SIGNAL AND IMAGE APPROXIMATION USING INTERVAL WAVELET TRANSFORM

A PROJECT REPORT

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BONAFIDE CERTIFICATE

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DECLARATION

We,

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hereby declare that the project entitled "SIGNAL AND IMAGE APPROXIMATION USING INTERVAL WAVELET TRANSFORM" , submitted in partial fulfillment to Anna University is a record of original work done by us under the supervision and guidance of the Department of Computer Science and Engineering, Kumaraguru College of Technology,Coimbatore.

Place:Coimbatore

Date: 27.04.09

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Abstract:

In signal approximation, classical wavelet syntheses are known to produce Gibbs-like phenomenon around discontinuities when wavelet coefficients in the cone of influence of the discontinuities are quantized. The wavelet transform is a powerful tool because it manages to represent both transient and stationary behaviors of a signal with few transform coefficients. By analyzing a function in a piecewise manner, filtering across discontinuities can be avoided. Any piecewise smooth signal can be expressed as a sum of a piecewise polynomial signal and a uniformly smooth residual. Using this principle, the interval wavelet transform is used to generate sparser representations in the vicinity of discontinuities than classical wavelet transforms. This work introduces two new constructions of interval wavelets and it is used for the purpose of image compression and up-scaling.

Introduction:

It is well known that wavelets can provide sparse and, hence, efficient representations of smooth functions. This property is especially advantageous to signal coding. In the vicinity of a signal discontinuity, large wavelet coefficients can be generated. When these coefficients are quantized or lost, Gibbs-like phenomenon can be observed in reconstructed signals. The problem worsens in the presence of higher dimensional discontinuities. For example in image compression, artifacts marked by *softness*, *ringings*, *halos*, and *color bleeding* can be seen along edges.The wavelet transform is a powerful tool because it manages to represent both transient and stationary behaviors of a signal with few transform coefficients. Discontinuities often carry relevant signal information, and therefore, they represent a critical part to analyze. The dependency across scales of the wavelet coefficients generated by discontinuities is studied. Any piecewise smooth signal can be expressed as a sum of a piecewise polynomial signal and a uniformly smooth residual. The notion of footprints, which are scale space vectors that model discontinuities in piecewise polynomial signals exactly.

Problem Description:

It is well known that wavelets can provide sparse and, hence efficient representations of smooth functions. This property is especially advantageous to signal coding. In the vicinity of a signal discontinuity, large wavelet coefficients can be generated. When these coefficients are quantized or lost, Gibbs-like phenomenon can be observed in reconstructed signals. The problem worsens in the presence of higher dimensional discontinuities. For example in image compression, artifacts marked by *softness*, *ringings*, *halos*, and *color bleeding* can be seen along edges.

Existing system:

To overcome this inadequacy of classical wavelets, several techniques have been proposed for more efficient analysis and synthesis of singularity structures. For example, in some techniques, directionality is introduced into existing 2-D wavelet functions. Separable 2-D wavelets gives only three orientations in the decomposition subbands—vertical, horizontal, and diagonal. Thus, edges with other orientation profiles will tend to have wavelet coefficients that span across all the subbands. By having directionality in wavelet bases, the wavelet coefficients for any edge can be contained in only one subband depending its orientation. Other works, such as *wavelet footprints*, *wavelet modulus maximas*, and the *essentially non-oscillatory* (ENO) wavelets, attempt to obtain sparser representations of 1-D discontinuities. By using a dictionary containing normalized wavelet coefficient sets belonging to the same *cone of influence* of some discontinuities, the wavelet footprints technique is able provide compact representations of discontinuities through dictionary indices. Wavelet maximas are wavelet coefficients that are strict local maximums on the decomposition map. The ENO schemes used in fluid dynamics to capture shocks, rarefaction, and contact discontinuities, are applied in the ENO-wavelet transform to avoid filtering across discontinuities. Other approaches to construct orthonormal bases for analyzing 2-D discontinuities include *beamlets*, *wedgelets,*, and *platelets.*

Proposed System:

The objective of our work is to show that more efficient edge representation can be obtained using *interval wavelet* transform. Given an ordered set of M+1 breakpoints

a N-length sequence X can be viewed as a cascade of M intervals

where $X_n = \{x_i\}$ $k_n = i = k_{n+1}$ is the n^{th} interval. Each interval can be analyzed independently using interval wavelets without filtering across the coefficient pairs $\{x_{i-1}, x_i\}$ $i \in K \setminus \{0, N\}$. If K is chosen such that it contains locations of discontinuities, it is possible for the interval wavelet transform to have sparser representations than the classical wavelet transform.

