



M.E DEGREE EXAMINATIONS: NOV/ DEC 2024

(Regulation 2024)

First Semester

STRUCTURAL ENGINEERING

24STT502: Theory of Elasticity and Plasticity

COURSE OUTCOMES

- CO1:** Mathematical formulation of elasticity problems in equilibrium and compatibility equations for 3D problems.
- CO2:** Formulating Boundary Value Problems in Linearized Elasticity and Solving 2D Problems with Airy's Stress Functions.
- CO3:** Solution to boundary value problems corresponding to end torsion of prismatic beams
- CO4:** Analyze with Plasticity Theories.
- CO5:** Analyze with fracture mechanics.

Time: Three Hours

Maximum Marks: 100

PART A (4*20 = 80 Marks)

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| 1. | a) | A cylindrical bar is subjected to a uniaxial tensile load of 50 kN. The bar has a diameter of 20 mm and a length of 300 mm. Using the generalized Hooke's law, calculate the axial stress, strain, and the change in length if the modulus of elasticity is $E = 200$ GPa and Poisson's ratio is $\nu = 0.3$. | 10 | CO1 | [K ₃] |
| | b) | A thick-walled cylinder is subjected to internal pressure $p_i = 100$ MPa and external pressure $p_o = 50$ MPa. Derive the expressions for radial and tangential stresses in cylindrical coordinates. Calculate these stresses at $r = 75$ mm. | 10 | CO1 | [K ₃] |
| 2. | a) | A plate is subjected to plane stress conditions with a stress distribution given by: $\sigma_x = ax^2 + by$, $\sigma_y = cy^2 + dx$, $\tau_{xy} = exy$. Verify whether this stress distribution satisfies the equilibrium equations. Derive the corresponding Airy's stress function. | 10 | CO2 | [K ₂] |
| | b) | A thick cylinder with inner radius $r_i = 50$ mm and outer radius $r_o = 150$ mm is subjected to an internal pressure $p_i = 20$ MPa. Determine the radial and hoop | 10 | CO2 | [K ₂] |

stresses at any radius r using Lamé's equations. Plot the stress distribution across the cylinder thickness.

3. a) A solid rectangular beam with dimensions $b \times h$ is subjected to pure torsion. 10 CO3 [K₃]
Using the Prandtl stress function method. Derive the expression for shear stress distribution in the cross-section.
- b) Explain the membrane analogy for torsion and apply it to find the torsional constant J for a thin-walled hollow rectangular section with dimensions $a = 100$ mm, $b = 100$ mm, and thickness $t = 2$ mm. Calculate the torsional rigidity GJ and shear stress for a given torque $T = 500$ Nm. 10 CO3 [K₃]
4. a) A material follows a linear stress-strain relationship up to the yield stress $\sigma_y = 250$ MPa and undergoes strain hardening thereafter, with the stress-strain curve given as $\sigma = \sigma_y + k\epsilon^p$, where $k = 1000$ MPa. Explain the significance of strain hardening in materials 10 CO4 [K₃]
- b) A pipeline is subjected to internal pressure and bending loads, causing localized plastic deformation. The pipeline material follows von Mises yield criterion with $\sigma_y = 300$ MPa. Calculate the equivalent stress under combined axial $\sigma_x = 150$ MPa and hoop stresses $\sigma_h = 200$ MPa. 10 CO4 [K₃]

Answer any ONE Question

PART B (1*20 = 20 Marks)

5. a) Analyze a real-life engineering failure due to fracture, apply fracture mechanics to identify the cause, and propose design modifications to prevent future failures, considering stress intensity and crack propagation. 12 CO5 [K₄]
- b) Evaluate the importance of fracture mechanics in modern engineering design. 8 CO5 [K₃]

OR

6. a) Given a plate with a thickness $t = 10$ mm with a crack length $a = 25$ mm, under an applied load of $P = 5$ kN. Calculate the energy release rate and analyze the likelihood of crack propagation, comparing it with the materials fracture toughness. 12 CO5 [K₄]
- b) Discuss the role of the stress intensity factor in characterizing fracture. 8 CO5 [K₃]
