus which crosses the negative real axis at a frequency $w = w_2$ with an intercept of a. Let a nunit circle centred at origin |obviously it passes through the point $(-1 + j0)$| intersect the $G(jw)H(jw)-$ locus at a frequency $w = w_1$ and let the phasor $G(jw1)H(jw1)$ make an angle of 0 with the negative real axis measured positively in counter-clockwise direction. It is immediately observed that as $G(jw)H(jw)$ - locus approaches $(-1+j0)$ point, the relative stability reduces. Simultaneously, the value of a approaches unity

and that of ϕ tends to zero. The relative stability could thus be measured in terms of the intercept a or the angle C. These concepts are used to define **gain margin** and **phase margin** as practical measures of relative stability.

2.8 CONDITIONALLY STABLE SYSTEMS:

If the open loop gain is increased sufficiently, the $G(jw)H(jw)$ locus encloses the $(-1+j0)$ point twice, and the system becomes unstable. If the open loop gain is decresed sufficiently, again the $G(jw)H(jw)$ locus encloses the $(-1+j0)$ point twice. The system is stable only for the limited range of the values of the open loop gain fo which the $(-1+j0)$ point is completely outside the $G(jw)H(jw)$ locus. Such a system is a conditionally stable one.

A conditionally stable system is stable for the value of the open loop gain lying between critical values, but it is unstable if the open loop gain is either increased or decreased sufficiently. Such a system becomes unstable when large input signals are applied since a large signal may cause saturations, which in turn reduces the open loop gain of the system. It is advisable to avoid drop such situations since the system may become unstable, should be the open loop gain drop beyond a critical value.

For stable operation of the conditionally stable system considered here, the critical point $(-1 +j0)$ must not be located in the regions between OA and BC shown fig 2.4.
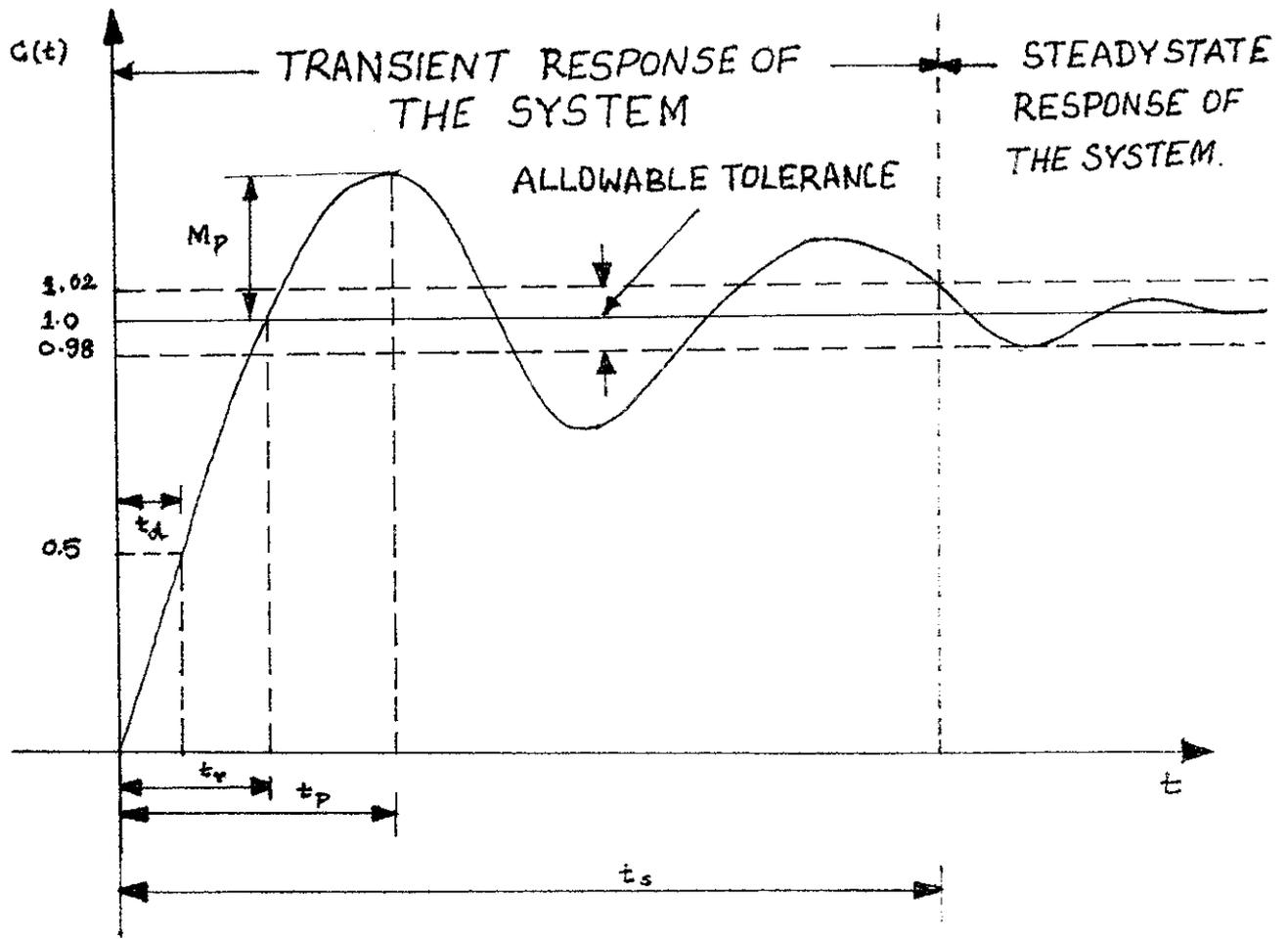
FIG. 2.1. TIME RESPONSE SPECIFICATIONS.
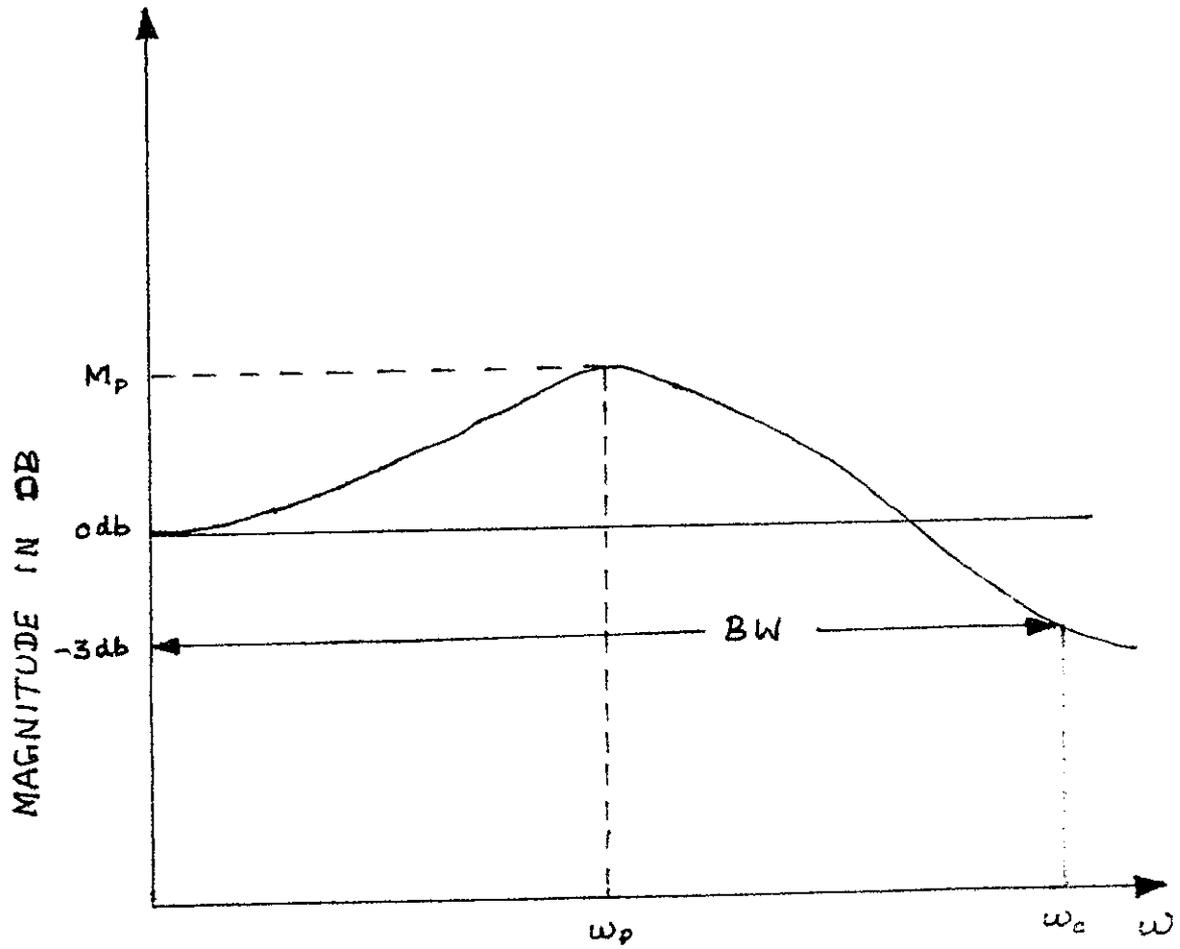
FIG. 2.2. MAGNITUDE CHARACTERISTICS OF
A CONTROL SYSTEM.

