



**B.E/B.TECH DEGREE EXAMINATIONS: APRIL /MAY 2024**

(Regulation 2018)

Sixth Semester

**AERONAUTICAL ENGINEERING**

U18AET6002: Finite Element Method

**COURSE OUTCOMES**

- CO1: Identify the mathematical model for simple and complex engineering problems using FEM approach.  
 CO2: Calculate stress, strain, and displacement value of simple 1-D problems.  
 CO3: Solve complex axisymmetric problems under various boundary conditions.  
 CO4: Apply finite element concept to Isoperimetric Element.  
 CO5: Analyze heat transfer and torsional problems.

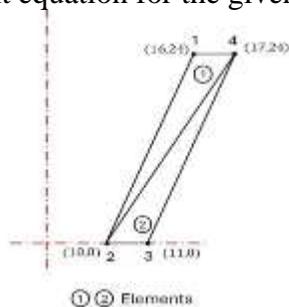
**Time: Three Hours**

**Maximum Marks: 100**

**Answer all the Questions:- PART A (10 x 2 = 20 Marks)**

**(Answer not more than 40 words)**

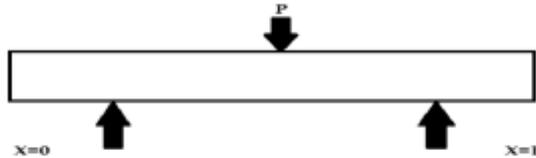
- Distinguish between the stiffness and force methodologies utilized in the Direct Equilibrium Method's analytical calculations. CO1 [K<sub>2</sub>]
- During discretization, mention the places where it is necessary to place a node? CO1 [K<sub>2</sub>]
- Is it possible to derive the finite element based analytical formulation without the support of transformation matrix? Briefly justify your answer. CO2 [K<sub>2</sub>]
- Why the stiffness matrix of the beam element is breaks its conventional properties? Briefly explain. CO2 [K<sub>2</sub>]
- Describe the shape function and express the formula for a constant strain triangular element under plane stress conditions. CO3 [K<sub>2</sub>]
- Write down the global finite element equation for the given axisymmetric elements. CO3 [K<sub>2</sub>]



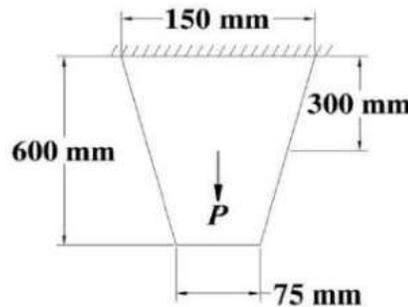
- Compare and contrast the isoparametric, super-parametric, and sub-parametric elements. CO4 [K<sub>2</sub>]
- Evaluate the integral  $I = \int_{-1}^{+1} [x^2 + \cos\left(\frac{x}{2}\right)] dx$  using 3-point Gaussian quadrature. CO4 [K<sub>2</sub>]
- List the three distinct processes that go into the finite element calculations for typical one-dimensional heat transfer problems. CO5 [K<sub>2</sub>]
- When solving heat transfer issues using the finite element method (FEM), what are the necessary initial and boundary conditions? CO5 [K<sub>2</sub>]

**Answer any FIVE Questions:- PART B (5 x 4 = 20 Marks)**  
**(Answer not more than 80 words)**

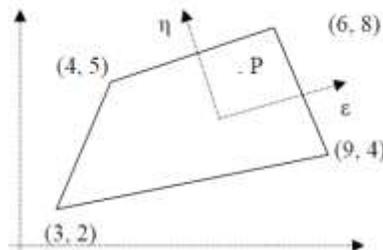
11. The Following Differential Equation is available for a physical phenomenon  $\frac{d^2y}{dx^2} - 10x^2 = 5$  Where “x” varies from 0 to 1. Boundary Conditions are:  $y(0) = 0, y(1) = 0$ . By using the two types of Weighted Residual Methods such as Least Squares Method and Galerkin Method, find an approximate solution for the above differential equation and also compare with exact solution. Take the trial function as  $y = a_1(x^2 - x)$ . CO1 [K<sub>3</sub>]
12. Mr. Karthi has decided to calculate the deflection of the beam element below using two distinct methods: Galerkin and Direct Equilibrium. Kindly provide a comparison and contrast of the principles, analytical methodologies, and outcomes of these two methods. CO1 [K<sub>3</sub>]



13. Consider a taper steel plate of uniform thickness,  $t = 20$  mm as shown in below Fig. The Young’s modulus of the plate, material is  $E = 200$  GPa and weight density  $\rho = 0.78 \times 10^{-4}$  N/mm<sup>3</sup>. In addition to its self weight, the plate is subjected to a point load  $P = 250$  N at its mid point. Evaluate the Global force vector  $\{F\}$  by modeling the plate with two finite elements. CO2 [K<sub>3</sub>]



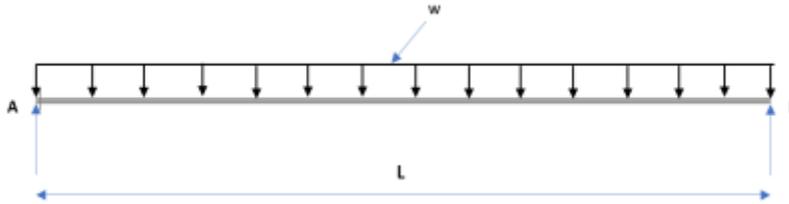
14. Compare and contrast the plane stress/plane strain concepts with axisymmetric approach. CO3 [K<sub>3</sub>]
15. A long hollow cylinder with inner diameter of 100 mm and outer diameter of 140 mm is subjected to external pressure of 40 N/mm<sup>2</sup>. By using 2 element discretization and axisymmetric conversion concept, determine the stress-strain relationship matrix where  $E=200$ GPa,  $\mu=0.3$ . CO3 [K<sub>4</sub>]
16. Evaluate the Cartesian coordinate of the point ‘P’ which has the local coordinate of (0.6,0.8) as shown in figure. CO4 [K<sub>4</sub>]