MODULES

1. Wavelets on Intervals [0, N]:

Most practical applications, such as image processing, involve signals f on finite support. However, filtering operations require values of the signal outside its supported range. Signals can be extrapolated by means of periodic, symmetric, or polynomial- based methods, which will allow filters to be applied on the signal as if $f \in L^2R$. Alternatively, special filters can be designed to replace original filters at the signal borders. Such filters are adapted to interval boundaries and, thus, do not require signal values outside the interval.

Here, the terms boundary and edge are used interchangeably to denote the signal boundaries while the interior denotes the region between signal boundaries. *Interval functions* is used as a collective term for both the boundary and interior functions. Thus, the interval scaling and wavelet functions are given by

where the superscripts of the boundary functions ϕ^{left}, ϕ^{right}, ψ^{left} and ψ^{right} denote adaptation to the appropriate interval boundaries. After vertical and horizontal detections of the given image, pre-processing and post-processing of Vertical-edge and Horizontal-edge matrices are followed.

2.Discrete Interval Wavelet Transform:

Signal synthesis using interval wavelets does not result in border distortions since filtering is not performed across boundaries. Furthermore, boundary wavelets possess vanishing moments which allows one to exploit the signal smoothness near the borders. Here, we propose utilizing the interval wavelets to yield sparser representations for piece-wise smooth signals. Essentially, the idea is to break up the signal into smaller intervals and perform independent analysis on each of the segments. If the intervals are chosen such that the breakpoints are at desired discontinuities within the signal, Gibbs-like artifacts could be reduced and even possibly eliminated.

a. Odd Length Decomposition

In dyadic wavelet decomposition, an even-length signal of 2^N gives 2^{N-1} coefficients in each low and high-pass subband. For an odd length 2^N+1 signal, the decomposition should ideally produce an unequal number of coefficients from each channel, such that $[(2^N+1)/2] + [(2^N+1)/2] = 2^N+1$. In practice, the usual treatment is to pad the signal to the nearest even length prior to decomposition at each scale so that dyadic decomposition could be applied. This introduces some redundancies in the decomposition. The interval wavelet transform has to adapt to different possible interval lengths.

b. Balanced Decomposition

For signals containing odd length intervals, it is possible to have a very unbalanced decomposition whereby there are significantly more low-pass than the high-pass coefficients. In case of an unbalanced decomposition of an even length signal containing odd length intervals, since the original signal is of even length, it is preferable to produce an equal number of low and high-pass output coefficients. To ensure that the number of scaling and wavelet coefficients are balanced in addition to having a non-expansive decomposition, the appropriate number of scaling and wavelet coefficients for each interval decomposition has to be determined.

c. Filter Misalignments between Scanlines

In a 2-D interval wavelet transform, the interior filters may be misaligned with respect to each decomposition row and column if only one type of boundary filter is used. Misaligned interior filters can lead to observable artifacts in the reconstructed images.

d. Discontinuities and Interval Breakpoints

Edge or discontinuity detection is a prerequisite step that enables the interval wavelet transform to divide a signal into intervals. Due to anti-aliasing effects in digital image processing, smoothed discontinuities are encountered more often than step discontinuities. Each detection scheme can give slightly different positions for the same discontinuities. Moreover, since different filters could be used for interval wavelet decomposition, these detected edge positions may not lead to sparse representations.

Implementation:

System implementation is the process of making the newly designed system fully operational and consistent in performance. The following steps has been followed in the implementation of the system.

As the part of implementation, the system is taken the site and Loaded on to client's computer. Some of the user's level, exposure to computer etc. these users is trained first and they run the system for a month. A detailed documentation is prepared for the employees and they trained to access the software. These users are trained first and they can run the system for a month.

