Register Number:……………..

B.E/B.TECH DEGREE EXAMINATIONS: APRIL /MAY 2024

(Regulation 2018)

Fourth Semester

ELECTRONICS AND COMMUNICATION ENGINEERING

U18MAT4103**:** Probability and Random Processes

COURSE OUTCOMES

CO1: Analyze random or unpredictable experiments and investigate important features of random experiments and analyze various distributions.

CO2: Construct probabilistic models for observed phenomena through distributions.

CO3: Analyze various random processes with practical applications.

CO4: Analyze correlation related to various random processes and establish the properties of spectral densities.

CO5: Analyze linear time invariant systems performance for random inputs.

Time: Three Hours **Maximum Marks: 100**

Answer all the Questions:-

PART A (10 x 2 = 20 Marks)

(Answer not more than 40 words)

1.	In a shooting test, the probability of hitting the target is 1/2 for A, 2/3 for B and 3/4 for C. If all of them fire at the target, find the probability that none of them hit the target and at least one of them hits the target.	CO1	[K_2]
2.	State Bayes theorem.	CO1	[K_1]
3.	Find the probability density function of a continuous random variable X when its cumulative distribution is given by $F(x)=\begin{cases} 0, & x<0 \\ 1-e^{-x/5}, & 0\le x<\infty. \end{cases}$	CO2	[K_2]
4.	If X follows the binomial distribution with mean and variance 4 and 4/3 respectively then find the probability mass function of X.	CO2	[K_2]
5.	Define SSS process.	CO3	[K_2]
6.	State any two properties of Poisson process.	CO3	[K_1]
7.	If the autocorrelation function of a stationary random process is $R(\tau)=16+\dfrac{9}{1+16\tau^2}$, then find the mean and variance of the process.	CO4	[K_2]

8. State Wiener-Khinchine theorem. CO4 [K_1]

9. Find the system transfer function, if a linear time invariant system has an impulse function CO5 [K_2]

$$H(t)=\begin{cases}\frac{1}{2c}, & |\tau|\le c\\ 0, & |\tau|\ge c.\end{cases}$$

10. If E_b, the energy bit of a binary digital signal is 10^{-5} Ws and the one-sided power spectral density of the white noise $N_0 = 10^{-6}$ W/Hz then find the output signal-to-noise(SNR) ratio of the matched filter. CO5 [K_2]

Answer any FIVE Questions:-
PART B (5 x 16 = 80 Marks)
(Answer not more than 400 words)

11. a) A college in a city shows the following analysis of its 200 teachers. 8 CO1 [K_3]

Age	Master's Degree	Ph.D Degree	Total
Under 30	90	10	100
30-40	20	30	50
Over 40	40	10	50
Total	150	50	200

If one teacher is selected at random from the college, find

(i) The probability that he has only a Master's degree

(ii) The probability that he has a Ph.D degree, given that he is over 40

(iii)The probability that he is under 30, given that he has only a Master's degree

b) A bag contains 5 balls and it is not known how many of them are white. Two balls are drawn at random from the bag and they are noted to be white. What is the chance that all the balls in the bag are white? 8 CO1 [K_3]

12. a) A random variable X has the following probability function: 8 CO2 [K2]

X	0	1	2	3	4	5	6	7
p(X)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

(i) Find the value of k,

(ii) Evaluate $P(X<6)$, $P(0<X<5)$

(iii) Find $P(1.5X<4.5/X>2)$.

b) It is known that the probability of an item produced by a certain machine will be defective is 0.05. If the produced items are sent to the market in packets of 20, find the number of packets containing at least, exactly and at most 2 defective items in a consignment of 1000 packets using Poisson distribution. 8 CO2 [K3]

13. a) Show that the random process $X(t)=A\cos(\omega_0 t+\theta)$, is wide-sense stationary, if A and ω_0 are constants and θ is a uniformly distributed random variable in $(0,2\pi)$. 8 CO3 [K2]

b) A man either drives a car or catches a train to go to office each day. He never goes 2 days in a row by train but if he drives one day, then the next day he is just as likely to drive again as he is to travel by train. Now suppose that on the first day of the week, the man tossed a fair die and drove to work if and only if a 6 appeared. Find (i) the probability that he takes a train on the third day, and (ii) the probability that he drives to work in the long run. 8 CO3 [K3]

14. a) The lifetime of a certain brand of an electric bulb may be considered a random variable with mean 1200 h and standard deviation 250 h. Find the probability, using central limit theorem, that the average lifetime of 60 bulbs exceeds 1250h. 8 CO4 [K3]

b) If the power spectral density of a wide sense stationary process is given by 8 CO4 [K3]

$$S(\omega)=\begin{cases}\frac{b}{a}(a-|\omega|), & |\omega|\le a\\ 0, & |\omega|>a\end{cases}$$

find the autocorrelation function of the process.

15.	a)	A wide sense stationary process $X(t)$ with $R_{xx}(\tau) = Ae^{-a\|\tau\|}$ where A and a are real positive constants is applied to the input of an linear time invariant system with $h(t) = e^{-bt}u(t)$ where b is a real positive constant, find the power spectral density of the output of the system.	8	CO5	[K3]
	b)	If a random process $X(t)$ is given as input to system with transfer function $H(\omega) = 1$ for $-\omega_0 < \omega < \omega_0$ and its autocorrelation function is $\frac{N_0}{2}\delta(t)$, then find the autocorrelation function of the output process.	8	CO5	[K3]
16.	a)	If the joint probability mass function of (X,Y) is given by $p(x,y) = k(2x+3y), x = 0,1,2;\ y = 1,2,3,$ then find the marginal probability distribution of X and Y.	8	CO2	[K2]
	b)	Obtain the equations of the lines of regression from the following data:	8	CO4	[K2]

X :	1	2	3	4	5	6	7
Y:	9	8	10	12	11	13	14

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