KUMARAGURU
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Register Number:……

M.TECH DEGREE EXAMINATIONS: NOV / DEC 2024

(Regulation 2018)

First Semester

DEFENCE TECHNOLOGY

P18DTT1003**:** Advanced Engineering Mathematics

COURSE OUTCOMES

CO1: Know the methods for solving differential equations, generating functions

CO2: Understand basic concepts of Fourier Transform, Laplace Transforms and solve problems with periodic functions, step functions, impulse functions and convolution.

CO3: Demonstrate MATLAB programming for engineering problems

CO4: Understand the utilization of mathematical methods for solving problems having relevance to defence applications.

Time: Three Hours **Maximum Marks: 100**

Answer all the Questions:-

PART A (10 x 1 = 10 Marks)

1. The steps involved in solving an ODE of the form $y'' + 4y = sin(2t)$ using Laplace transform are CO2 [K_1]

   1) Find $L[y]$and apply inverse Laplace transform on both sides.
   2) Take Laplace transform on both sides of the given ordinary differential equation.
   3) Using suitable method, evaluate inverse Laplace transform to get y.
   4) Apply the given initial conditions in the equation obtained.

   a) 3-1-2-4 b) 2-1-3-4

   c) 3-4-2-1 d) 2-4-1-3

2. The integrating factor of $(2x+3)\frac{dy}{dx} - y = e^{2x}(2x+3)$is CO1 [$K_2$]

   a) $\frac{1}{2x+3}$ b) $\log(2x+3)$

   c) $\frac{1}{\sqrt{(2x+3)}}$ d) $e^{\frac{1}{2x+3}}$

3. Which type of variable is used in Linear Programming Problems to ensure the feasibility of an initial solution, especially when dealing with equality constraints? CO2 [K_1]

   a) Slack variable b) Unrestricted variable

   c) Surplus variable d) artificial variable

4. Match list I with list II and select the correct answer from the table given below CO4 [K_3]

List I (Test)	List II (Test Statistic)
A. Independence of attributes	1. t - test
B. Difference between means of independent samples	2. Paired t - test
C. Equality of two variances	3. Chi- square test
D. Difference of means of dependent samples	4. F – test

a) A-4,B-2, C-3, D-1 b) A-4,B-1, C-2, D-3

c) A-3,B-1,C-4,D-2 d) A-3,B-2, C-4, D-1

5. The Laplace transform of $\sin(3t)$ is CO2 [K2]

a) $\frac{1}{s^2+9}$ b) $-\frac{1}{(s+3)^3}$

c) $\frac{1}{(s+3)^3}$ d) $-\frac{1}{s^3}$

6. Which of the following properties is true for both the Laplace transform L and its inverse L^{-1} CO2 [K3]

a) They are linear transformations. b) They are differential operators.

c) They are nonlinear transformations. d) They are scalar operators.

7. In a 3×33 \times 33×3 Latin Square Design, how many degrees of freedom are there for error? CO4 [K1]

a) 0 b) 1

c) 2 d) 3

8. To test whether the average amount spent per customer has increased from the previous average of \$50, a company conducts a hypothesis test with $H_0: \mu = 50$ and $H_1: \mu > 50$. What type of test is this? CO4 [K3]

a) Left-tailed test b) Right-tailed test

c) Two-tailed test d) None of these

9. To test a hypothesis about a single mean, it is known that $n = 100,\ \bar{x} = 1580,\ s = 90,\ \mu = 1600.$ The value of the test statistic z is CO4 [K2]

a) -2.22 b) 3.54

c) 1.96 d) -1.87

10. Examine the two statements carefully and select the answer using the codes given below: CO4 [K2]

Assertion (A): The probability of a certain event is 1.
Reason (R): The sum of probabilities of all outcomes of a random experiment is 1.

a) Both A and R are true and R is the correct explanation for A b) A is true but R is false

c) Both A and R are true but R is not the correct explanation for A d) A is false but R is true

PART B (10 x 2 = 20 Marks)

11. A card is drawn form a well-shuffled pack of playing cards. What is the probability that it is either a spade or an ace? CO4 [K1]

