



**M.E DEGREE EXAMINATIONS: DEC 2022**

(Regulation 2018)

First Semester

**ENVIRONMENTAL ENGINEERING**

P18MAT0001: Statistical Methods for Environmental Engineers

(Statistical Tables Required)

**COURSE OUTCOMES**

- CO1:** Discriminate theory of Statistical estimation.  
**CO2:** Test hypothesis using various tests for small and large samples.  
**CO3:** Gain knowledge in Multiple and partial correlation and regression.  
**CO4:** Analyse experiments based on one-way, two – way and Latin square classifications.  
**CO5:** Analyse multivariate data.

**Time: Three Hours**

**Maximum Marks: 100**

**Answer all the Questions:-**

**PART A (10 x 1 = 10 Marks)**

1. Examine the two statements carefully and select the answer using the codes given below: CO1 [K<sub>4</sub>]  
**Assertion (A):** A consistent Estimator is unbiased in the limit.  
**Reason (R) :** An unbiased estimator is may or may not be a consistent estimator.
- a) Both A and R are Individually true and R is the correct explanation of A      b) A is true but R is false  
c) Both A and R are Individually true but R is not the correct explanation of A      d) A is false but R is true
2. If a sufficient estimator exists, then it is a function of CO1 [K<sub>2</sub>]  
a) Bernoulli's Trial      b) Maximum Likelihood Estimator  
c) Uniform Estimator      d) invariance
3. In a F-distribution, the value of F lies between CO2 [K<sub>2</sub>]  
a)  $(0, \infty)$       b)  $(-\infty, 0)$   
c)  $(-\infty, \infty)$       d)  $(0, \pi)$
4. Probability of rejecting a lot when it is good is called CO2 [K<sub>1</sub>]  
a) Student's risk      b) Producer's risk  
c) Critical Error      d) Consumer's risk

5. The Normal Equations for fitting the straight line  $y = ax + b$  are CO3 [K<sub>1</sub>]

- |  |  |
|--|--|
| a) $\Sigma xy = a\Sigma x^2 + n\Sigma y$<br>$\Sigma xy = a\Sigma x^2 + b\Sigma xy$ | b) $\Sigma x^2 y = a\Sigma x^2 + n\Sigma x$<br>$\Sigma xy^2 = a\Sigma x^4 + b\Sigma x^2$ |
| c) $\Sigma y = a\Sigma x + nb$<br>$\Sigma xy = a\Sigma x^2 + b\Sigma x$            | d) $\Sigma y^2 x = a\Sigma x^2 + n\Sigma x$<br>$\Sigma xy = a\Sigma x^4 + b\Sigma x$     |

6. The equation of the plane of regression of  $X_1$  on  $X_2$  and  $X_3$  is ..... CO3 [K<sub>2</sub>]

- |                                      |                                      |
|--------------------------------------|--------------------------------------|
| a) $X_1 = b_{12.3}X_2 + b_{13.2}X_3$ | b) $X_1 = b_{12.3}X_2 + b_{13.2}X_3$ |
| c) $X_3 = b_{2.3}X_2 + b_{1.2}X_1$   | d) $X_1 = b_{12.3}X_2$               |

7. The correction factor for the below data is ..... CO4 [K<sub>2</sub>]

A	B	C	D
8	6	14	20
9	8	12	22
11	10	18	25
12	4	9	23

- |            |            |
|------------|------------|
| a) 1453.52 | b) 2782.56 |
| c) 4128.69 | d) 1245.33 |

8. The three basic principles of Experimental Designs are CO4 [K<sub>1</sub>]

- |                                   |  |
|-----------------------------------|--|
| a) Replication, Error, Treatments | b) Blocks, fields, Treatments                |
| c) Randomization, fields, yield   | d) Randomization, Replication, Local Control |

9. In principal component analysis, a smaller eigenvalue indicates that CO5 [K<sub>1</sub>]

- |  |  |
|--|--|
| a) A given principal component is more important | b) A given variable in the original data set is more important |
| c) A given principal component is less important | d) A given variable in the original data set is less important |

10. The eigenvalues of a matrix A are 1, 3, 4, then the eigenvalues of  $A^3$  are CO5 [K<sub>5</sub>]

- |           |             |
|-----------|-------------|
| a) 1,3,4  | b) 1,27, 64 |
| c) 1,9,16 | d) 3,9,12   |

**PART B (10 x 2 = 20 Marks)**

11. What are the criteria for the best estimator? CO1 [K<sub>2</sub>]

12. Write any two assumptions for Maximum likelihood estimators. CO1 [K<sub>2</sub>]

13. Write any two assumptions for t-test. CO2 [K<sub>1</sub>]

14. For a 2x2 contingency table with cell frequencies  $a, b, c, d$  CO2 [K<sub>2</sub>]

a	b	a + b
c	d	c + d
a + c	b + d	N

Then write down the value of  $\chi^2$ .