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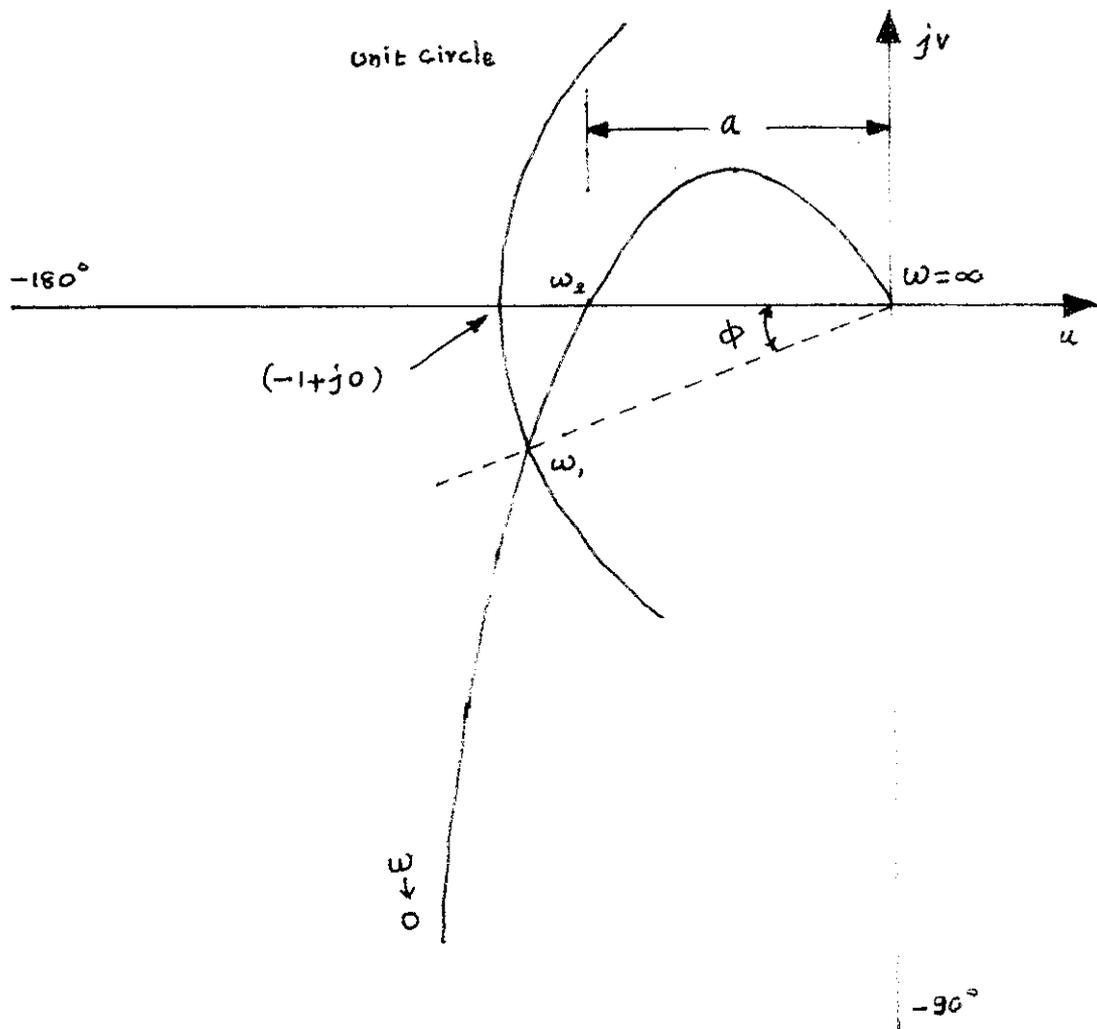
FIG. 2.3. A TYPICAL $G(j\omega)H(j\omega)$.
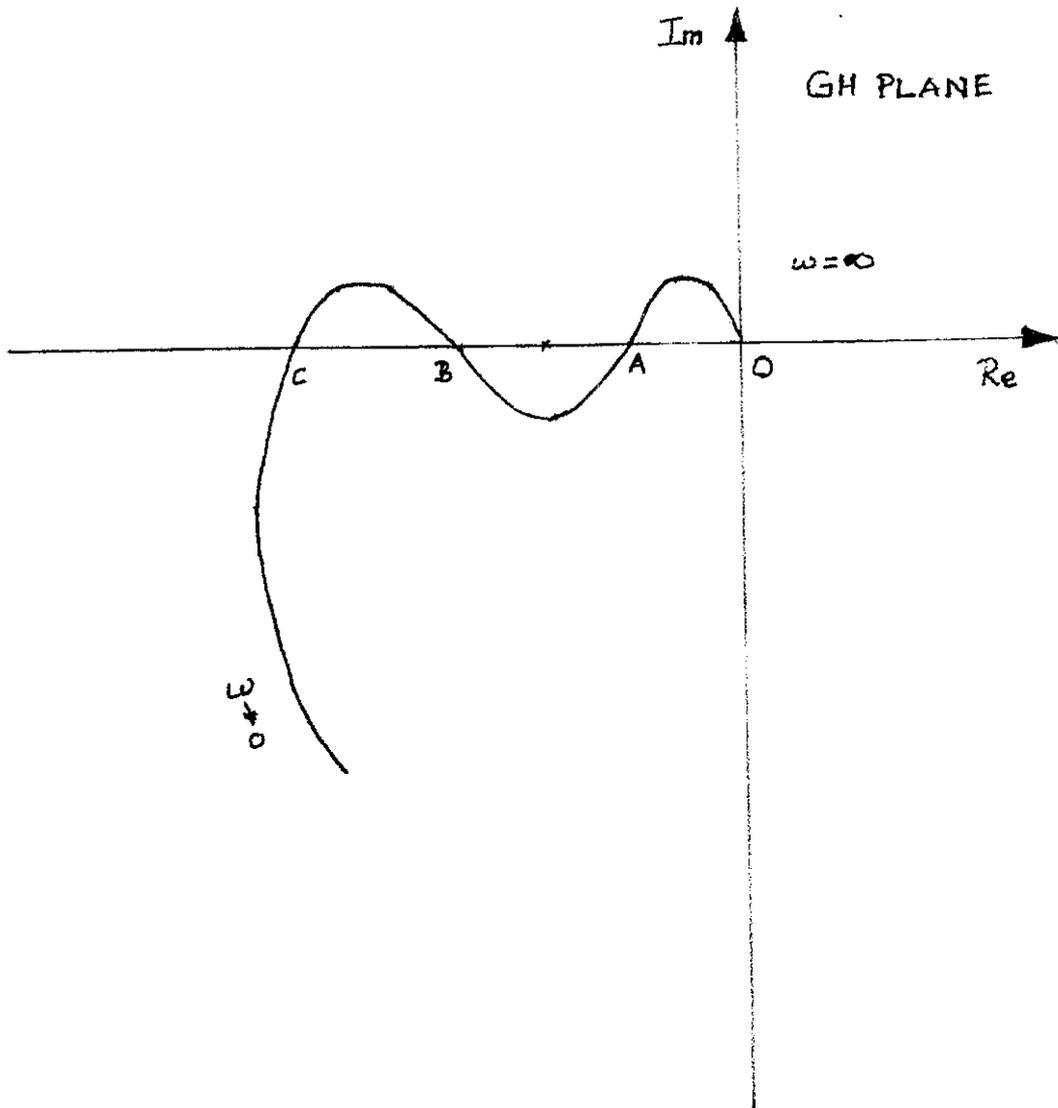
FIG. 2.4. POLAR PLOT OF A CONDITIONALLY STABLE SYSTEM.

Software Development

mulx	-	To convert x and y to integer.
pm	-	Phase margin.
gam	-	Gain margin.
gcf	-	Gain cross over frequency.
xw	-	Phase cross over frequency.
pq	-	To check the presence of second order in denomiator
zq	-	To check the presence of second order equation in the numerator.
p	-	A array in which first order roots of denomiator are stored.
z	-	A array in which first order roots of numerator arre stored.
x3,y3	-	To centre the x axis (column) and y axis (row) respectively.
pa,ps	-	The arrays that are used in finding the magnitude of first order roots in denomiator.
za,zs	-	The arrays that are used in finding the magnitude of first order roots in numerator.
ty	-	To find the type of the system.
mx	-	Maximium value of frequency.
mn	-	Minimium value of frequency.
in	-	Increment of frequency.
delay	-	To check the presence of transportation lag in the system.

3.2 FLOW CHART

The Flow chart in Fig. 3.1 depicts the steps in detail to be followed to obtain the polar plots.

3.3 COMPUTER PROGRAM

A computer program in 'C' language is developed. The listing of the program is given at the end of this chapter.

START

READ THE FIRST
ORDER ROOTS IN
NR & DR

READ THE VALUE
OF TOV AND T
(IF QUADRATIC
EQUATION
PRESENT)

READ THE TYPE
OF THE SYSTEM
&VALUE OF L
(IF DELAY
IS PRESENT)

6

READ THE MN,
MX AND INC OF
FREQUENCY

W = MN

5

IS
W <MX

NO → C

YES

A

F I G . 3 . 1 . F L O W   C H A R T

3c

```
        ( A )
          │
          ▼
┌─────────────────────┐
│  FIND THE MAGNITUDE │
│  OF FIRST ORDER ROOTS│
│  IN NR&DR AND STORE IT│
│  IN THE VARIABLE NR&DR│
└─────────────────────┘
          │
          ▼
┌─────────────────────┐
│  FIND THE ARGUEMENT OF│
│  FIRST ORDER ROOTS ON│
│  NR&DR AND STORE IT IN│
│  NR_THETA&DR_THETA   │
└─────────────────────┘
          │
          ▼
┌─────────────────────┐
│  MULTIPLY TY TIMES W │
│  WITH DRAND ADD TY   │
│  TIMES 90 DEGREES TO │
│  DR_THETA.           │
└─────────────────────┘
          │
          ▼
        ╱ IS ╲    YES
       ╱ PQ=1 ╲────────►
       ╲      ╱
        ╲    ╱
          │ NO
          ▼
        ( B )
```

$$DR = DR * SQRT\{(1 - W^2 T^2)^2 + (2*TOU*T*W)^2$$

$$DR\_THETA = DR\_THETA + + ATAN \left[\frac{(2*TOU*T*W)}{(1- W^2 T^2)} \right]$$

3.

B

IS
ZQ=1 —YES→

$$NR=NR*SQRT\{(1-W^2T^2)^2 \\ (2*TOU*T*W)^2\}$$
$$NR\_THETA=NR\_THETA$$
$$+ATAN\left[\frac{(2*TOU*T*W)}{(1 - W^2T^2)}\right]$$

NO

IS
DELAY=1 —YES→

$$NR=NR*1$$
$$NR\_THETA=NR\_THETA$$
$$- W*T$$

NO

$$R=(NR*CONST)/DR$$
$$THETA=NR\_THETA-DR\_THETA$$
$$X = R*COS(THETA)$$
$$Y = R*SIN(THETA)$$

CONVERT THE VALUE OF X&Y
TO NEGATIVE AND MULTIPLY IT
WITH THE MULTIPLICATION
FACTOR TO GET X&Y IN
INTEGER VALUE.

IS
X<0 —NO→ E

YES

D

32

```
    ( D )                    ( E )
       │                        │
       ▼                        │
    ╱IS ╲        NO             │
   ◄ Y=0 ►──────────────►───────┤
    ╲   ╱                       │
       │ YES                    │
       ▼                        │
  ┌─────────┐                   │
  │ XW=X    │                   │
  │ GM=-1/X │                   │
  │ XS=W    │                   │
  └─────────┘                   │
       │◄──────────────────────┘
       ▼
    ╱IS ╲        NO
   ◄ X<0 ►──────────────►───────┐
    ╲   ╱                       │
       │ YES                    │
       ▼                        │
  ┌─────────────┐               │
  │ K_E=X²+Y²   │               │
  └─────────────┘               │
       │                        │
       ▼                        │
    ╱   IS    ╲    NO           │
   ◄0.991<1.009►────────►───────┤
    ╲         ╱                 │
       │ YES                    │
       ▼                        │
  ┌──────────────┐              │
  │ PM=ATAN(Y/X) │              │
  │ GCF=W        │              │
  └──────────────┘              │
       │◄──────────────────────┘
       ▼
    ( F )
```

33

```
                    ( F )
                      │
                      ▼
         ┌──────────────────────┐
         │  PUT A DOT WITH      │
         │  THE VALUE OF X&Y    │
         └──────────────────────┘
                      │
                      ▼
         ┌──────────────────────┐
         │      W = W+INC        │
         └──────────────────────┘
                      │
                      ▼
                    ( 5 )
                      │                      ( C )
                      │                        │
                      ◄────────────────────────┘
                      ▼
                    ╱IS ╲            NO
                   ╱ XW<0 ╲ ──────────────►  ( 3 )
                    ╲     ╱
                     ╲   ╱
                      YES
                      │
                      ▼                    ┌──────────────────────┐
                    ╱IS ╲       YES        │  DISPLAY THAT        │
                   ╱ GM>1 ╲ ──────────────►│  THE SYSTEM IS       │
                    ╲     ╱                │  STABLE              │
                     ╲   ╱                 └──────────────────────┘
                      NO                              │
                      │                               │
                      ▼                               │
         ┌──────────────────────┐                     │
         │  DISPLY THAT THE     │                     │
         │  SYSTEM IS           │                     │
         │  UNSTABLE.           │                     │
         └──────────────────────┘                     │
                      │       ◄──────────────────────┘
                      ▼
         ┌──────────────────────────────┐
         │  DISPLAY PHASE MARGIN,        │
         │  GAIN MARGIN, PHASE CROSS     │
         │  OVER FREQUENCY AND           │
         │  GAIN CROSS OVER              │
         │  FREQUENCY                    │
         └──────────────────────────────┘
                      │
                      ▼
                    ( G )
```

34

```
                              ( 3 )
                                │
                                ▼
                         ╱──────────╲                    ┌──────────────────────┐
                        ╱    IS       ╲      YES          │   DISPLAY THE        │
                        ╲   GCF=0     ╱ ──────────────▶   │  SYSTEM IS STABLE    │
                         ╲──────────╱                     └──────────────────────┘
                               │                                    │
                               │ NO                                 │
                               ▼                                    │
┌──────────────────────┐    ╱──────────╲                           │
│ DISPLAY SYSTEM IS    │ NO ╱    IS      ╲                          │
│ UNSTABLE WITH        │◀───╲   PM>0     ╱                          │
│ INFINITY GAIN        │     ╲──────────╱                          │
└──────────────────────┘          │                                │
            │                      │ YES                            │
            │                      ▼                                │
            │            ┌──────────────────────┐                  │
            │            │ DISPLAY THAT THE      │                  │
            │            │ SYSTEM IS STABLE      │                  │
            │            │ WITH INFINITY         │                  │
            │            │ GAIN                  │                  │
            │            └──────────────────────┘                  │
            │                      │        ◀───────────────────────┘
            │                      │   ◀────────────  ( G )
            └──────────▶           │
                         ┌──────────────────────┐
                         │ CHECK WHETHER USER   │
                         │ WANT TO GIVE THE     │
                         │ DIFFERENT VALUE FOR  │
                         │ MX,MN,INC. IF YES GOTO│
                         │ STEP 6 ELSE          │
                         └──────────────────────┘
                                   │
                                   ▼
                              (  END  )
```
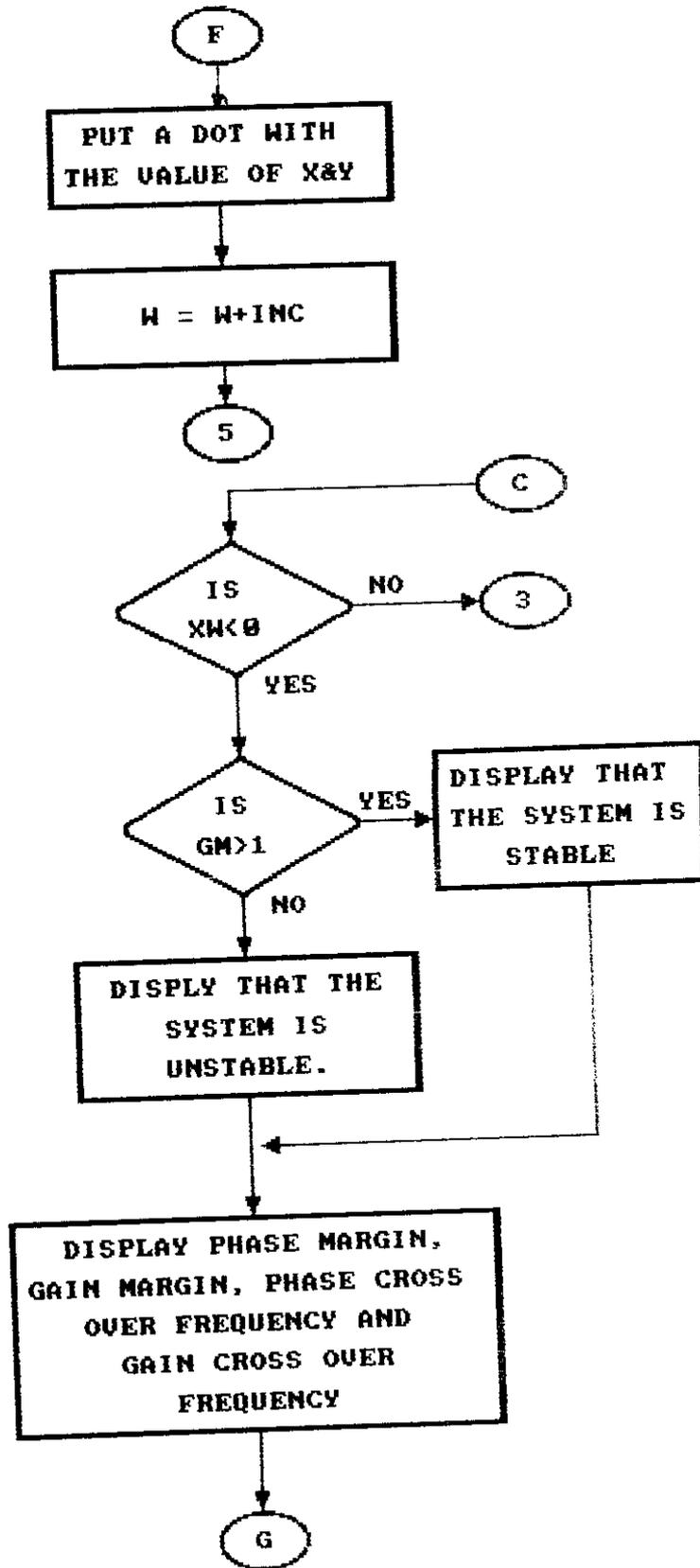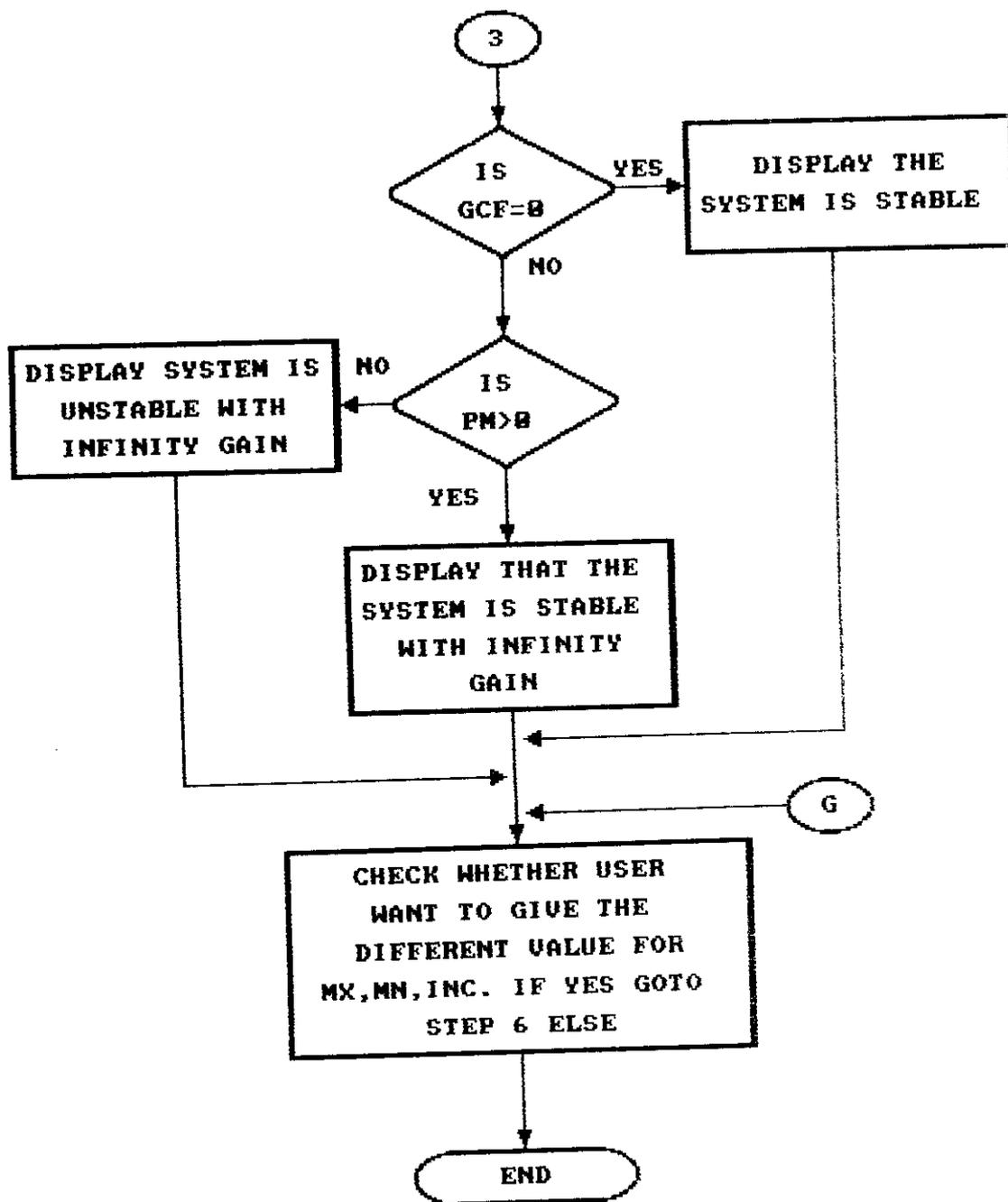
```c
#include<stdio.h>
#include<math.h>
#include<float.h>
#include<graphics.h>
#include "print.c"
main()
{
double p[10],pa[200],w,ps[200],sen,psa=0.0,dr;
double z[10],gam,za[200],zs[200],zdr=0.0,nr;
double mag[500],arg[500],omega[500],vx[500],vy[500];
int gd=DETECT,gm=CGAC0,i,pq,zq,n,x,m=0,j=0,b=0,k=0,my=0,me=0;
int zn,y,pn,aa,aa1,bb,bb1,xag,yag,st,hz,kj;
double dr_theta=0.0,nr_theta=0.0,theta,r;
double xa,ya,xs=0.0,xw=0.0,yw;
int delay,x1,x2,x3,y1,y2,y3;
double sem,ke,pm,gcf=0.0,ty,in,mx,mn,t;
double con,pt,pe,zt,ze,xas=0.0,yas=0.0;
int ssam,exa,cam,mulx,maxx,maxy,zx=0;
clrscr();
printf("enter the no. of roots in dr:");
scanf("%d",&pn);
printf("enter the roots now\n");
for(i=0;i<pn;++i)
scanf("%lf",&p[i]);
printf("enter the no. of roots in nr:");
scanf("%d",&zn);
printf("enter the roots now\n");
for(i=0;i<zn;++i)
scanf("%lf",&z[i]);
printf("enter the constant in nr:");
scanf("%lf",&con);
printf("if quadratic equation  present in  dr.enter 1 else 0:");
scanf("%d",&pq);
if(pq==1)

 {
 printf("enter the value of e:");
 scanf("%lf",&pe);
 printf("enter the value of pt: ");
 scanf("%lf",&pt);

}

printf("if quadratic eqn.  present in nr. enter 1 else 0:");
scanf("%d",&zq);

if(zq == 1)

{
  printf("enter the value of e: ");
  scanf("%lf",&ze);
  printf("enter the value of zt: ");
  scanf("%lf",&zt);

}

printf("enter the type of system:");
scanf("%lf",&ty);
printf("do you have any delay if yes enter 1 else 0: ");
scanf("%d",&delay);
```

36

```c
if(delay == 1)

{

 printf("enter the value of t: ");
 scanf("%lf",&t);


}
printf("\n\n");
printf("so the given tr.fn.is  in form given below\n");
printf("           ");
printf("%lf",con);
for(i=0;i<zn;++i)
{
  if(z[i] !=0)
    {
      printf("(1 + ");
      printf("%lf",z[i]);
      printf("s)");
    }
}
if(zq == 1)
{
 printf("(%lf)(%lf)",zt,zt);
 printf("(s*s +");
 printf("(2)(%lf)(%lf)(s) + 1",ze,zt);
 printf(")");
}
if(delay == 1)
{
 printf("exp(-(%lf)(s)",t);
}
printf("\n");
printf("G(s) = ");
printf("------------------------------------------------------------\n");
printf("           ");
if(ty ==1)
{
printf("(s) ");
}
else if(ty>1)
{
   printf("(s ");
   for(i=1;i<ty;++i)
     {
       printf("*s");
     }
   printf(")");
}
for(i=0;i<pn;++i)
{
  if(p[i] != 0)
  {
   printf("(1 + ");
   printf("%lf",p[i]);
   printf("s)");
  }
}
if(pq == 1)
{
```

37

```c
printf("{[(%lf)(%lf)",pt,pt);
printf("(s*s)] +");
printf("[(2)(%lf)(%lf)(s)] +1",pe,pt);
printf("}");
}
printf("\n");
printf("press any key to continue\n");
getch();
onceagain:
clrscr();
initgraph(&gd,&gm," ");
if(ty > 0)
{
 printf(" the minimium value of w(>0):");
}
else
{
 printf("the minimium value of w:");
}
scanf("%lf",&mn);
printf(" the maximium value of w:");
scanf("%lf",&mx);
printf(" the increment of w:");
scanf("%lf",&in);
maxx = getmaxx();
maxy = getmaxy();
printf("the maximium no. of pixels in horizontal is %d\n",maxx);
printf("the maximium no. of pixels in vertical is %d\n",maxy);
printf(" enter the centre co ordinates:");
scanf("%d%d",&x3,&y3);
printf(" the multiplication factor:");
scanf("%d",&mulx);
printf("do you the values of x,y,w,theta,magnitude (1/0): \n");
scanf("%d",&y2);
printf("\n");
printf("press any key to continue\n");
cleardevice();
getch();
line(x3,0,x3,maxy);
line(0,y3,maxx,y3);
outtextxy(x3+1,y3+1,"0");
outtextxy(x3-mulx-3,y3,"-1");
if(ty == 0)
{
outtextxy(x3+mulx+5,y3,"1");
}
if(x3<400)
{
outtextxy(x3+175,y3+1," RE(G(jw))-------> ");
}
else
{
outtextxy(x3-300,y3+1," RE(G(jw)) ");
}
if(x3<100)
{
outtextxy(x3,y3+100,"IM(G(jw))");
}
else
{
```

38

```c
outtextxy(x3-100,y3+100,"  IM(G(jw))   ");
}

for(w=mn;mn<mx;  )
{

  /*    TO FIND THE MAGNITUDE OF DR. ROOTS          */

pa[j] = 1.0;
++j;
pa[j] = w*w*p[m]*p[m];
++m;
for(i=0;i<pow(2,m);++i)
{
 ps[i] = pa[i];
}
for(m=1;pn>m;       )
{
  x = pow(2,m);
  for(i=0;i<x;++i)
  {
   ps[i] = pa[i];
  }
  for(i=x;i<(x*2);++i)
   {
     ps[i]= pa[i-x]*w*w*p[m]*p[m];
   }
  ++m;
  for(i=0;i<pow(2,m);++i)
   {
     pa[i] = ps[i];
   }
}

for(i=0;i<pow(2,m);++i)
  {
  psa+=pa[i];
   }
   if(psa<0)
   {
   psa = -1*psa;
   }
   dr = sqrt(psa);


  /*            TO FIND THE MAGNITUDE OF NR. ROOTS          */

za[b] = 1.0;
++b;
za[b] = w*w*z[k]*z[k];
++k;
for(i=0;i<pow(2,k);++i)
{
 zs[i] =za[i];
}
for(k=1;zn>k; )
{
 y = pow(2,k);
 for(i=0;i<y;++i)
  {
```

```c
      zs[i] = za[i];
    }
    for(i=y;i<(y*2);++i)
    {
      zs[i]= za[i-y]*w*w*z[k]*z[k];
    }
    ++k;
    for(i=0;i<pow(2,k);++i)
    {
      za[i] = zs[i];
    }
}
for(i=0;i<pow(2,k);++i)
{
  zdr+=za[i];
}
if(zdr<0)
{
  zdr= -1*zdr;
}
nr = sqrt(zdr);


/*     TO FIND THE ARGUMENT OF DR      */

for(i=0;i<pn;++i)
dr_theta+=atan(w*p[i]);


/*     TO FIND THE ARGUMENT OF NR      */

for(i=0;i<zn;++i)
nr_theta+=atan(w*z[i]);
if(ty>0)
{
  for(i=0;i<ty;++i)
  {
    dr*=w;
    dr_theta+=1.570796327;
  }
}

if(pq ==1)
{
sen = (1-pt*pt*w*w);
dr= dr * sqrt(((1-pt*pt*w*w)*(1-pt*pt*w*w))+(4*pe*pe*pt*pt*w*w));
dr_theta = dr_theta + atan((2*pe*w*pt)/sen);
}
if(zq == 1)
{
nr = nr * sqrt(((1-zt*zt*w*w)*(1-zt*zt*w*w))+(4*w*w*ze*ze*zt*zt));
nr_theta = nr_theta + atan((2*ze*zt*w)/(1-zt*zt*w*w));
}
if(delay == 1)
{
nr_theta+=-1*w*t;
}
r = (nr*con)/dr;
theta = nr_theta - dr_theta;
xa = r*cos(theta);
```

```c
ι = r*sin(theta);
ι = -1*ya;

?(sen<0)

ι = -1*xa;
ι = -1*ya;

ag =xa*mulx;
ag = ya*mulx;
f(y2 == 1)

nega[zx] = w;
y [zx] = -1*ya;
x[zx] = xa;
ag[zx] = r;
rg[zx] = theta;



   TO  FIND  THE  PHASE  MARGIN      */

if((xag+x3<x3) && (yag+y3 != y3-1))
  {
   if(me==0)
     {
        ke = (xa*xa + ya*ya);

        if(ke>.991  && ke<1.009)
        {
          circle(x3,y3,mulx);
          xas = xa;
          yas = ya;
          ya = -1*ya;
          pm = ((atan(ya/xa)*180)/3.141593);
          gcf = w;
          ++me;
        }
     }
  }


/*     TO FIND THE GAIN  MARGIN     */

if(yag+y3 == y3)
{
   if(xag+x3<x3)
    {
     if(my==0)
       {
          if(ya<0)
           {
             ya= -1*ya;
           }
          ssam= ya*5000;
          if(ssam == 0)
           {
             xs =xs + w;
             xw =xw +  xa;
```

```c
            gam =-1.0*(1/xs);
            /*circle(x3,y3,mulx);*/
            my = my+1;
          }
        }
    }
}
putpixel(xag+x3,yag+y3,15);
mn+=in;
zdr =0.0;
psa = 0.0;
k=0;
j=0;
m=0;
b=0;
w = mn;
dr_theta = 0.0;
nr_theta = 0.0;
zx = zx+1;
}
finish:
outtextxy(200,2," P  O  L  A  R    P  L  O  T ");
outtextxy(200,190,"DO you want to take printout (y / n) ");
if(toupper(getch())=='Y')print_graph('e',0,1);
closegraph();
clrscr();
if(xw != 0 )
{
  if(((xw*mulx)+x3<x3) && ((ya*mulx) != 0))
    {
      if(gam>1)
        {
        printf("                    SYSTEM IS STABLE                    \n");
        }
        else
        {
          if(gam<1)
          {
            printf("              SYSTEM IS UNSTABLE  \n");
          }
        }
printf("  THE PHASE CROSS OVER FREQUENCY  =  %lf\n",xs);
printf("  THE VALUE OF X = %lf\n",xw);
printf("  THE GAIN MARGIN (GM) =  %lf\n",gam);
printf("  THE VALUE OF X  =  %lf\n",xas);
printf("  THE VALUE OF Y  =  %lf\n",yas);
printf("  THE PHASE MARGIN =  %lf degrees:\n", pm);
printf("  THE GAIN CROSS OVER FREQUENCY  = %lf\n",gcf);
    }
}
else
{
  if( gcf == 0)
    {
      printf(" SYSTEM IS STABLE \n");
    }
  else
    {
      if(pm<0)
```

```c
          {
          printf("SYSTEM IS UNSTABLE WITH INFINITY GAIN\n");
           printf("PHASE MARGIN =  %lf",pm);
          }
        else
        {
         if(pm>0)
         {
          printf("SYSTEM IS STABLE WITH INFINITY GAIN\n");
           printf("PHASE MARGIN  = %lf",pm);
         }
   }
   }
}
if(toupper(getch()) == 'y' ) */
printf("------------------------------------------------------------
printf(" w         mag         theta         x              y
printf("------------------------------------------------------------
if(y2 ==1)
{
one:
for(i=0;i<zx;++i)
{
printf("%13.6lf",omega[i]);
printf("%13.6lf",mag[i]);
printf("%13.6lf",arg[i]);
printf("%13.6lf",vx[i]);
printf("%13.6lf",vy[i]);
printf("\n\n");
++j;
if(j == 12)
{
printf("press any key to continue\n");
getch();
j = 0 ;
goto one;
}
}
}
last:
getch();
printf("\n\n");
printf("DO YOU WANT TO PLOT AGAIN WITH OTHER PARAMETERS ? \n");
if(toupper(getch()) == 'y' )
{
clrscr();
goto onceagain;
}
}
```

Simulation Results and Analysis

3. Adding a Pole to 2.

i.e.

$$G(s) = \frac{5}{(1 + s)(1 + 2s)(1 + 3s)}$$

SYSTEM IS STABLE

GAIN MARGIN = 2

PHASE MARGIN = 24.92°

PHASE CROSS OVER FREQUENCY = 1

GAIN CROSS OVER FREQUENCY = 0.71

4. Adding a Zero to 2.

i.e.

