KUMARAGURU
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B.E DEGREE EXAMINATIONS: APRIL / MAY 2023

(Regulation 2018)

Fourth Semester

(COMMON TO AERO/AUTO/CIVIL/EEE/MECH/MCE)

U18MAT4101: Numerical Methods and Probability

COURSE OUTCOMES

CO1: Apply various numerical techniques for solving non-linear equations and systems of linear equations.

CO2: Analyze and apply the knowledge of interpolation and determine the integration and differentiation of the functions by using the numerical data.

CO3: Predict the dynamic behaviour of the system through solution of ordinary differential equations by using numerical methods.

CO4: Solve PDE models representing spatial and temporal variations in physical systems through numerical methods.

CO5: Apply the concepts of probability to random variables.

CO6: Construct probabilistic models for observed phenomena through distributions which play an important role in many engineering applications.

Time: Three Hours   **Maximum Marks: 100**

Answer all the Questions:-

PART A (10 x 2 = 20 Marks)

(Answer not more than 40 words)

1. Find an iterative formula to find the reciprocal of a given number N by Newton's method.	CO1	[K₂]

1. Find an iterative formula to find the reciprocal of a given number N by Newton's method.   CO1   [K_2]

2. Solve the following system of equations by Gauss elimination method
$10x + y + z = 12$, $2x + 10y + z = 13$, $x + y + 5z = 7$.   CO1   [K_2]

3. Obtain the difference table for the following data: CO2   [K_3]

x	-1	1	2	3
f(x)	-21	15	12	3

4. From the following table, which gives the velocity of a body at time t, find the acceleration at t = 1.1.   CO2   [K_3]
t: 1.0   1.1   1.2   1.3   1.4
v: 43.1   47.7   52.1   56.4   60.8

5. Find the values of k_1 and l_1 if $\frac{d^2y}{dx^2} - x\left(\frac{dy}{dx}\right)^2 + y^2 = 0$, $y(0) = 1, y'(0) = 0$ is solved by Runge – Kutta method of 4th order [h = 0.02].   CO3   [K_1]

6. What is single step method and multi-step method. Give examples.   CO3   [K_2]

7. Classify the Laplace equation $u_{xx} + u_{yy} = 0$.   CO4   [K_2]

8. Write down the standard five-point formula to solve $\nabla^2 u = 0$.   CO4   [K_1]

9. Three coins are tossed together. Find the probability that there are exactly 2 heads.   CO5   [K_3]

10. For usual notation with binomial variant X find p if $9 P(X=4)=P(X=2)$ and n=6.   CO6   [K_4]

Answer any FIVE Questions:-
PART B (5 x 16 = 80 Marks)
(Answer not more than 400 words)

11. a) Find a root of $x \log_{10} x - 1.2 = 0$ by False position method correct to 4 decimal places. 8 CO1 [K₃]

    b) Find the dominant eigen value and the corresponding eigen vector of the matrix 8 CO1 [K₂]
    $A = \begin{pmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{pmatrix}$ by using power method.

12. a) Using Lagrange's interpolation formula, find y(10) from the following data 8 CO2 [Kₗ]
    x: 5   6   9   11
    y: 12   13   14   16

    b) By dividing the range into 10 equal parts, find the approximate value of 8 CO2 [Kₗ]
    $\int_0^\pi \sin x \, dx$ by trapezoidal rule and Simpson's 1/3ʳᵈ rule. Verify the answers by actual integration.

13. Given $y' = 1 - y, y(0) = 0$, find 16 CO3 [K₃]

    (i) $y(0.1)$ by Taylor's series method

    (ii) $y(0.2)$ by Euler's method

    (iii) $y(0.3)$ by Improved Euler's method and

    (iv) $y(0.4)$ by Milne's method.

14. a) Solve $u_{xx} + u_{yy} = 0$ correct to 2 decimal places over the square mesh of side 4 8 CO4 [K₄]

    units, satisfying the following boundary conditions:

    $(i) u(0, y) = 0$ for $0 \le y \le 4$
    $(ii) u(4, y) = 12 + y$ for $0 \le y \le 4$
    $(iii) u(x, 0) = 3x$ for $0 \le x \le 4$
    $(iv) u(x, 4) = x^2$ for $0 \le x \le 4$

    b) Using Crank Nicholson's scheme, solve $u_{xx} = 16u_t,$ $0 < x < 1, t > 0$ given 8 CO4 [K₂]

    $u(0, t) = 0$ $u(1, t) = 100t; u(x, 0) = 0$. Compute u for one-time step-in t direction taking h = ¼.

15. a) The mileage in thousands of miles which car owners get with a certain kind of tyre is a random variable having probability density function 8   CO5 [K₃]

$f(x) = \frac{1}{20}e^{-x/20}$, for $x > 0$ and 0 for $x \leq 0$. Find the probabilities that one of these tyres will last (i) atmost 10,000 miles (ii) anywhere from 16,000 to 24,000 miles (iii) at least 30,000 miles.

b) In a test of 2000 electric bulbs it was found that the life of a particular make was normally distributed with average life of 2040 hrs and standard deviation of 60 hrs. Estimate 8   CO6 [K₂]

i) the number of bulbs likely to burn for more than 2150 hrs

ii) between 1920 and 2160 hrs .

16. a) Using Runge-Kutta method of fourth order, solve $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$, given $y(0) = 1$ at 8   CO3 [K₄]

$x = 0.2$.

b) The contents of urns I, II, III are as follows: 1 white, 2 black and 3 red balls; 2 white, 1 black and 1 red balls; 4 white, 5 black and 3 red balls respectively. One urn is chosen at random, and two balls are drawn from it. They happen to be white and red. What is the probability that they come from urns I, II, or III? 8   CO5 [K₃]

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