15. Write down the normal equations in fitting the curve  $y = px^2 + qx + r$ . CO3 [K<sub>1</sub>]
16. Obtain the possible values of  $r_{12}$  if  $r_{23} = 0.4, r_{13} = 0.2$  and  $R_{1,2,3} = 0.75$ . CO3 [K<sub>3</sub>]
17. Compare Randomized Block Design with Completely Randomized Design. CO4 [K<sub>4</sub>]
18. Is Latin square Design applicable to 2\*2 square field? Justify your answer. CO4 [K<sub>2</sub>]
19. Write down the characteristic equation and find the eigenvalues of the matrix  $\begin{bmatrix} 1 & 0 \\ 2 & 4 \end{bmatrix}$ . CO5 [K<sub>3</sub>]
20. If X is a normal variate with mean 30 and standard deviation 5. Find  $P(26 \leq X \leq 40)$ . CO5 [K<sub>3</sub>]

**PART C (10 x 5 = 50 Marks)**

21. Obtain the maximum likelihood estimators of  $\lambda$  in the Poisson distribution. CO1 [K<sub>3</sub>]
22. If  $x_1, x_2, \dots, x_n$  is a random sample from a normal population  $N(\mu + 1)$ . Show that  $t = \frac{1}{n} \sum_{i=1}^n x_i^2$ , is an unbiased estimator of  $\mu^2 + 1$ . CO1 [K<sub>4</sub>]
23. A buyer of electric bulbs bought 100 bulbs each of two famous brands. Upon testing these he found that brand A had a mean life of 1500 hours with a standard deviation of 50 hours whereas brand B had a mean life of 1530 hours with a standard deviation of 60 hours. Can it be concluded at 5% level of significance, that the two brands differ significantly in quality? CO2 [K<sub>3</sub>]
24. Ten persons were appointed in the officer cadre in an office. Their performance was noted by giving a test and the marks were recorded out of 100. CO2 [K<sub>4</sub>]

Employee:	A	B	C	D	E	F	G	H	I	J
Before training:	80	76	92	60	70	56	74	56	70	56
After training:	84	70	96	80	70	52	84	72	72	50

By applying t test, can it be concluded that the employees have been benefited by the training?

25. Fit a straight line to the following data. CO3 [K<sub>2</sub>]
- |    |     |     |     |     |     |     |
|----|-----|-----|-----|-----|-----|-----|
| x: | 1   | 2   | 3   | 4   | 5   | 6   |
| y: | 0.5 | 2.5 | 2.0 | 4.0 | 3.5 | 6.0 |
26. From the data relating to the yield of dry bark(X1), height (X2) and girth (X3) for 18 cinchona plants, the following correlation coefficients were obtained:  $r_{12} = 0.77, r_{13} = 0.72, r_{23} = 0.52$ . Find the value of  $R_{1,2,3}$ . CO3 [K<sub>4</sub>]
27. A random sample is selected from each of 3 makes of ropes and their breaking strength are measured with the following results. CO4 [K<sub>3</sub>]

I	70	72	75	80	83		
II	100	110	108	112	113	120	107
III	60	65	57	84	87	73	

Test whether the breaking strength of the ropes differ significantly.

28. The following data represents the number of units of productions per day turned out by different workers using four different types of machines. CO4 [K<sub>4</sub>]

		Machine Type			
		A	B	C	D
	1	44	38	47	36
Workers	2	46	40	52	43
	3	34	36	44	32
	4	43	38	46	33
	5	38	42	49	39

Test whether the 5 men differ with respect to mean productivity and test whether the mean productivity is the same for the four different machine types.

29. Suppose  $\Sigma = \begin{bmatrix} 4 & 1 & 2 \\ 1 & 9 & -3 \\ 2 & -3 & 25 \end{bmatrix}$ . Find the standard deviation matrix and population correlation matrix. CO5 [K<sub>5</sub>]
30. Compute the principal component analysis to the matrix  
 $A = \begin{bmatrix} 1 & 4 \\ 4 & 100 \end{bmatrix}$  CO5 [K<sub>5</sub>]

**Answer any TWO Questions  
PART D (2 x 10 = 20 Marks)**

31. In a tri-variate distribution  $r_{12} = 0.8, r_{23} = -0.56, r_{31} = -0.40$ .  
 Find  $r_{12.3}, r_{13.2}$  and  $r_{23.1}$ . CO3 [K<sub>4</sub>]

32. Two random samples gave the following results: CO2 [K<sub>5</sub>]

Sample	Size	Sample mean	Sum of the square of deviations from the mean
1	10	15	90
2	12	14	108

Examine whether the samples come from the same normal population at 5% level of significance.

33. Analyze the variance in the Latin square of yields (in kgs.) of paddy where P, Q, R, S denote the different methods of cultivation. CO4 [K<sub>4</sub>]

S122 P121 R123 Q122  
 Q124 R123 P122 S125  
 P120 Q119 S120 R121  
 R122 S123 Q121 P122

Examine whether the different methods of cultivation have given significantly different yields.

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