Register Number:……………..

KUMARAGURU
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B.E DEGREE EXAMINATIONS: NOV/DEC 2024

(Regulation 2018)

Fourth Semester

ELECTRONICS AND COMMUNICATION ENGINEERING

U18MAT4103 : Probability and Random Processes

COURSE OUTCOMES

CO1: Analyze random or unpredictable experiments and investigate important features of random experiments and analyses various distributions.
CO2: Construct probabilistic models for observed phenomena through distributions.
CO3: Analyze various random processes with practical applications.
CO4: Analyze correlation related to various random processes and establish the properties of spectral densities.
CO5: Analyze linear time invariant systems performance for random inputs.

Time: Three Hours **Maximum Marks: 100**

Answer all the Questions:-

PART A (10 x 2 = 20 Marks)

(Answer not more than 40 words)

1.	Among 40 CSE students interviewed in a class for a job, 25 knew Java programming, 28 knew Oracle and 7 did not know any one of the languages. Find out how many knew both languages.	CO1	[K_2]
2.	If the probability that a communication system has high selectivity is 0.54 and the probability that it will have high fidelity is 0.81 and the probability that it will have high fidelity and high selectivity is 0.18, what is the probability that a system with high fidelity will also have high selectivity?	CO1	[K_3]
3.	If a random variable X takes the value 1, 2, 3, 4 such that $2P(X=1)=3P(X=2)=P(X=3)=5P(X=4)$. Find the probability distribution of X.	CO2	[K_2]
4.	The time (in hours) required to repair a machine is exponentially distributed with parameter λ= ½. What is the conditional probability that the repair time takes atleast 10 hours given that its duration exceeds 9 hours?	CO2	[K_3]
5.	Define WSS process.	CO3	[K_3]
6.	State any two properties of Gaussian process.	CO3	[K_2]
7.	Find mean and variance of a stationary random process whose auto correlation function is given by $R_{XX}(\tau)=18+\frac{2}{6+\tau^2}$.	CO4	[K_3]
8.	State Wiener - Khinchine relation.	CO4	[K_2]
9.	Define Linear time invariant system.	CO5	[K_1]
10.	Define White noise.	CO5	[K_1]

Answer any FIVE Questions:-

PART B (5 x 16 = 80 Marks)

(Answer not more than 400 words)

11. a) The contents of urns I, II, III are as follows: 1 white, 2 black and 3 red balls; 2 white, 1 black and 1 red balls; 4 white, 5 black and 3 red balls respectively. One urn is chosen at random and two balls are drawn from it. They happen to be white and red. What is the probability that they come from urn I. 8 CO1 [K_3]

b) From a well shuffled deck of 52 playing cards, 4 cards are selected at random. Find the probability that the selected cards are 8 CO1 [K_3]

- i) 3 spades and 1 heart
- ii) 2 kings, 1 ace and 1 queen
- iii) All are diamonds
- iv) There is one card of each suit.
- v) All the four are hearts and one of them is a jack.
- vi) A spade, a heart, a diamond and a club in that order.

12. a) A discrete random variable X has the probability function given below: 10 CO2 [K_2]

X:	0	1	2	3	4	5	6	7
$P(X = x)$:	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$

Find (i) the value of k (ii) $P(X < 6),\ P(X \geq 6),\ P(0 < X < 4)$

iii) the distribution function of X. (iv) the minimum value of 'a' such that $P(X \leq a) > \frac{1}{2}$

b) The average number of traffic accidents on a certain section of a highway is two per week. Assume that the number of accidents follows a Poisson distribution. Find the probability of (i) no accident in a week (ii) atmost two accidents in a 2 week period. 6 CO2 [K_3]

13. a) The probability distribution of the process $\{X(t)\}$ is given by 8 CO3 [K_3]

$$P(X(t) = n) = \begin{cases} \frac{(at)^{n-1}}{(1+at)^{n+1}}, n = 1,2,\ldots\ldots \\ \frac{at}{1+at}, n = 0 \end{cases}.$$

Show that $\{X(t)\}$ is not stationary.

b) Let $\{X_n; n = 1,2,3,\ldots.\}$ be a Markov chain with state space S = {1, 2, 3} with one step transition probability matrix. 8 CO3 [K_4]

$$P = \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 1 & 0 & 0 \end{bmatrix}$$

- i) Sketch the transition diagram.
- ii) Is the chain irreducible? Explain.
- iii) Is the chain Ergodic? Explain.

14.	a)	Find the power spectral density of Binary Transmission process, whose auto correlation function is given by $\mathrm{R}(\tau) = \begin{cases} 1 - \frac{\lvert\tau\rvert}{T}, & \lvert\tau\rvert \leq T \\ 0, & otherwise \end{cases}$.	8	CO4	[K$_4$]
	b)	The cross-power spectrum of a real random processes X(t) and Y(t) is given by $S_{XY}(\omega) = \begin{cases} a + jb\omega; & \lvert\omega\rvert < 1 \\ 0; & elsewhere \end{cases}$. Find the cross-correlation function.	8	CO4	[K$_3$]
15.	a)	The input to the RC filter is a White noise process with auto correlation function $R_{XX}(\tau) = \frac{N_0}{2}\delta(\tau)$. If the frequency response $H(\omega) = \frac{1}{1+j\omega RC}$, find the auto correlation and the mean square value of the output process Y(t).	8	CO5	[K$_3$]
	b)	If $\{N(t)\}$ is a band limited white noise such that $S_{NN}(\omega) = \begin{cases} \frac{N_o}{2}, & \lvert\omega\rvert < W_B \\ 0, & elsewhere \end{cases}$, find the auto correlation function.	8	CO5	[K$_2$]
16.	a)	The joint p.d.f of the two dimensional random variable (X, Y) is given by $f(x,y) = \begin{cases} \frac{8xy}{9} : & 1 \leq x \leq y \leq 2 \\ 0 : & otherwise \end{cases}$ Find (i) Marginal densities of X and Y. (ii) The conditional density functions f (x / y) and f (y / x).	8	CO2	[K$_2$]
	b)	Suppose that customers arrive at a bank according to a Poisson process with a mean rate of 3 per minute. Find the probability that during a time interval of 2 minutes. i. Exactly 4 customers arrive ii. Less than 4 customers arrive iii More than 4 customers arrive	8	CO3	[K$_3$]

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