After installation of software, the hardware specifications are checked. If hardware specifications are satisfactory, then the software is loaded for pilot run. User training starts at this time itself. Users will be given a user manual, which documents how to use the system and all the exception handling procedures.

a. One-Dimensional Signal Approximation

In 1-D signal linear approximation using only scaling coefficients, the Laplacian operator is used to obtain the edge locations for the interval wavelet transform. The reconstructed signal using classical wavelets will exhibit the usual ringing artifacts in the vicinity of discontinuities. In contrast, these artifacts are absent in the signal approximated using the ENO and the interval wavelet transform. In case of the approximated signal with additive Gaussian white noise, modulus wavelet maximas are used to obtain the edge positions to avoid false edges due to noise. It is observed that where the edges are not detected, the usual artifacts are introduced at the discontinuities.

b. Image Approximation

The image decomposition and reconstruction scheme using interval wavelet transform is illustrated where the binary maps, E_v and E_h, represent positions of vertical and horizontal image edges respectively. The multiscale binary edge maps are denoted by E'_v and E'_h. Each image row and column data is transformed using the boundary filter assignment rules. Comparing nonlinear approximations of images using classical wavelet, wavelet maxima and interval wavelet transforms, it is observed that, as expected, approximation by classical wavelets blurs the edges and introduces ringing artifacts along edges. Both the wavelet maxima and interval wavelet transform are able to produce sharper image edges with fewer artifacts. However, as expected, PSNR values are not consistent with the observed quality of the reconstructed images.

c. Image Upscaling

Well-known image scaling techniques, such as *bicubic* and *bilinear* interpolation, tend to increase the transient width of an edge profile, which leads to blurring. The bicubic interpolation also introduces overshoots, or peaking, around discontinuities, which is also known as the *halo* effect. Using the interval wavelet transform, it is possible to obtain sharper upscaled images. Consider 2*upsampling of an image. The original image is assumed to be the scaling coefficients and all other wavelet coefficients are zeroed. Performing an inverse transform with appropriate scaling yields a 2*upsampled image. However, the edge locations in the finer subbands have to be predicted prior to the inverse transform to avoid edge aliasing artifacts. Compareing image upsampling using bicubic and interval wavelets, as expected, images produced using the interval wavelet interpolation have sharper edges.

Results and Discussions:

There is a similarity between the interval wavelet and the ENO wavelet transform—both techniques do not filter across discontinuities of interest. Hence, the two methods would give similar sparse representations and nonlinear approximation of piecewise smooth signals. Compared to the computation cost of the classical wavelet transform, which is of order O(N.L) where N is size of data and L the length of filter, the ENO wavelet transform has an additional cost of O(K.L) for extrapolation about K discontinuities. In contrast, the interval wavelet transform does not incur additional computational costs since the wavelet filters near the discontinuities are simply replaced by boundary filters. Therefore, a fast forward and inverse interval wavelet transform can be implemented using Mallat's algorithm with only complexity O(N). Similarly, the wavelet maxima reconstruction can be obtained using the *à trous* algorithm with $O(N \log_2 N)$ calculations per iteration.

it is shown that interval wavelets can be exploited to obtain sparser representations of signal discontinuities by splitting a signal into intervals at the discontinuities and transforming each interval independently. The two new sets of boundary functions are introduced to enable a more robust decomposition. The potential of applying interval wavelet transform is demonstrated to image compression and upscaling. In both cases, images are reconstructed using interval wavelets are sharper and have less artifacts along the edges than conventional methods.

Conclusion:

This work shows that interval wavelets can be exploited to obtain sparser representations of signal discontinuities by splitting a signal into intervals at the discontinuities and transforming each interval independently. two new sets of boundary functions are introduced to enable a more robust decomposition. The potential of applying interval wavelet transform is demonstrated to image compression and upscaling.

In both cases, Images that are reconstructed using interval wavelets are sharper and have less artifacts along the edges than conventional methods.

Literature Survey:

Contourlets: A Directional Multiresolution Image Representa.ion

A new scheme is proposed, named *contourlet*, which provides a flexible multiresolution, local and directional image expansion. The contourlet transform is realized efficiently via a double iterated filter bank structure. Furthermore, it can be designed to satisfy the anisotropy scaling relation for curves, and thus offers a fast and structured curvelet-like decomposition. As a result, the contourlet transform provides a sparse representation for two-dimensional piecewise smooth signals resembling images.

The Contourlet Transform: An Efficient Directional Multiresolution Image Representation

The limitations of commonly used separable extensions of one-dimensional transforms, such as the Fourier and wavelet transforms, in capturing the geometry of image edges are well known. In this work, we pursue a "true" two-dimensional transform that can capture the intrinsic geometrical structure that is key in visual information. The main challenge in exploring geometry in images comes from the discrete nature of the data. Thus, unlike other approaches, such as curvelets, that first develops a transform in the continuous domain and then discretizes for sampled data, our approach starts with a discrete-domain construction and then studies its convergence to an expansion in the continuous domain. Specifically, we construct a discrete-domain multiresolution and multidirection expansion using non-separable filter banks, in much the same way that wavelets were derived from filter banks. This construction results in a flexible multiresolution, local, and directional image expansion using contour segments, and thus it is named the *contourlet* transform. The discrete contourlet transform has a fast iterated filter bank algorithm that requires order N operations for N-pixel images. Furthermore, we establish a precise link between the developed filter bank and the associated continuous domain contourlet expansion via a directional multiresolution analysis framework. We show that with parabolic scaling and sufficient directional vanishing moments, contourlets achieve the optimal approximation rate for piecewise smooth functions with discontinuities along twice continuously differentiable curves.