**Answer any FIVE Questions:- PART C (5 x 12 = 60 Marks)**  
**(Answer not more than 300 words)**

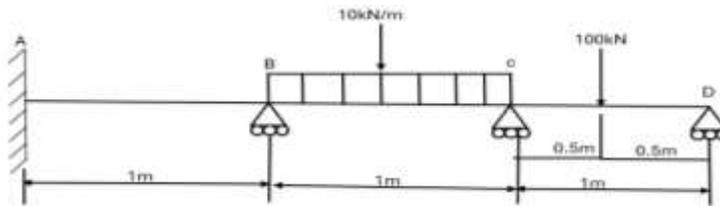
17. As per the below figure, a simply supported beam subjected to uniformly distributed load over entire span. Determine the bending moment and deflection at mid span by Rayleigh Ritz method. For this problem, use the trial function as,

$$y = a_1 \sin\left(\frac{\pi x}{l}\right) + a_2 \sin\left(\frac{3\pi x}{l}\right).$$



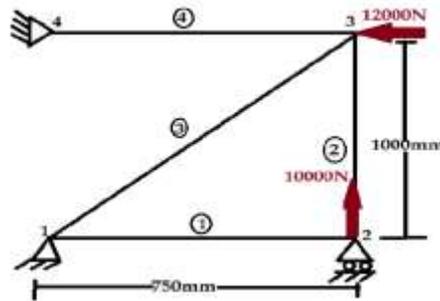
16 CO1 [K4]

18. For a Beam and loading shown in figure, calculate the rotation at B, D and C. Take  $E=210\text{GPa}$ ,  $I=6 \times 10^{-6}\text{m}^4$ .



16 CO2 [K4]

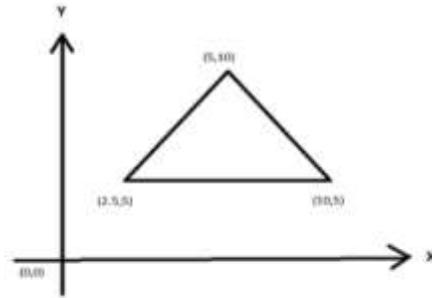
19. Consider a four-bar truss as shown in figure. It is given that  $E=200\text{GPa}$ ,  $A=625\text{mm}^2$  for all the elements. Determine the element stiffness matrix for each element, assemble global stiffness matrix for truss, and solve nodal displacements.



16 CO2 [K4]

20. The 2D plane strain element is shown in below figure and the coordinates of the same CST elements are in mm with the values of (5, 10), (2.5, 5), (10, 5) respectively. The nodal displacement are  $u_1=0.0025\text{mm}$ ,  $u_2=0.05\text{mm}$ ,  $u_3=0.05\text{mm}$ ,  $v_1=0.025\text{mm}$ ,  $v_2 = 0.025\text{mm}$ ,  $v_3 = 0.02 \text{ mm}$ . Calculate the elemental stress, elemental strain, principle stresses and principle angle. Assume the element is in plane strain condition and also take  $\mu=0.3$  and  $E=70 \times 10^3 \text{ N/mm}^2$ .

16 CO3 [K4]



21. For four noded rectangular element shown in the figure determine the following: a) Jacobian Matrix; b) Strain displacement matrix; c) Elemental stress; d) Elemental

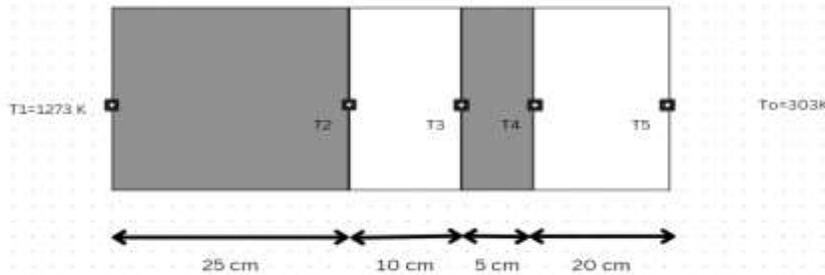
strain; Take  $E = 200\text{GPa}$ ,  $\mu = 0.25$ ;  $u = \begin{bmatrix} 0 \\ 0 \\ 0.003 \\ 0.004 \\ 0.006 \\ 0.004 \\ 0 \\ 0 \end{bmatrix}$ ;  $\varepsilon = 0$ , and  $\eta = 0$ . Assume plane

stress condition and all the dimensions mentioned in the figure are in mm.

16 CO4 [K<sub>4</sub>]



22. A furnace wall is made up of four layers (refer figure), in which the inside layer with the temperature of 1273 K and thermal conductivity of 8.5 W/mK. The first middle layer with thermal conductivity of 0.08 W/mK, the second middle layer with thermal conductivity of 0.25 W/mK, the outer layer with thermal conductivity of 0.30 W/mK. The respective layers thicknesses are 25cm, 10cm, 5cm, and 20cm. The outside of the wall is exposed to atmospheric air at 303 K with the heat transfer co-efficient of 45 W/m<sup>2</sup>K. Determine the nodal Temperatures.



\*\*\*\*\*

16 CO5 [K<sub>4</sub>]