Register Number:……………..

KUMARAGURU
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B.E/B.TECH DEGREE EXAMINATIONS: NOV/DEC 2022

(Regulation 2018)

Fifth Semester

INFORMATION TECHNOLOGY

U18MAT5101: PARTIAL DIFFERENTIAL EQUATIONS AND TRANSFORMS

COURSE OUTCOMES

CO1:	Form partial differential equations and solve certain types of partial differential equations.
CO2:	Determine the Fourier Series and half range Fourier Series of a function
CO3:	Solve one dimensional wave equation, one dimensional heat equation in steady state using Fourier series
CO4:	Apply Fourier series to solve the steady state two-dimensional heat equation in cartesian coordinates.
CO5:	Identify Fourier transform, Fourier sine and cosine transform of certain functions and use Parseval's identity to evaluate integrals.
CO6:	Evaluate Z – transform of sequences and inverse Z – transform of functions and solve difference equations.

Time: Three Hours **Maximum Marks: 100**

Answer all the Questions:-

PART A (10 x 1 = 10 Marks)

1. Matching type item with multiple choice code (CO 2) [K_1]

List I	**List II**
A). Odd function	i. $f(x) = -x + x^2$
B). Even Function	ii. $f(x) = \sum_{n=1}^{\infty} b_n \sin n x$
C). Neither odd nor even	iii. $f(x) = \cos x$
D). Half Range Sine series	iv. $f(x) = \sin x$

	A	B	C	D
a)	ii	i	iii	iv
b)	iii	iv	ii	i
c)	iv	iii	i	ii
d)	iii	i	ii	iv

2. The number of initial conditions needed to solve one dimensional wave equation is (CO 3) [K_1]

a)	1	b)	2

	c)	3	d)	4		
3.	Which of the following statements are correct? 1. If a periodic function f(x) is even, then its Fourier expansion contains only constant and cosine terms. 2. The value of b_n for $f(x) = \|x\|$in the interval $(-1,1)$ is zero 3. $\tan x$ and $\cot x$ are periodic function with period 2π. 4. The sum of the Fourier series at a point $x = x_0$when the function f(x) has a finite discontinuity is $f(x) = \frac{f(x_0+)+f(x_0-)}{2}$.				CO 2	[K_2]
	a)	1,3	b)	2,3		
	c)	1,2,4	d)	1,2,3		
4.	Which of the following statements are correct? 1. One dimensional heat equation is parabolic. 2. One dimensional wave equation is hyperbolic. 3. Two-dimensional heat equation is elliptic. 4. Laplace equation is hyperbolic.				CO 3	[K_2]
	a)	1,2	b)	2,3		
	c)	1,2,3	d)	1,3,4		
5.	Examine the two statements carefully and select the answer using codes given below: **Assertion (A):** The function $f(x) = e^{\frac{-x^2}{2}}$ is self-reciprocal under Fourier transform. **Reason (R):** If the Fourier transform of $f(x)$ is $F(s)$, then $f(x)$is said to be self-reciprocal.				CO 5	[K_2]
	a)	Both A and R are Individually true and R is the correct explanation of A	b)	Both A and R are Individually true but R is not the correct explanation of A		
	c)	A is true but R is false	d)	A is false but R is true		
6.	The steps involved in finding $F_S\left[xe^{-a^2x^2}\right]$: (1) Find $F_C\left[e^{-a^2x^2}\right]$. (2) Substitute the value of $F_C\left[e^{-a^2x^2}\right]$ in the property (3) Write the property connecting FST and FCT of a function. (4) Differentiate with respect to s to get the desired result.				CO 5	[K_3]
	a)	2-1-3-4	b)	3-2-1-4		
	c)	1-2-3-4	d)	3-1-2-4		
7.	Arrange in a sequence to obtain the general solution of a PDE through method of grouping. 1. Form the subsidiary equations 2. Rewrite the equation in the form$Pp + Qq = R$. 3. Solve to get $u(x,y) = c_1$ and $v(x,y) = c_{2.}$				CO 1	[K_3]

