



M.E/ M.TECH DEGREE EXAMINATIONS: NOV / DEC 2024

(Regulation 2024)

First Semester

EMEDDED SYSTEM TECHNOLOGIES

24MAI503: Applied Mathematics for Embedded Systems

The use of statistical tables is permitted

COURSE OUTCOMES:

- CO1: Analyze and filter noise in communication systems by applying Fourier transforms, ensuring clearer signal transmission in embedded systems.
- CO2: Apply PDE solutions to model and solve heat distribution problems in the design of embedded thermal management systems for electronic devices.
- CO3: Apply Dijkstra's and Prim's algorithms to optimize routing in wireless sensor networks for smart cities, ensuring efficient data flow.
- CO4: Apply linear programming to optimize resource allocation in supply chain management for embedded systems in industrial automation.
- CO5: Apply probability distributions to model and predict the reliability of embedded control systems in autonomous vehicles under varying conditions.
- CO6: Apply queuing models to optimize server load and response times in cloud-based embedded systems for IoT applications, ensuring minimal latency.

Time: Three Hours

Maximum Marks: 100

PART A (4*20 = 80 Marks)

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|----|----|--|---|-----|-------------------|
| 1. | a) | Obtain infinite Fourier transform of Dirac-Delta function. | 2 | CO1 | [K ₁] |
| | b) | Determine the Fourier transform of $f(x)$ defined by | 8 | CO1 | [K ₂] |

$$f(x) = 1, |x| \leq a$$

$$= 0, |x| > a > 0 \text{ Hence evaluate } \int_0^{\infty} \frac{\sin x}{x} dx$$

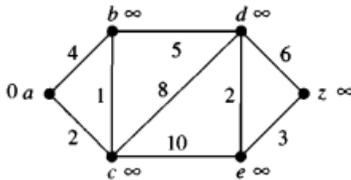
Scenario: Thermal Dissipation in a Microcontroller: An embedded system's microcontroller generates heat while operating. The temperature distribution $u(x,t)$ in the microcontroller at time t and position x is governed by the one-dimensional heat equation:

$$\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$$

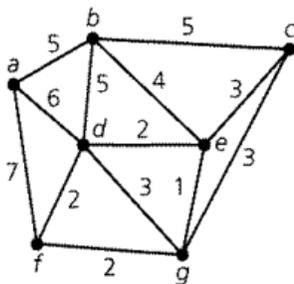
with α being the thermal diffusivity. The initial temperature distribution is $u(x,0) = f(x)$.

- c) Apply the Fourier transform to the heat equation $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$ and derive the transformed equation in the frequency domain. 5 CO2 [K₃]
- d) Use the Fourier transform of the initial condition $u(x,0) = f(x)$. to express the final temperature distribution $u(x, t)$ at any time t . 5 CO2 [K₃]

2. a) Define path in a graph G. 2 CO3 [K₁]
- b) Using Dijkstra's Algorithm to find a Shortest Path from a to z for the following graph. 8 CO3 [K₃]



- c) Define chromatic number of a graph. 2 CO3 [K₁]
- d) Use kruskal's algorithm to obtain the optimum tree for the following graph. 8 CO3 [K₃]



3. a) Use the graphical method to solve the following LP problem. 5 CO4 [K₃]
 Maximize $Z = 15x_1 + 10x_2$
 subject to the constraints (i) $4x_1 + 6x_2 \leq 360$, (ii) $3x_1 + 0x_2 \leq 180$,
 (iii) $0x_1 + 5x_2 \leq 200$ and $x_1, x_2 \geq 0$.
- b) A company has three production facilities, S1, S2 and S3 with production capacity of 7, 9 and 18 units (in 100s) per week of a product, respectively. These units are to be shipped to four warehouses, D1, D2, D3 and D4 with requirement of 5, 8, 7 and 14 units (in 100s) per week, respectively. The 10 CO4 [K₄]

transportation costs (in rupees) per unit between factories to warehouses are given in the table below:

	D1	D2	D3	D4	Supply
S1	19	30	50	10	7
S2	70	30	40	60	9
S3	40	8	70	20	18
Demand	5	8	7	14	34

Apply MODI method to obtain optimal solution of transportation problem.

- c) A computer Centre has three expert programmers. The Centre wants three application programs to be developed. The head of the computer Centre, after carefully studying the programs to be developed, estimates the computer time in minutes required by the experts for the application programs as follows

		Programs		
		A	B	C
Programmers	1	120	100	80
	2	80	90	110
	3	110	140	120

Assign the programmers to the programs in such a way that the total computer time is minimum.

4. a) In a shooting test, the probability of hitting the target is $\frac{1}{2}$ for A, $\frac{2}{3}$ for B and $\frac{3}{4}$ for C. If all of them fire at the target, find the probability that none of them hits the target. 2 CO5 [K₃]
- b) State any two properties of moment generating function (MGF). 2 CO5 [K₁]
- c) A computing system manager states that the rate of interruptions to the internet service is 0.2 per week. Use the Poisson distribution to find the probability of (i) one interruption in 3 weeks (ii) at least two interruptions in 5 weeks (iii) at most one interruption in 15 weeks. 8 CO5 [K₃]
- d) The local authorities in a certain city install, 10,000 electric lamps in the streets of the city. If these lamps have an average life of 1000 burning hours with a standard deviation of 200 hours, assuming normality, what number of lamps might be expected to fail (i) in the first 800 burning hours? (ii) between 800 and 1.200 burning hours? 8 CO5 [K₃]

Answer any ONE Question
PART B (1*20 = 20 Marks)

5. a) A road transport company has one reservation clerk on duty at a time. He handles information of bus schedules and makes reservations. Customers arrive at a rate of 8 per hour and the clerk can, on an average, service 12 customers per hour. After stating your assumptions, answer the following: 10 CO6 [K4]
(i) What is the average number of customers waiting for the service of the clerk? (ii) What is the average time a customer has to wait before being served?
- b) Let there be an automobile inspection situation with three inspection stalls. Assume that cars wait in such a way that when a stall becomes vacant, the car at the head of the line pulls up to it. The station can accommodate almost four cars waiting (seven in the station) at one time. The arrival pattern is Poisson with a mean of one car every minute during the peak hours. The service time is exponential with a mean of 6 minutes. Obtain the average number of customers in the system during the peak hours. 10 CO6 [K4]

OR

6. a) Consider a single server queuing system with Poisson input and exponential service times. Suppose the mean arrival rate is 3 calling units per hour, the expected service time is 0.25 hour and the maximum permissible calling units in the system is two. Derive the steady-state probability distribution of the number of calling units in the system, and then calculate the expected number in the system. 10 CO6 [K4]
- b) A tax consulting firm has 4 service counters in its office for receiving people who have problems and complaints about their income, wealth and sales taxes. Arrivals average 80 persons in an 8-hour service day. Each tax adviser spends an irregular amount of time servicing the arrivals, which have been found to have an exponential distribution. The average service time is 20 minutes. Calculate the average number of customers in the system, average number of customers waiting to be serviced. 10 CO6 [K4]