12. Solve $y'' + 4y' + 4y = 0$. CO1 [K3]

13. Apply Euler's method to find y(0.2) at $y' = x - y,\ y(0) = 1.$ CO1 [K2]

14. Determine whether the following function is periodic. $f(x) = 5sin(2x) + 3cos(2x)$ CO2 [K2]

15. A company manufactures two products, A and B. The profit per unit for Product A is $40, and for Product B is $30. Each unit of Product A requires 2 hours of labor and 3 hours of machine time, while each unit of Product B requires 1 hour of labor and 1 hour of machine time. The company has a maximum of 60 labor hours and 75 machine hours available per week. Formulate a linear programming problem to determine how many units of each product the company should produce to maximize profit, subject to labor and machine time constraints. CO2 [K_3]

16. Define Type I and II error CO4 [K_2]

17. Find the Laplace transform of e^{-at} CO2 [K_4]

18. Write down the indicial equation of $2xy'' + (x+1)y' + y = 0$. CO1 [K_3]

19. Write down the Bessel's function of first kind. CO1 [K_3]

20. State Polya lemma CO4 [K_1]

PART C (6 x 5 = 30 Marks)

21. A company implemented a new training program to improve employee productivity. The following table shows the results of an assessment after the training: CO4 [K_3]

	Productivity Improved	**Productivity Improved**
Trained	40	10
Not Trained	20	30

Use a significance level of α=0.05 to determine if there is a significant association between treatment with the drug and symptom reduction.

22. Solve $y'' + 4y = sec\,2\,x$ CO1 [K_2]

23. Find the Laplace transform of the following square wave function: CO2 [K_3]

$$f(t) = \begin{cases} 2 & 0 < t < \pi \\ -2 & \pi < t < 2\pi \end{cases}$$

This pattern repeats with period $T = 2\pi$.

24. Solve the following L.P.P graphically CO2 [K_3]

$$\text{Max } Z = 100x_1 + 40x_2$$

Subject to $5x_1 + 2x_2 \leq 1000$, $3x_1 + 2x_2 \leq 900$, $x_1 + 2x_2 \leq 500$ and $x_1, x_2 \geq 0$.

25. Solve the following differential equation by power series method $y' + y = 0$. CO1 [K_3]

26. State and prove Burnside lemma CO4 [K_3]

Answer any FOUR Questions

PART D (4 x 10 = 40 Marks)

27. Using Laplace transform , Solve the following IBVP: CO1 [K4]

$$\text{PDE} : u_t = u_{xx}\,, 0 < x < l\,, t > 0$$

$$\text{BCs} : u(0,t) = u(l,t) = 0\,, t > 0$$

$$\text{ICs} : \; u(x,0) = sin\pi x\,,\; u_t(x,0) = -sin\pi x \;\; 0 < x < l.$$

28. An experiment consists of three boxes (A, B, and C) containing balls of different colors. CO1 [K4]

- **Box A** contains 2 white, 3 black, and 4 green balls.
- **Box B** contains 3 white, 1 black, and 3 green balls.
- **Box C** contains 1 white, 4 black, and 5 green balls.

A box is selected at random, and two balls are drawn from it without replacement. The balls drawn are one white and one green. What is the probability that these balls came from Box B?

29. Use Simplex to solve the LPP CO2 [K3]

$$Max\ Z = 5x_1 + 8x_2$$

subject to

$$3x_1 + 2x_2 \leq 50, x_1 + 4x_2 \leq 40, 2x_1 + 5x_2 \leq 60, x_1, x_2 \geq 0$$

30. Derive the Bessel's function. CO1 [K4]

31. A company conducts a study to evaluate the efficiency of five different assembly line teams (T1, T2, T3, T4, T5) on assembling five different types of products (P1, P2, P3, P4, P5) using five unique assembly techniques (A, B, C, D, E). The data below shows the time taken (in minutes) by each team to complete an assembly for each product with a particular technique. Using a significance level of $\alpha = 0.05$, test if there are significant differences in mean assembly times across teams, products, and techniques. CO4 [K4]

	P_1	P_2	P_3	P_4	P_5
T_1	A31	B24	C20	D20	E18
T_2	B21	C27	D23	E25	A31
T_3	C21	D27	E25	A29	B21
T_4	D21	E25	A33	B25	C22
T_5	E21	A37	B24	C24	D20

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