Ridgelets: a key to higher-dimensional intermittency?

In dimensions two and higher, wavelets can efficiently represent only a small range of the full diversity of interesting behavior. In effect, wavelets are well adapted for point-like phenomena, whereas in dimensions greater than one, interesting phenomena can be organized along lines, hyperplanes, and other non-pointlike structures, for which wavelets are poorly adapted. A new subject is discussed in this paper, ridgelet analysis, which can effectively deal with line-like phenomena in dimension 2, plane-like phenomena in dimension 3 and so on. It encompasses a collection of tools which all begin from the idea of analysis by ridge functions $\psi(u_1x_1 + \ldots + u_nx_n)$ whose ridge profiles ψ are wavelets, or alternatively from performing a wavelet analysis in the Radon domain. The paper reviews recent work on the continuous ridgelet transform (CRT), ridgelet frames, ridgelet orthonormal bases, ridgelets and edges and describes a new notion of smoothness naturally attached to this new representation.

Curvelets, Multiresolution Representation, and Scaling Laws

Curvelets provide a new multiresolution representation with several features that set them apart from existing representations such as wavelets, multiwavelets, steerable pyramids, and so on. They are based on an anisotropic notion of scaling. The frame elements exhibit very high direction sensitivity and are highly anisotropic. In this paper these properties and indicate why they may be important for both theory and applications are described.

Image compression with geometrical wavelets

A sparse image representation is introduced that takes advantage of the geometrical regularity of edges in images. A new class of one-dimensional wavelet orthonormal bases, called foveal wavelets, are introduced to detect and reconstruct singularities. Foveal wavelets are extended in two dimensions, to follow the geometry of arbitrary curves. The resulting two dimensional "bandelets " define orthonormal families that can restore close approximations of regular edges with few non-zero coefficients. A double layer image coding algorithm is described. Edges are coded with quantized bandelet coefficients, and a smooth residual image is coded in a standard two-dimensional wavelet basis. 1. GEOMETRICAL COMPRESSION Currently, the most efficient image transform codes are obtained in orthonormal wavelet bases. For a given distortion associated to a quantizer, at high compression rates the bit budget is proportional to the number of non-zero quantized coefficients. For images decomposed in wavelet orthonormal bases, these non-zero coefficients are created by singularities and contours. When the contours are along regular curves, this bit budget can be reduced by taking advantage of this regularity. Many image compressions with edge coding have already been proposed, but they rely on ad-hoc algorithms to represent the edge information, which makes it difficult to compute and optimize the distortion rate. In this paper, "bandelet " orthonormal bases, are constructed that carry all the edge information and take advantage of their regularity by concentrating their energy over few coefficients. An application to image compression is studied. 2. FOVEAL WAVELET BASES Contours are considered here as one-dimensional singularities that move in the image plane.

Sparse Geometric Image Representations with Bandelets

This paper introduces a new class of bases, called bandelet bases, which decompose the image along multiscale vectors that are elongated in the direction of a geometric flow. This geometric flow indicates directions in which the image grey levels have regular variations. The image decomposition in a bandelet basis is implemented with a fast subband filtering algorithm. Bandelet bases lead to optimal approximation rates for geometrically regular images. For image compression and noise removal applications, the geometric flow is optimized with fast algorithms, so that the resulting bandelet basis produces a minimum distortion. Comparisons are made with wavelet image compression and noise removal algorithms.

