Register Number:……………..

KUMARAGURU college of technology
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B.E DEGREE EXAMINATIONS: APRIL / MAY 2023

(Regulation 2018)

Fourth Semester

ELECTRONICS AND COMMUNICATION ENGINEERING

U18MAT4103**:** Probability and Random Processes

COURSE OUTCOMES

CO1: Analyze random or unpredictable experiments and investigate important features of random experiments and analyse various distributions.
CO2: Construct probabilistic models for observed phenomena through distributions.
CO3: Analyze various random processes with practical applications.
CO4: Analyze correlations related to various random processes and establish the properties of spectral densities.
CO5: Analyze linear time invariant systems performance for random inputs.

Time: Three Hours **Maximum Marks: 100**

Answer all the Questions:-
PART A (10 x 2 = 20 Marks)
(Answer not more than 40 words)

1. Find the power set for $A = \{1, 2, 3\}$. CO1 [K_2]
2 If at least one child in a family of three children is a boy, find the probability that all three are boys. CO1 [K_3]
3. A bus arrives every 20 minutes, at a specified stop, beginning at 6.40 a.m. and continuing until 8.40 a.m. A passenger arrives randomly between 7.00 a.m. and 7.30 a.m. Find the probability that the passenger must wait for more than 5 minutes for a bus. CO2 [K_3]
4. Suppose that during a rainy season on a tropical island, the length of the shower has an exponential distribution, with an average of 2 minutes. If the shower has already lasted for 2 minutes, what is the probability that it will last for at least one more minute? CO2 [K_3]
5. A housewife buys 3 kinds of cereals A, B, and C. She never buys the same cereal in successive weeks. If she buys cereal A, the next week she buys B. However, if she buys B or C, the next week she is 3 times as likely to buy A as the other cereal. Find the transition probability matrix. CO3 [K_4]
6. The message arrives at a telegraph office according to a Poisson process with $\lambda = 3$ / hour. What is the probability that no message arrives between 8 a.m. and 12 p.m.? CO3 [K_3]

7. Two regression lines are given as $8x - 10y + 66 = 0$ and $40x - 18y - 214 = 0$. Find the regression coefficients b_{xy}, b_{yx} and correlation coefficient. CO4 [K4]

8. Find the mean square value of the process whose power density spectrum is $\frac{4}{4+\omega^2}$. CO4 [K4]

9. Define linear time invariant system. CO5 [K1]

10. If $h(t) = \begin{cases} \lambda e^{-\lambda t} & ; \quad t \geq 0 \\ 0 & ; \quad t < 0 \end{cases}$, find $H(\omega)$. CO5 [K3]

Answer any FIVE Questions:-
PART B (5 x 16 = 80 Marks)
(Answer not more than 400 words)

11. a) A problem is given to 3 students $A, B,$ and C whose chances of solving it are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ respectively. What is the probability that 8 CO1 [K3]

(i) The problem is solved.

(ii) Exactly one of them solves the problem.

b) For a certain binary, communication channel, the probability that a transmitted '0' is received is 0.95 and the probability that a transmitted '1' is received as 1 is 0.90. If the probability that a '0' is transmitted s 0.4, find the probability that 8 CO1 [K3]

(i) a '1' is received and

(ii) a '1' was transmitted given that a '1' was received.

12. The joint probability mass function (X, Y) is given by $P(x, y) = k(2x + 3y)$, where $x = 0,1,2; y = 1,2,3$. 16 CO2 [K3]

(i) Find the marginal distributions of X and Y.

(ii) Find all conditional probability distributions.

13. a) Let X be a continuous random variable with a probability density function. 8 CO2 [K3]

$$f(x) = \begin{cases} ax & : \ 0 \leq x \leq 1 \\ a & : \ 1 \leq x \leq 2 \\ -ax + 3a & : \ 2 \leq x \leq 3 \\ 0 & : \ Otherwise \end{cases}$$

(i) Determine the constant 'a'.

(ii) Compute $P(X \leq 1.5)$.

(iii) The cumulative distribution function of X.

b) The mean yield for one-acre plots is 662 kgs with standard deviation 32. Assuming normal distribution, how many one-acre plots in a batch of 1000 plots would you expect to yield? 8 CO2 [K3]

(i) Over 700 kgs

(ii) Below 650 kgs

(iii) What is the lowest yield of the best 100 plots?

14. Consider a Markov chain with state space $S = \{0,1,2\}$ and one-step transition probability matrix is given by $P = \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & 1 & 0 \end{bmatrix}$. 16 CO3 [K4]

(i) Is the chain Ergodic? Explain

(ii) Find the invariant probabilities.

15. a) Find the correlation coefficient and the equations of the regression lines for the following data: 8 CO4 [K3]

X:	1	2	3	4	5
Y:	2	5	3	8	7

b) Find the power spectral density of Binary Transmission process, where auto correlation function is $R(\tau) = \begin{cases} 1 - \frac{|\tau|}{T} & ; \ |\tau| \leq T \\ 0 & ; \ Otherwise \end{cases}$ 8 CO4 [K4]

16. a) Show that power spectral densities of the input and output processes in the system are connected by the relation $S_{YY}(\omega) = |H(\omega)|^2 S_{XX}(\omega)$ where $H(\omega)$ is the Fourier transform of a unit impulse response function h(t). 8 CO5 [K4]

b) L et $X(t)$ be the input voltage to a circuit and $Y(t)$ be the output voltage. $\{X(t)\}$ is a stationary random process with $\mu_x = 0$ and $R_{xx}(\tau) = e^{-\alpha|\tau|}$. Find μ_y and $S_{yy}(\omega)$ if the power transfer function is $H(\omega) = \frac{R}{R+i\omega}$. 8 CO5 [K4]

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