Register Number:……………..

KUMARAGURU
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B.E/B.TECH DEGREE EXAMINATIONS: NOV/DEC 2024

(Regulation 2018)

Second Semester

ARTIFICIAL INTELLIGENCE AND DATA SCIENCE

U18MAI2203**:** Probability and Statistics

COURSE OUTCOMES

CO1: Understand and apply the concept of probability and random variables and predict probabilities of events in models following normal distribution.

CO2: Apply the concepts of two dimensional random variables, central limit theorem and estimation, which lay the foundation for Machine Learning and Data Science.

CO3: Perform hypothesis testing and interpret the results which will form the basis for Data Analysis.

CO4: Understand the principles of design of experiments and perform analysis of variance which will help in Data Analysis.

CO5: Learn and apply multivariate analysis necessary for Principal Component Analysis.

Time: Three Hours **Maximum Marks: 100**

Answer all the Questions:-

PART A (10 x 2 = 20 Marks)

(Answer not more than 40 words)

1. If the discrete random variable X has the probability function given by the table CO1 [K_2]

x	1	2	3	4
p(x)	k/3	k/6	k/3	k/6

then find the value of k.

2. State the formula for finding mean and variance of a binomial random variable X. CO1 [K_1]

3. If b_{xy}= 0.45, b_{yx}= 0.8, find the correlation coefficient and also state its nature. CO2 [K_L]

4. State Central limit theorem. CO2 [K_1]

5. Define Type I error and Type II error. CO3 [K_1]

6. Give two applications of χ^2-test. CO3 [K_1]

7. Define Mean square. CO4 [K_1]

8. List any two differences between completely randomized design and randomized block design. CO4 [K_1]

9. Find the expected value of a random vector $X = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$ where X_1 and X_2 have CO5 [K2]

X_1	-1	0	1
$P(X_1)$	0.3	0.3	0.4

and

X_2	0	1
$P(X_2)$	0.8	0.2

10. Write the formula of a covariance matrix and population correlation matrix in principal component analysis. CO5 [K1]

Answer any FIVE Questions:-
PART B (5 x 16 = 80 Marks)
(Answer not more than 400 words)

11. a) A bolt is manufactured by 3 machines A, B and C. A turns out twice as many items as B, and machines B and C produce equal number of items. 2% of bolts produced by A and B are defective and 4% of bolts produced by C are defective. All bolts are put into 1 stock pile and 1 is chosen from this pile. What is the probability that it is defective? 8 CO1 [K3]

b) A random variable X has the following probability function: 8 CO1 [K2]

X	0	1	2	3	4	5	6	7
P(X)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

(i)Find k.

(ii)Evaluate $\mathbf{P[X < 6]}, P[X \geq \mathbf{6}]$.

(iii)Find the minimum value of c for $\mathbf{P[X \leq c]} > \frac{1}{2}$.

12. a) The joint probability function (x,y) is given by P(x,y) = K(2x+3y), x=0,1,2; y=1,2,3 8 CO2 [K3]

i) Find the marginal distribution of x

ii) Find the marginal distribution of y

b) Compute the mean value of X and the correlation coefficient when the regression equation of X on Y is $3y - 5x + 108 = 0$, the mean of Y is 44, and the variance of X is 9/16th of the variance of Y. 8 CO2 [K3]

13. a) The mean breaking strength of the cables supplied by a manufacturer is 1800, with a standard deviation of 100. By a new technique in the manufacturing process, it is claimed that the breaking strength of the cable has increased. To test this claim, a sample of 50 cables is tested and it is found that the mean breaking strength is 1850. Can we support the claim at 1% level of significance? 8 CO3 [K3]

b) The following data give the number of aircraft accidents that occurred during the various days of a week. 8 CO3 [K3]

Day	Mon	Tue	Wed	Thur	Fri	Sat
No. of accidents	15	19	13	12	16	15

Test whether the accidents are uniformly distributed over the week.

14. Three varieties of a crop are tested in a randomized block design with four replications, the layout being as given below: The yields are given in kilograms. Analyze for significance. 16 CO4 [K3]

C 48	A 51	B 52	A 49
A 47	B 49	C 52	C 51
B 49	C 53	A 49	B 50

15. a) If X_1 and X_2 have the joint probability mass function $p(x_1, x_2) = \frac{x_1 + x_2}{18}$, $x_1 = 1,2$ and $x_2 = 1,2$ then find 8 CO5 [K3]

(i) Marginal probability mass functions of x_1 and x_2

(ii) Mean vector

(iii) Variance-covariance matrix.

b) Compute the principal components to the variance covariance matrix $\Sigma = \begin{bmatrix} 1 & 4 \\ 4 & 100 \end{bmatrix}$. 8 CO5 [$K_3$]

16. a) The number of monthly breakdown of a computer is a random variable having a Poisson distribution with mean equal to 1.8. Find the probability that this computer will function for a month (i) without a breakdown (ii) with only one breakdown (iii) with at least one break down. 8 CO1 [K_3]

b) A completely randomized design experiment with 10 plots and 3 treatments gave the following results: 8 CO4 [K_3]

Plot No.	1	2	3	4	5	6	7	8	9	10
Treatment	A	B	C	A	C	C	A	B	A	B
Yield	5	4	3	7	5	1	3	4	1	7

Analyze the results for treatment effects.

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