



**M.TECH DEGREE EXAMINATIONS: NOV/ DEC 2024**

(Regulation 2024)

First Semester

**DEFENCE TECHNOLOGY**

24DTT501: Advanced Engineering Mathematics

**COURSE OUTCOMES**

- CO1:** Know the methods for solving differential equations, generating functions.
- CO2:** Understand basic concepts of Fourier Transform, Laplace Transforms and solve problems with periodic functions, step functions, impulse functions and convolution.
- CO4:** Understand the utilization of mathematical methods for solving problems having relevance to defence applications.

**Time: Three Hours**

**Maximum Marks: 100**

**PART A (4\*20 = 80 Marks)**

1. a) Scenario: A rocket engine's thrust response is modeled by the transfer function:  $F(s) = \frac{20}{s(s+3)(s+5)}$  Find the inverse Laplace transform to determine the time-domain thrust response. 10 CO2 [K<sub>2</sub>]
  
- b) Scenario: The vibration of an aircraft component (e.g., wing) after it experiences an aerodynamic force is described by:  $\frac{d^2y(t)}{dt^2} + 4\frac{dy(t)}{dt} + 13y(t) = f(t)$  where  $y(t)$  is the displacement, and  $f(t)=0$  for the free vibration (unforced) case. Solve for  $y(t)$ , the displacement of the component, using the Laplace transform and given initial conditions  $y(0)=2$  and  $\frac{dy(0)}{dt} = 0$ . 10 CO4 [K<sub>4</sub>]
  
2. a) A military bunker experiences an explosive shock wave, modeled by the equation:  $M\frac{d^2u}{dt^2} + C\frac{du}{dt} + Ku = P_0e^{-\alpha t}$  where  $M=1000$  kg,  $C = 50N\cdot s/m$ ,  $K = 2000N/m$ ,  $P_0 = 5000N$ , and  $\alpha = 0.2$ . Find the displacement  $u(t)$  of the bunker over time. 10 CO4 [K<sub>4</sub>]

- b) A targeting system for a military vehicle uses feedback control, described by: 10 CO1 [K<sub>3</sub>]

$$\frac{d^2x}{dt^2} + 2\zeta\omega_n \frac{dx}{dt} + \omega_n^2 x = G(t),$$

where:

- $x(t)$  is the position of the target,
- $\omega_n = 4$  rad/s is the natural frequency,
- $\zeta = 0.5$  is the damping ratio,
- $G(t) = 5\cos(t)$  N is a feedback gain.

1. Solve the homogeneous equation for the system.
2. Use the variation of parameters method to find a particular solution.

3. a) A jammer emits a signal with the frequency spectrum:  $F(\omega) = \text{rect}\left(\frac{\omega}{2\pi}\right)$ . 10 CO4 [K<sub>4</sub>]  
Determine the corresponding time-domain signal to understand its effect on communication systems.

- b) A radar system sends out a short pulse modeled by the function  $p(t) = \text{sinc}(t)$ . Find its Fourier Transform to determine the frequency spectrum and bandwidth. 10 CO4 [K<sub>4</sub>]

4. a) The noise from an aircraft propeller is modeled as a periodic step function: 10 CO2 [K<sub>2</sub>]

$$f(x) = \begin{cases} 1, & 0 \leq x < \pi, \\ -1, & \pi \leq x < 2\pi. \end{cases}$$

Find the Fourier series for  $f(x)$  over  $(0, 2\pi)$  to identify noise harmonics and develop noise-canceling systems.

- b) The temperature of a tank's armor under cyclic thermal stress is given by: 10 CO2 [K<sub>3</sub>]

$$f(x) = x^2 - L^2, x \in (-L, L).$$

Find the Fourier series of f(x) to study the distribution of stress and predict failure points.

**Answer any ONE Question**

**PART B (1\*20 = 20 Marks)**

5. a) In electronic warfare, a jamming signal is used to disrupt the communication of enemy equipment. Suppose the jamming signal is a periodic sinusoidal wave given by: 10 CO4 [K<sub>4</sub>]

$$f(t) = A \sin(\omega t) \text{ where } A \text{ is the amplitude and } \omega \text{ is the angular frequency.}$$

Find the Laplace transform of the jamming signal, considering it as a periodic function with period  $T = \frac{2\pi}{\omega}$ .

- b) The vibrations in a tank's engine can be modeled as a damped sinusoidal function:  $f(t) = e^{-\alpha t} \cos(\omega_0 t)$  Calculate its frequency spectrum to identify resonance frequencies that may impact performance. 10 CO4 [K<sub>3</sub>]

OR

6. a) The barrel of a gun experiences periodic heating during firing, with heat flux modeled by: 10 CO2 [K<sub>3</sub>]

$$f(x) = \begin{cases} H, & 0 \leq x \leq L, \\ 0, & L < x \leq 2L. \end{cases}$$

where H is the heat intensity. Find the Fourier series for f(x) over (0,2L) to model heat distribution

- b) Sonar systems onboard submarines rely on wave propagation characteristics 10 CO1 [K<sub>2</sub>]  
analyzed using eigenvalues and eigenvectors of the propagation matrix :

$$P = \begin{bmatrix} 5 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 3 \end{bmatrix} \text{ Find the eigenvalues and eigenvectors of P.}$$

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