KUMARAGURU
college of technology
character is life

B.E/B.TECH DEGREE EXAMINATIONS: APRIL / MAY 2023

(Regulation 2018)

Second Semester

ALL BRANCHES EXCEPT AI&DS

U18MAI2201: Advanced Calculus and Laplace Transforms

COURSE OUTCOMES

CO1: Evaluate double and triple integrals in cartesian coordinates and apply them to calculate area and volume.

CO2: Apply various integral theorems for solving engineering problems involving cubes and rectangular parallelepipeds.

CO3: Construct analytic functions of complex variables and transform functions from z-plane to w-plane and vice-versa, using conformal mappings.

CO4: Apply the techniques of complex integration to evaluate real and complex integrals over suitable closed paths or contours.

CO5: Determine solution of linear differential equations using Laplace transform technique.

CO6: Determine multiple integrals, vector differentials, vector integrals and Laplace transforms using MATLAB.

Time: Three Hours   **Maximum Marks: 100**

Answer all the Questions:-
PART A (10 x 2 = 20 Marks)
(Answer not more than 40 words)

1. Evaluate $\int_1^2 \int_0^1 (x^2 + xy)dxdy$ CO1   [K_2]

2. Sketch the region of integration of $\int_0^a \int_{x^2/a}^x \left(\frac{x}{x^2+y^2}\right) dydx$ CO1   [K_2]

3. Determine the normal derivative of $\emptyset = xz + yz + xyz$ at the point $(-1, -1, 0)$ CO2   [K_2]

4. Prove that $\vec{F} = (x + 2y)\vec{i} + (y + 3z)\vec{j} + (x - 2z)\vec{k}$ is a solenoidal vector.   CO2   [K_2]

5. Verify whether the function $w = \bar{z}$ is analytic everywhere.   CO3   [K_2]

6. What are the possible invariant points of the bilinear transformation $w = \frac{2z+6}{z+7}$ CO3   [K_2]

7. Evaluate $\int_C \frac{1}{z-2}dz$ where C is the circle $|z - 2| = 4$ using Cauchy's integral formula.   CO4   [K_2]

8. Expand $f(z) = log(1 + z)$ as a Taylor's series upto second degree terms in the region $|z| < 1$.   CO4   [K_2]

9. State first shifting property of Laplace transform and hence find $L[e^{-2t} \cos 6t]$ CO5   [K_2]

10. State initial value and final value theorem of Laplace transform.   CO5   [K_1]

11. a) Change the order of integration in $\int_0^{4a} \int_{x^2/4a}^{2\sqrt{ax}} dy\,dx$ and hence evaluate. (6)   CO1   [K₅]

   b) Determine the volume of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ (10)   CO1   [K₅]

12.   Verify Gauss divergence theorem for a vector field defined by the vector field (16)   COL   [K₃]

     $\vec{F} = 4xz\vec{i} - y^2\vec{j} + yz\vec{k}$ over the cube bounded by the lines x = 0, x = 1, y = 0, y = 1, z = 0 and z = 1

13. a) Show that the function $u = e^x[x\cos y - y\sin y]$ is a harmonic function. Using (10)   CO3   [K₃] Milne Thomson method, derive the analytic function W such that $W = u + iv$ is analytic if $u = e^x[x\cos y - y\sin y]$.

   b) Find the Bilinear transformation which maps the points $z_1 = 0$; $z_2 = 1$; $z_3 = \infty$ of (6)   CO3   [K₂] the Z-plane into the points $w_1 = i$; $w_2 = 1$; $w_3 = -i$ of the W-plane.

14. a) Evaluate $\int_C \frac{4-3z}{z(z-1)(z-2)} dz$ where C is the circle $|z| = 3/2$ by Cauchy's residue (6)   CO4   [K₃] theorem.

   b) Evaluate $\int_0^{2\pi} \frac{\cos 3\theta}{5-4\cos\theta} d\theta$ using contour integration. (10)   CO4   [K₂]

15. a) Find the Laplace transform of the triangular wave function f(t) defined by (8)   CO5   [K₃]

     $f(t) = \begin{cases} t, & 0 < t < a \\ 2a - t, & a < t < 2a \end{cases}$ with period $2a$

   b) Using Laplace transform, solve the differential equation $y'' - 2y' + y = e^t$ with (8)   CO5   [K₃] $y(0)=2$, $y'(0) = 1$

16. a) Verify Green's theorem in the XY plane where C is the region bounded by the (8)   CO2   [K₃] lines x =0, y=0, x+y=1 for $\int_C [3x^2 - 8y^2]dx + [4y - 6xy]dy$

   b) Using Convolution theorem, find the inverse Laplace (8)   CO5   [K₃]

     transform of $\frac{2}{(s+1)(s^2+4)}$

****