Footprints and edgeprints for image denoising and compression

In recent years wavelets have been quite successful in compression or denoising applications. To further improve the performance of wavelet based algorithms, we have recently introduced the notion of footprint, which is a data structure which contains all the wavelet coefficients generated by a discontinuity. The combined use of wavelets and footprints leads to very efficient algorithms for compression and denoising of 1-D piecewise smooth signals

Wavelets footprints: Theory, algorithms and applications

Wavelet-based algorithms have been successful in different signal processing tasks. The wavelet transform is a powerful tool because it manages to represent both transient and stationary behaviors of a signal with few transform coefficients. Discontinuities often carry relevant signal information, and therefore, they represent a critical part to analyze. The dependency across scales of the wavelet coefficients generated by discontinuities is studied. Any piecewise smooth signal can be expressed as a sum of a piecewise polynomial signal and a uniformly smooth residual. The notion of footprints, which are scale space vectors that model discontinuities in piecewise polynomial signals exactly. The footprints form an over-complete dictionary and develop efficient and robust algorithms to find the exact representation of a piecewise polynomial function in terms of footprints. This also leads to efficient approximation of piecewise smooth functions.

Characterization of signals from multiscale edges

A multiscale Canny edge detection is equivalent to finding the local maxima of a wavelet transform. The authors study the properties of multiscale edges through the wavelet theory. For pattern recognition, one often needs to discriminate different types of edges. They show that the evolution of wavelet local maxima across scales characterize the local shape of irregular structures. Numerical descriptors of edge types are derived. The completeness of a multiscale edge representation is also studied. The authors describe an algorithm that reconstructs a close approximation of 1-D and 2-D signals from their multiscale edges. For images, the reconstruction errors are below visual sensitivity. As an application, a compact image coding algorithm that selects important edges and compresses the image data by factors over 30 has been implemented

Application of wavelets and neural networks to diagnostic system development, 1, feature extraction

An integrated framework for process monitoring and diagnosis is presented which combines wavelets for feature extraction from dynamic transient signals and an unsupervised neural network for identification of operational states. Multiscale wavelet analysis is used to determine the singularities of transient signals which represent the features characterising the transients. This simultaneously reduces the dimensionality of the data and removes noise components. A modified version of the adaptive resonance theory is developed, which is designated ARTnet and uses wavelet feature extraction as the substitute of the data pre-processing unit. ARTnet is proved to be more effective in dealing with noise contained in the transient signals while retains being an unsupervised and recursive clustering approach. The work is reported in two parts. The first part is focused on feature extraction using wavelets. The second part describes ARTnet and its application to a case study of a refinery fluid catalytic cracking process.

Beamlets and multiscale image analysis

A framework for multiscale image analysis in which *line segments* play a role analogous to the role played by *points* in wavelet analysis is described.

The framework has 5 key components. The *beamlet dictionary* is a dyadically organized collection of line segments, occupying a range of dyadic locations and scales, and occurring at a range of orientations. The *beamlet transform* of an image $f(x, y)$ is the collection of integrals of f over each segment in the beamlet dictionary; the resulting information is stored in a *beamlet pyramid.* The *beamlet graph* is the graph structure with pixel corners as vertices and beamlets as edges; a path through this graph corresponds to a polygon in the original image. By exploiting the first four components of the beamlet framework, we can formulate *beamlet-based algorithms* which are able to identify and extract beamlets and chains of beamlets with special properties.

In this work a four-level hierarchy of beamlet algorithms is described. The first level consists of simple procedures which ignore the structure of the beamlet pyramid and beamlet graph; the second level exploits only the parent-child dependence between scales; the third level incorporates collinearity and co-curvity relationships; and the fourth level allows global optimization over the full space of polygons in an image.

These algorithms can be shown in practice to have suprisingly powerful and apparently unpreceden.ed capabilities, for example in detection of very faint curves in very noisy data.

Platelets: A multiscale approach for recovering edges and surfaces in photon-limited medical imaging

The nonparametric multiscale platelet algorithms presented in this paper, unlike traditional wavelet-based methods, are both well suited to photon-limited medical imaging applications involving Poisson data and capable of better approximating edge contours. This paper introduces platelets, localized functions at various scales, locations, and orientations that produce piecewise linear image approximations, and a new multiscale image decomposition based on these functions. Platelets are well suited for approximating images consisting of smooth regions separated by smooth boundaries. For smoothness measured in certain Holder classes, it is shown that the error of m-term platelet approximations can decay significantly faster than that of m-term approximations in terms of sinusoids, wavelets, or wedgelets. This suggests that platelets may outperform existing techniques for image denoising and reconstruction. Fast, platelet-based, maximum penalized likelihood methods for photon-limited image denoising, deblurring and tomographic reconstruction problems are developed. Because platelet decompositions of Poisson distributed images are tractable and computationally efficient, existing image reconstruction methods based on expectation-maximization type algorithms can be easily enhanced with platelet techniques. Experimental results suggest that platelet-based methods can outperform standard reconstruction methods currently in use in confocal microscopy, image restoration, and emission tomography.

