Register Number:……..

KUMARAGURU
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B.E/B.TECH DEGREE EXAMINATIONS: APRIL / MAY 2023

(Regulation 2018)

Fourth Semester

COMMON TO AERO / CIVIL / EEE / MECH

U18MAT4101: Numerical Methods and Probability

COURSE OUTCOMES

CO1: Apply various numerical techniques for solving non-linear equations and systems of linear equations.

CO2: Analyze and apply the knowledge of interpolation and determine the integration and differentiation of the functions by using the numerical data.

CO3: Predict the dynamic behavior of the system through solution of ordinary differential equations by using numerical methods.

CO4: Solve PDE models representing spatial and temporal variations in physical systems through numerical methods.

CO5: Apply the concepts of probability to random variables.

CO6: Construct probabilistic models for observed phenomena through distributions which play an important role in many engineering applications.

Time: Three Hours **Maximum Marks: 100**

Answer all the Questions:-

PART A (10 x 2 = 20 Marks)

(Answer not more than 40 words)

1. Solve the system of equations $x-2y=0, 2x+y=5$ by Gauss elimination method. CO1 [K_2]

2. Evaluate $\sqrt{15}$ using Newton-Raphson's formula. CO1 [K_1]

3. Using Lagrange interpolation find $y(2)$ from the following data. CO2 [K_1]

x	0	1	3
y	5	6	50

4. Find the first order derivative of $f(x)$ at $x=1$ if CO2 [K_2]

x	1	2	3	4
y	1	8	27	64

5. Solve $\frac{dy}{dx}=x+y$, given $y(0)=1$ for $x=0.2$ by Euler's method. CO3 [K_2]

6. Compare Runge-Kutta methods and Predictor-Corrector methods for solution of initial value problem. CO3 [K_1]

7. Write the diagonal five-point formula to solve the Laplace's equation $u_{xx}+u_{yy}=0$. [K1] CO4

8. Find the value of k to solve $\frac{\partial u}{\partial t} = \frac{1}{2}u_{xx}$ by Bender-Schmidt method with $h = 1$ if h and k are the increments of x and t respectively. CO4 [K2]

9. A random variable X has the following probability function. CO5 [K2]

X=x	0	1	2	3	4	5	6	7	8
P(X=x)	k	3k	5k	7k	9k	11k	13k	15k	17k

Find the value of 'k'.

10. The mean of a Binomial distribution is 20 and SD is 4. Determine the parameters of the distribution. CO5 [K2]

Answer any FIVE Questions:-
PART B (5 x 16 = 80 Marks)
(Answer not more than 400 words)

11. a) Solve the following equations by Gauss-Seidal method (8) CO1 [K3]

$4x + 2y + z = 14, x + 5y - z = 10, x + y + 8z = 20.$

b) Find the numerically largest eigenvalue of $A = \begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix}$ and the corresponding eigenvector. (8) CO1 [K3]

12. a) From the following data, find θ at $x = 43$ and $x = 84$. (8) CO2 [K3]

x	40	50	60	70	80	90
θ	184	204	226	250	276	304

b) Evaluate $\int_0^6 \frac{1}{1+x}dx$ using Trapezoidal rule and Simpson's rule. Check up by direct integration. (8) CO2 [K3]

13. a) Using Taylor's method compute $y(0.2)$ and $y(0.4)$ correct to 4 decimal places given $\frac{dy}{dx} = 1 - 2xy$ and $y(0) = 0$. (6) CO3 [K3]

b) Given $\frac{dy}{dx} = x^3 + y, y(0) = 2.$ Compute $y(0.2), y(0.4), y(0.6)$ by Runge-Kutta method of fourth order. Also find $y(0.8)$ by Milne's predictor-corrector method taking $h = 0.2.$ (10) CO3 [K3]

14. a) By iteration method, solve the Laplace equation $u_{xx} + u_{yy} = 0,$ over the square region, satisfying the boundary conditions. (10) CO4 [K3]

$u(0, y) = 0, \quad 0 \le y \le 3$

$u(3, y) = 9 + y, \quad 0 \le y \le 3$

$u(x, 0) = 3x, \quad 0 \le x \le 3$

$u(x, 3) = 4x, \quad 0 \le x \le 3.$

b) Solve by Crank-Nicholson method the equation $u_{xx} = u_t$ subject to $u(x, 0) = 0,\ u(0, t) = 0$ and $u(1, t) = t,$ for one time steps. (6) CO4 [K3]

15. a) A first step towards identifying spam is to create a list of words that are more likely to appear in spam than in normal messages. For instance, words like buy or the brand name of an enhancement drug are more likely to occur in spam messages than in normal messages. Suppose a specified list of words is available and that your data base of 5000 messages contains 1700 that are spam. Among the spam messages, 1343 contain words in the list. Of the 3300 normal messages, only 297 contain words in the list. Obtain the probability that a message is spam given that the message contains words in the list. (8) CO5 [K3]

b) If a random variable has the probability density $f(x) = \begin{cases} 2e^{-2x} & \text{for } x \succ 0 \\ 0 & \text{for } x \le 0 \end{cases}.$ (8) CO5 [K3]

Find

(i) the probability that the values of x between 1 and 3

(ii) the probability that the values of x greater than 0.5

(iii) Mean and variance.

16. a) It has been claimed that in 60% of all solar-heat installations the utility bill is reduced by at least one-third. Accordingly, Find the probabilities that the utility bill will be reduced by at least one-third in (6) CO6 [K_3]

(i) four of five installations

(ii) at least four of five installations.

b) With an eye toward improving performance, industrial engineers study the ability of scanners to read the bar codes of various food and household products. The maximum reduction in power, occurring just before the scanner cannot read the bar code at a fixed distance, is called the maximum attenuation. This quantity, measured in decibels, varies from product to product. After collecting considerable data, the engineers decided to model the variation in maximum attenuation as a normal distribution with mean 10.1 dB and standard deviation 2.7 dB. (10) CO6 [K_3]

(i) For the next food or product, find the probability that its maximum attenuation is between 8.5 dB and 13.0 dB.

(ii) According to the normal model, find proportion of the products have maximum attenuation between 8.5 dB and 13.0 dB.

(iii) Find proportion of the products have maximum attenuation greater than 15.1 dB.

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