	4. General solution is $\varphi(u,v)=0$.					
	a)	2-1-3-4	b)	1-3-2-4		
	c)	3-4-2-1	d)	4-1-3-2		
8.	The Z-Transform of a^n is				CO 6	[K_2]
	a)	$\frac{z}{z-a}$	b)	$\frac{z}{z-1}$		
	c)	$\frac{z}{z+a}$	d)	$\frac{1}{z-a}$		
9.	Examine the two statements carefully and select the answer using the codes given below: **Assertion (A)**: A Partial differential equation may have a large number of entirely different solutions. **Reason (R)** : A solution of a PDE in a region R is a function of the independent variables whose partial derivatives satisfy the PDE at every point in R.				CO 1	[K_1]
	a)	Both A and R are Individually true and R is the correct explanation of A	b)	Both A and R are Individually true but R is not the correct explanation of A		
	c)	A is true but R is false	d)	A is false but R is true		
10.	The poles of the equation $\frac{z}{(z+2)(z-1)^2}$ are				CO 6	[K_2]
	a)	2,-4	b)	-2,-1		
	c)	-2,1	d)	2,-1		

PART B (10 x 2 = 20 Marks)
(Answer not more than 40 words)

11.	Form a partial differential equation by eliminating the constants a and b from $z=ax^n+by^n$.	CO1	[K_2]
12.	Solve $\sqrt{p}+\sqrt{q}=1$.	CO1	[K_2]
13.	State Dirichlet's conditions for Fourier series.	CO2	[K_1]
14.	Find the R.M.S value of the function $f(x)=x\ in\ (0,l)$.	CO2	[K_2]
15.	Give three possible solutions of the equation $\frac{\partial u}{\partial t}=a^2\frac{\partial^2 u}{\partial x^2}$.	CO3	[K_1]
16.	When the ends of a rod length 20 cm are maintained at the temperature 10° C and 20° C respectively until steady state is prevailed. Determine the steady state temperature of the rod.	CO4	[K_3]
17.	Find the Fourier Transform of $f(x)=\begin{cases}1 in \lvert x\rvert < a\\ 0 in \lvert x\rvert > a\end{cases}$	CO5	[K_3]
18.	Find the Fourier Cosine transform of e^{-x}.	CO5	[K_2]
19.	Find the z-transform of (n + 1) (n + 2).	CO6	[K_2]

20.	Solve $y_{n+1} - 2y_n = 0$given $y_0 = 2$.	CO6	[K_3]

Answer any FIVE Questions:-

PART C (5 x 14 = 70 Marks)

(Answer not more than 350 words)

21.	a)	Solve $(D^2 + 4DD' - 5D'^2)z = 3e^{2x-y} + sin(x - 2y)$.	7	CO1	[K_2]
	b)	Solve $z = px + qy + \sqrt{p^2 + q^2 + 16}$.	7	CO1	[K_3]
22.	a)	Obtain the Fourier series expansion of $f(x) = \begin{cases} 1 + \frac{2x}{\pi}, -\pi \le x \le 0 \\ 1 - \frac{2x}{\pi}, 0 \le x \le \pi \end{cases}$. Deduce $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots = \frac{\pi^2}{8}$.	7	CO2	[K_3]
	b)	Expand f (x) in a Fourier series upto 2nd harmonic using the following table x: 0, 1, 2, 3, 4, 5 $f(x)$: 9, 18, 24, 28, 26, 20	7	CO2	[K_2]
23.		A rod of length $'l'$ has its ends A and B kept at 0^0c and 100^0c respectively until steady state conditions prevail. If the temperature at B is reduced suddenly to 0^0c and kept so, while that of A is maintained, find the temperature u(x, t) at a distance x from A and at time$'t'$.	14	CO4	[K_4]
24.		A string is tightly stretched and its ends are fastened at two points x = 0 and $x = l$. The mid point of the string is displaced transversely through a small distance 'b' and the string is released from rest in that position. Find an expression for the transverse displacement of the string at any time during the subsequent motion.	14	CO3	[K_4]
25.	a)	Find the Fourier sine transform of e^{-ax} and e^{-bx} and hence evaluate $\int_0^\infty \frac{x^2}{(x^2+a^2)(x^2+b^2)} dx$, by using Parseval's identity.	7	CO5	[K_3]
	b)	Find the F.T of $f(x) = \begin{cases} 1 - \lvert x \rvert & if \quad \lvert x \rvert < 1 \\ 0 \quad if & \lvert x \rvert > 1 \end{cases}$ and hence find $\int_0^\infty \frac{sin^4 t}{t^4} dt$.	7	CO5	[K_3]
26.	a)	Find the inverse Z – Transform by using convolution theorem $\frac{8z^2}{(2z-1)(4z-1)}$.	7	CO6	[K_2]
	b)	Solve $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$ given that $y_0 = y_1 = 0$.	7	CO6	[K_4]

Table for 22 b):

x	0	1	2	3	4	5
$f(x)$	9	18	24	28	26	20

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