The construction of orthonormal wavelets using symbolic methods and a matrix analytical approach for wavelets on the interval

The construction of orthonormal wavelets using symbolic methods and a matrix analytical approach for wavelets on the interval.

Approximation power of biorthogonal wavelet expansions

This paper looks at the effect of the number of vanishing moments on the approximation power of wavelet expansions. The Strang-Fix conditions imply that the error for an orthogonal wavelet approximation at scale $a=2^{-i}$ globally decays as a^N, where N is the order of the transform. This is why, for a given number of scales, higher order wavelet transforms usually result in better signal approximations. We prove that this result carries over for the general biorthogonal case and that the rate of decay of the error is determined by the orc.er properties of the synthesis scaling function alone. We also derive asymptotic error formulas and show that biorthogonal wavelet transforms are equivalent to their corresponding orthogonal projector as the scale goes to zero. These results strengthen Sweldens earlier analysis and confirm that the approximation power of biorthogonal and (semi-)orthogonal wavelet expansions is essentially the same. Finally, we compare the asymptotic performance of various wavelet transforms and briefly discuss the advantages of splines. We also indicate how the smoothness of the basis functions is beneficial in reducing the approximation error.

Morphological contour coding using structuring functions optimized by genetic algorithms

Shape representation is an important image analysis task which can be used for contour coding and feature extraction. The morphological skeleton is a geometrical shape description by means of maximal inscribed structuring elements. The form of the structuring element is usually chosen a priori, and we show how genetic algorithms can be used for an automatic optimization of an arbitrary shaped structuring element. It permits improved progressive contour transmission and the extraction of shape features.

Singularity detection and processing with wavelets

The mathematical characterization of singularities with Lipschitz exponents is reviewed. Theorems that estimate local Lipschitz exponents of functions from the evolution across scales of their wavelet transform are reviewed. It is then proven that the local maxima of the wavelet transform modulus detect the locations of irregular structures and provide numerical procedures to compute their Lipschitz exponents. The wavelet transform of singularities with fast oscillations has a particular behavior that is studied separately. The local frequency of such oscillations is measured from the wavelet transform modulus maxima. It has been shown numerically that one- and two-dimensional signals can be reconstructed; with a good approximation; from the local maxima of their wavelet transform modulus. As an application, an algorithm is developed that removes white noises from signals by analyzing the evolution of the wavelet transform maxima across scales. In two dimensions, the wavelet transform maxima indicate the location of edges in image

Wavelet Multi-scale Edge Detection for Extraction of Geographic Features *to Improve Vector Map Databases*

Although numerous at smaller geographic scales, vector databases often do not exist at the more detailed, larger scales. A possible solution is the use of image processing techniques to detect edges in high-resolution satellite imagery. Features such as roads and airports are formed from the edges and matched up with similar features in existing low-resolution vector map databases. By replacing the old features with the new more accurate features, the resolution of the existing map database is improved. To accomplish this, a robust edge detection algorithm is needed that will perform well in noisy conditions. This paper studies and tests one such method, the Wavelet Multi-scale Edge Detector. The wavelet transform breaks down a signal into frequency bands at different levels. Noise present at lower scales smoothes out at higher levels. It is demonstrated that this property can be used to detect edges in noisy satellite imagery. Once edges are located, a new method will be proposed for storing these edges geographically so that features can be formed and paired with existing features in a vector map database.

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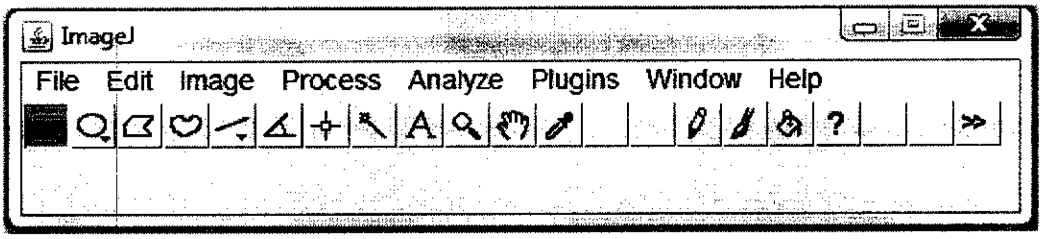
matrix analytical approach for wavelets on the interval," *Exp. Math.*, vol. 10, no. 124, pp. 66–86, 2001.

