Register Number:……………..

KUMARAGURU
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B.E/B.TECH DEGREE EXAMINATIONS: APRIL / MAY 2023

(Regulation 2018)

Second Semester

COMMON TO ALL BRANCHES EXCEPT AI&DS

U18MAI2201**:** Advanced Calculus and Laplace Transforms

COURSE OUTCOMES

CO1: Evaluate double and triple integrals in Cartesian coordinates and apply them to calculate area and volume.

CO2: Apply various integral theorems for solving engineering problems involving cubes and rectangular parallelepipeds.

CO3: Construct analytic functions of complex variables and transform functions from z-plane to w-plane and vice-versa, using conformal mappings.

CO4: Apply the techniques of complex integration to evaluate real and complex integrals over suitable closed paths or contours.

CO5: Determine solution of linear differential equations using Laplace transform technique.

Time: Three Hours **Maximum Marks: 100**

Answer all the Questions:-

PART A (10 x 2 = 20 Marks)

(Answer not more than 40 words)

1. Change the order of integration in $\int_0^a \int_0^x f(x,y)\,dydx$ CO1 [K_2]

2. Evaluate the triple integral $\int_0^1 \int_0^2 \int_0^e dydxdz$ CO1 [K_2]

3. If $\vec{F} = yz\vec{i} + zx\vec{j} + xy\vec{k}$, then prove that curl $\vec{F} = 0$. CO2 [K_2]

4. State Stoke's theorem. CO2 [K_1]

5. Find the analytic function whose real part is $x^4 - 6x^2y^2 + y^4$. CO3 [K_2]

6. Find the invariant points of the bilinear transformation $2 - \frac{2}{z}$. CO3 [K_2]

7. Expand $f(z) = \frac{1}{z-2}$ at $z = 0$ in a Taylor's series. CO4 [K_2]

8. Find the residue of $f(z) = \frac{z^2}{(z-1)^2}$ at its pole. CO4 [K_2]

9.	Using Laplace transform, find $L\left[e^{-2t}\sin 3t\right]$		CO5	[K2]
10.	Find $L^{-1}\left(\dfrac{1}{s^2+4s+4}\right)$		CO5	[K2]

Answer any FIVE Questions:-
PART B (5 x 16 = 80 Marks)
(Answer not more than 400 words)

11.	a)	Change the order of integration in $\int_0^4 \int_{\frac{x^2}{4}}^{2\sqrt{x}} dydx$ and hence evaluate.	08	CO1	[K2]
	b)	Find the volume of the tetrahedron bounded by the planes $x=0, y=0, z=0$ and $\dfrac{x}{a}+\dfrac{y}{b}+\dfrac{z}{c}=1$.	08	CO1	[K3]
12.	a)	Verify Green's theorem in a plane with respect to $\int_C \left(x^2dx+xydy\right)$, where C is the boundary of the square formed by $x=0,\ x=a,\ y=0,\ y=a,\ (a>0)$.	12	CO2	[K3]
	b)	Obtain the values of the constants a, b, c so that $\vec{F}=(axy+bz^3)\vec{i}+(3x^2-cz)\vec{j}+(3xz^2-y)\vec{k}$ is irrotational.	04	CO2	[K2]
13.		Verify the Gauss divergence theorem for $\vec{F}=4xz\,\vec{i}-y^2\,\vec{j}+yz\,\vec{k}$ over the cube bounded by $x=0,\ x=1,\ y=0,\ y=1,\ z=0,\ z=1$.	16	CO2	[K3]
14.	a)	Prove that an analytic function with constant modulus is constant.	08	CO3	[K2]
	b)	Find the bilinear transformation that maps the points ∞, i, 0 in the z-plane on to the points 0, i, ∞ in the w plane.	08	CO3	[K2]
15.	a)	Find the Laurent's series of $f(z)=\dfrac{1}{(z+1)(z+3)}$ valid in the regions $1<\lvert z\rvert<3$.	08	CO4	[K2]
	b)	Evaluate $\int_C \dfrac{\sin\pi z^2+\cos\pi z^2}{(z-2)(z-3)}dz$, where C is $\lvert z\rvert=4$ by using Cauchy's integral formula.	08	CO4	[K2]
16.	a)	Find $L\left[e^{-t}t\cos t\right]$	06	CO5	[K2]
	b)	Solve $y''+5y'+6y=2,\ y(0)=0\ and\ y'(0)=0$ by using Laplace transform method.	10	CO5	[K3]

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