$$G(s) = \frac{5(1 + 3s)}{(1 + s)(1 + 2s)}$$

SYSTEM IS STABLE

5. Quadratic Factor

$$G(s) = \frac{1}{0.1 s^2 + 0.19 s + 1}$$

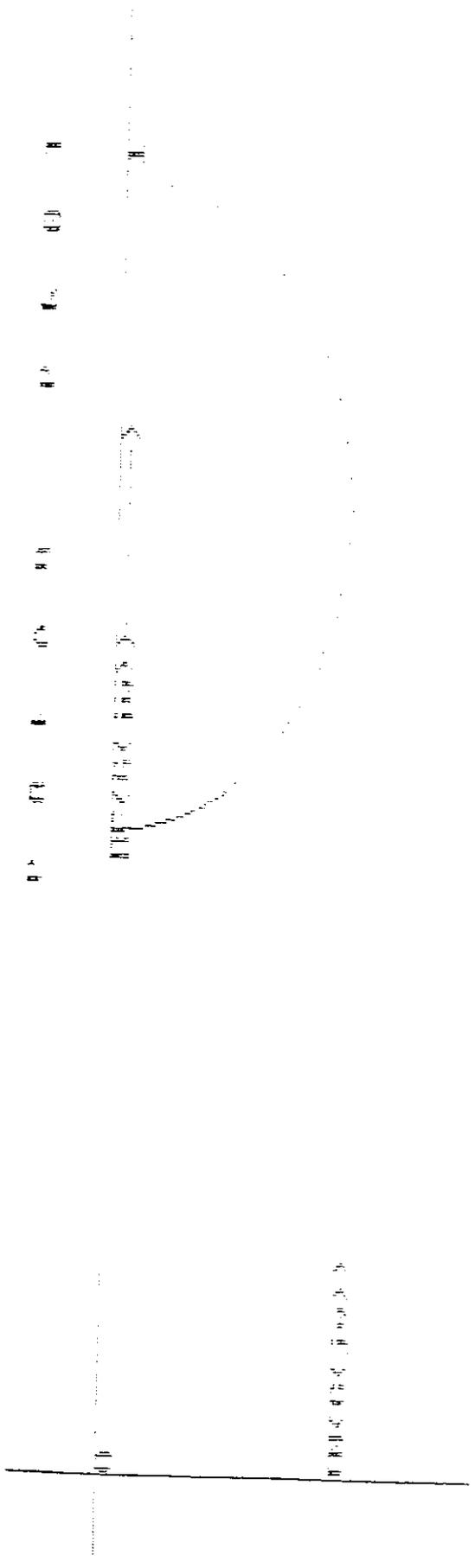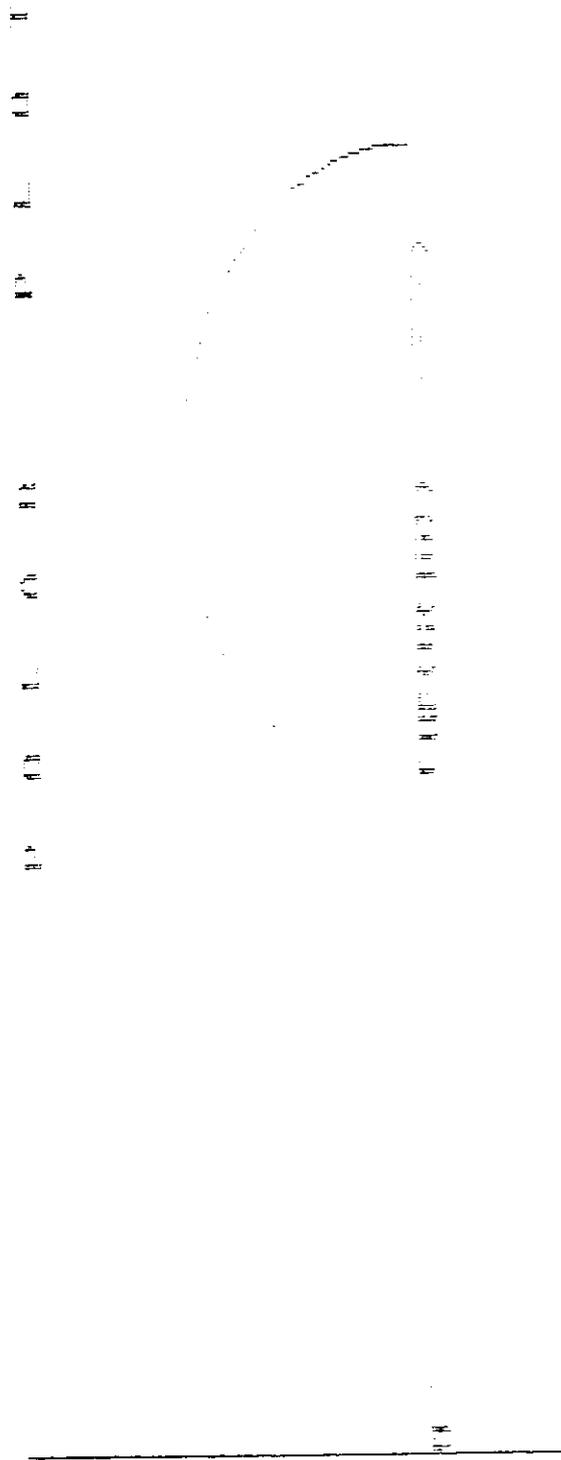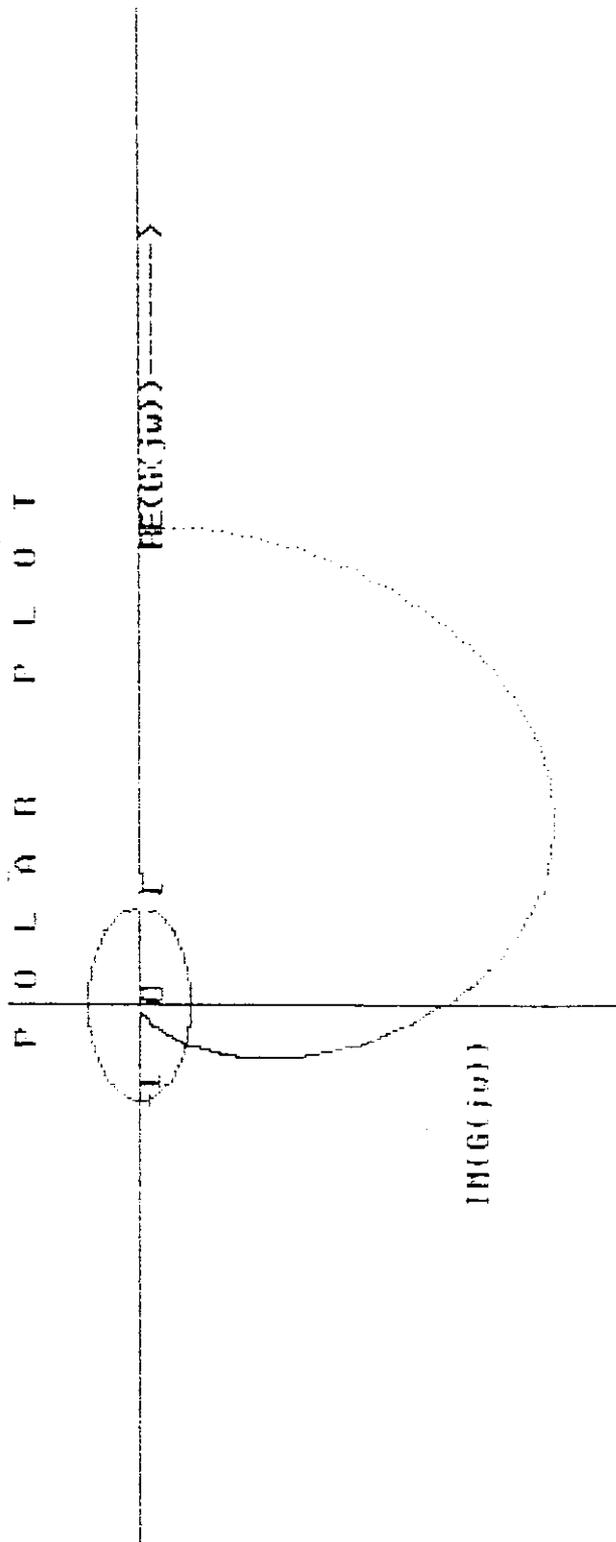SYSTEM IS STABLE WITH INFINITY GAIN MARGIN

PHASE MARGIN = 50.26°.

Fig. 4.1a

Fig 4.1b

P O L A R   P L O T

RE(G(jω))————>

IMG(G(jω))

Fig.4.2

POLAR PLOT

Fig.4.3

RE(G(jw)) ------->

IM(G(jw))

1

0

-1
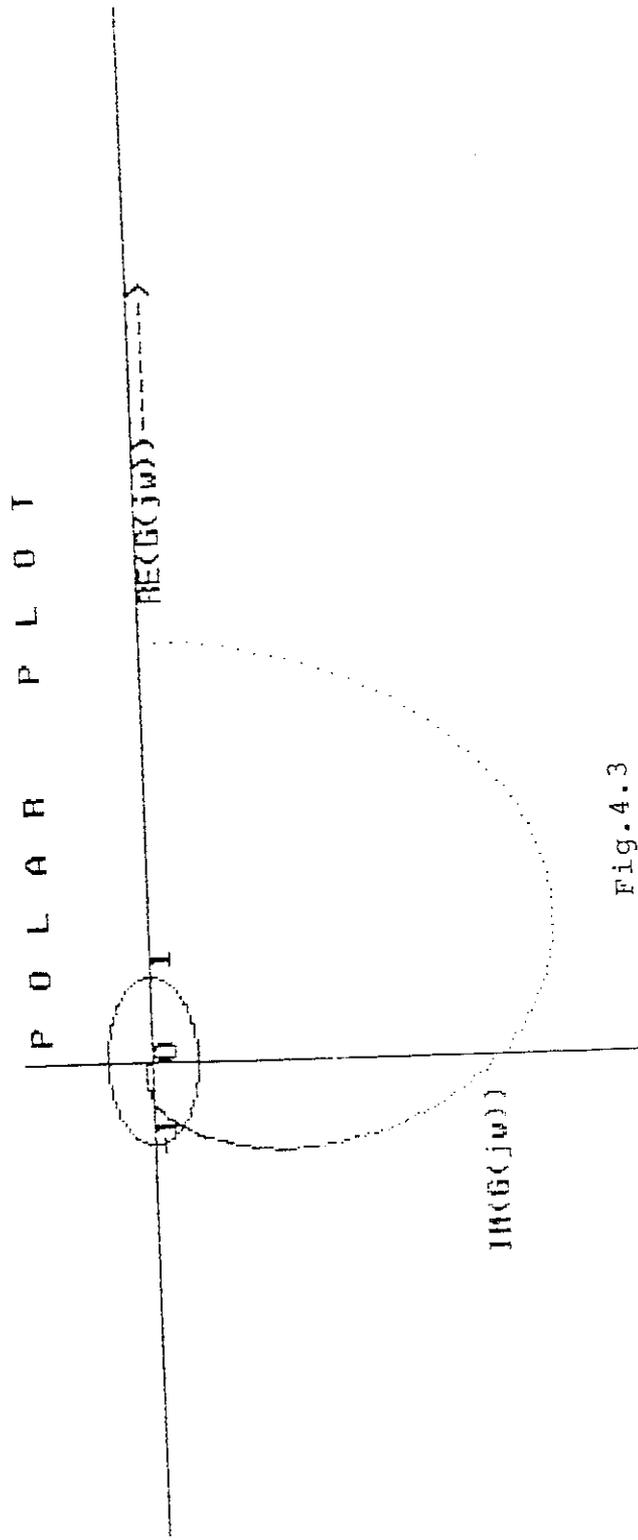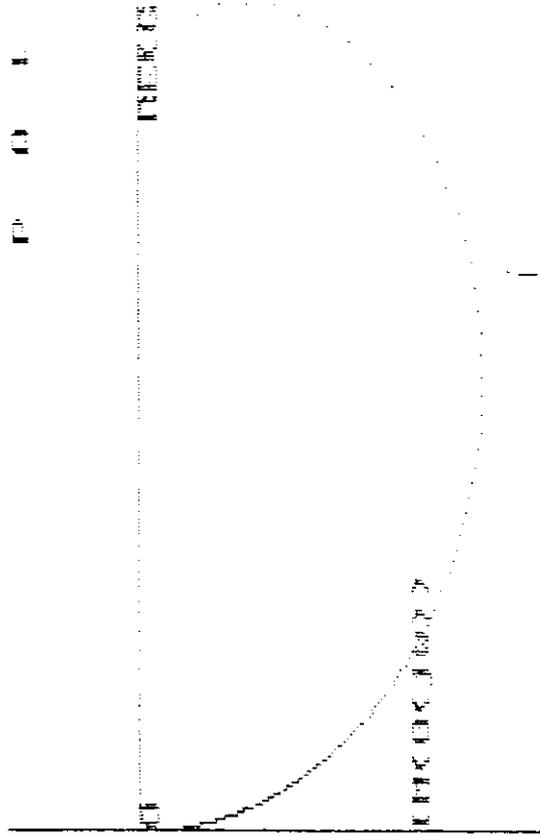
Fig.4.4
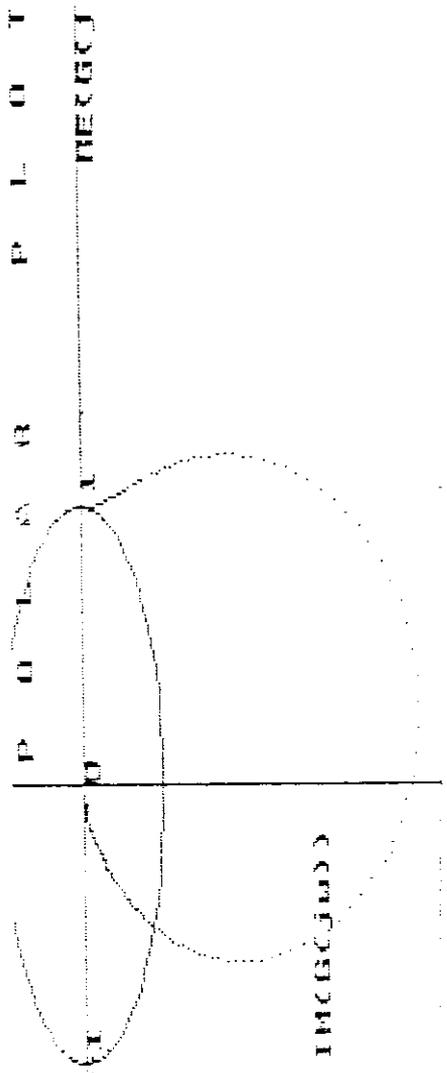
POLAR PLOT

Fig.4.5

Conclusion

C H A P T E R - V

CONCLUSION

In this project, a software package has been developed to draw the polar plot without the aid of the polar sheet and accessories. The polar plots obtained for type 0 & 1 systems are presented. Instead of time consuming manual analysis, just by giving the transfer function, maximimum value, minimium value and increment of frequencies, it is possible to obtain the curve within a short time. As the analysis leads to design which involves inclusion or modification of the system parameters it becomes a time consuming process. Hence the modification like adding a pole or zero, including an integral function can be easily done using this software. Further analysis after modification is done within a few minutes. It will reduce human work.

The accuracy of the output (i.e) the phase margin and gain margin is closely related to the increment in frequency.

It is found that the software produces satisfactory results.

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APPENDIX A — DEFINITIONS

SYSTEM:

A system is a combination of components that act together and perform a certain objective.

CONTROL SYSTEM:

The control system is that means by which any quantity of in interest in a machine, mechanism or other equipment is maintained or altered in accordance with a desired manner.

OPEN LOOP CONTROL SYSTEM:

Open — loop control systems are control systems in which the output has no effect upon the control action.

CLOSED LOOP CONTROL SYSTEM:

A Closed — loop control system is one in which the output signal has a direct effect upon the control action.

LINEAR SYSTEM:

When the magnitude of the signals in a control system are limited to a range in which system exhibits linear characteristics (i.e. the principle of superposition applies), the system is linear system.

NON LINEAR SYSTEM:

When the magnitudes of the signals are extended outside the range of the linear operation (i.e. does not obeys superposition theorem), the system is non linear system.

POLE:

If a function $G(s)$ is analytic and single valued in the neighborhood of s_i, except at s_i, it is said to be have a pole of order r at $s = s_i$ if limit

$$\lim_{s \to s_i} (s-s_i)^r \, G(s)$$

has a finite non zero value.

ZERO:

If the function $G(s)$ is analytic at $s = s_i$, it is said to be have a zero of order r at $s = s_i$, if the limit ,

$$\lim_{s \to s_i} (s - s_i)^{-r} \, G(s)$$

TIME INVARIANT SYSTEM:

When the parameters of a control systems are stationary with respect to time during the operation of the system is called a time invariant system.

APPENDIX AN INTRODUCTION TO 'C'

C was written by Dennis Ritchie in the early 70 's to support the development and implementation of Bell Laboratories UNIX operating system. C is often the language of choice for the development of systems software, by the developers of personnel computers and is encountered in real time applications.

In any programming language, programmers often simplify a difficult task by breaking the task into several smaller, manageable ones. In C, we call these smaller tasks functions. By combining functions, we create programs. A major benefit of separating large programs into functions is that several programmers can work on the different parts of a problem at the same time and later combine these to come to a final solution of the problem. Once a function is created we can reuse this in other programs without having to change the code and this saves a great deal of time and effort.

C is a free format language, which means that we do not have to specify line numbers or place our statements in specific locations of a line.

ADVANTAGE OF C LANGUAGE

1. C is one of the most portable programming languages in existence. A program written in C on one machine will normally run on other machines with little or no modification.

2. C provides us with a large set of data structures, an economy of expression and a robust collection of operators and thus helps in case of development.

3. C provides access to operations that are normally restricted to assembly language programs. The advantages gained by employing a high level language for these functions include case of development and testing, increased protability and modifiability.

4. UNIX one of the most operating systems in use today and it may becomes an industry - wide standard operating system. 85% of UNIX code was written in C and this fetches many advantages.