APPENDIX: SNAPSHOTS

IJ TOOLBAR:

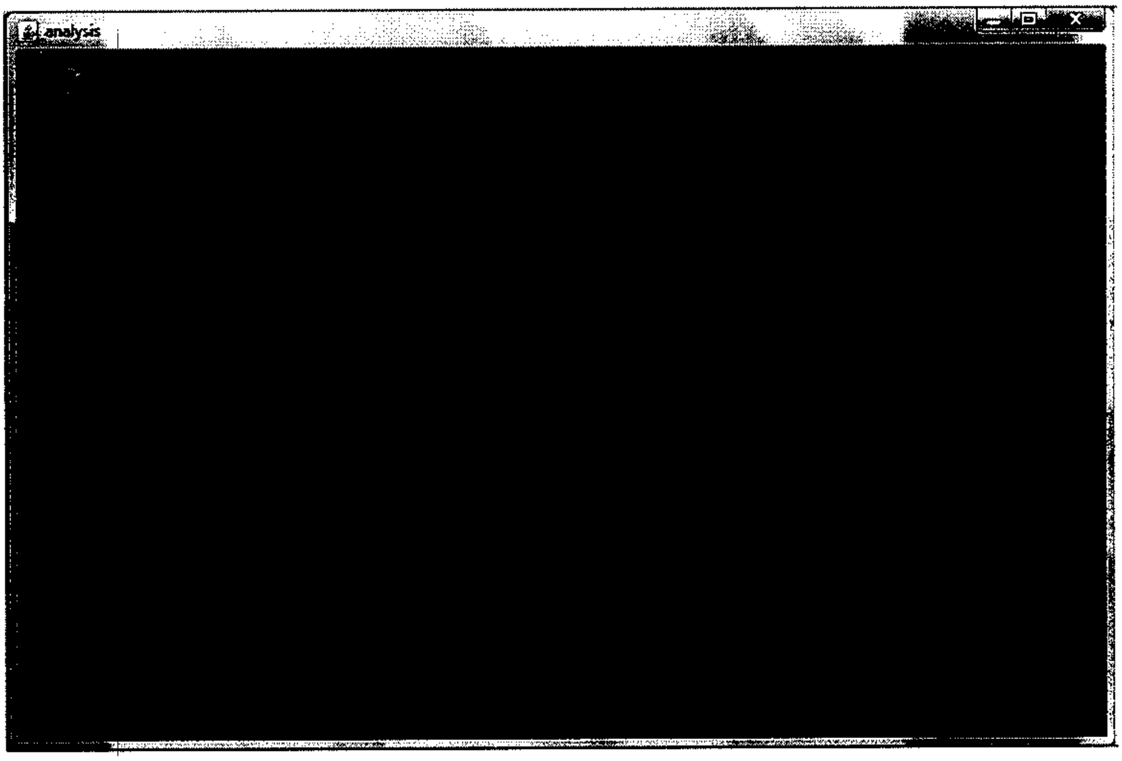


INPUT IMAGE:

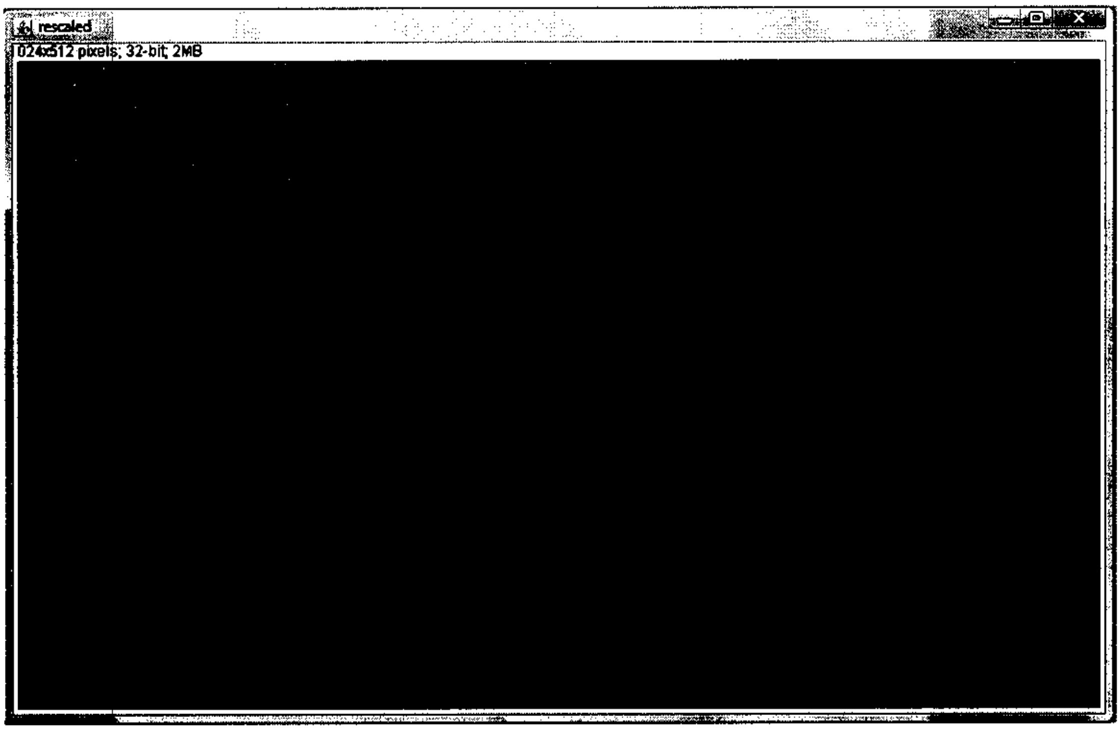
AFTER GRAY SCALE CONVERSION:

CROPPED IMAGE:

IMAGE AFTER COMPRESSION:

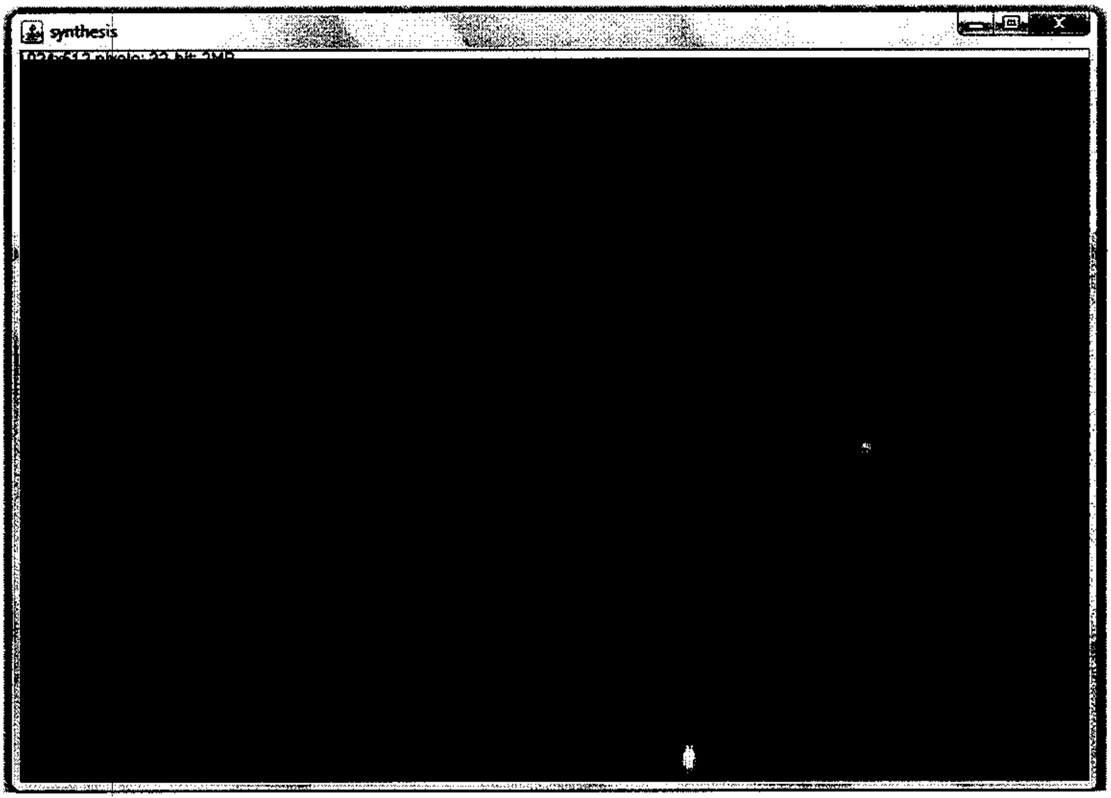


RESCALED IMAGE:



THRESHOLD:

SYNTHESIS:



PERFORMANCE RECORDED: