Register Number:……………..

KUMARAGURU
college of technology
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B.E/B.TECH DEGREE EXAMINATIONS: NOV /DEC 2024
(Regulation 2014)
Fifth Semester
U14MAT506: Probability and Queuing Theory
(Common To CSE/IT)

COURSE OUTCOMES

CO1: Analyze random or unpredictable experiments and investigate important features of random experiments
CO2: Construct probabilistic models for observed phenomena through distributions which play an important role in many engineering applications
CO3: Associate random variables by designing joint distributions and correlate the random variables.
CO4: Know about random processes, in particular about Markov chains which have applications in engineering.
CO5: Identify the queueing model in the given system, find the performance measures and analyze the result

Time: Three Hours **Maximum Marks: 100**

Answer all the Questions:-

PART A (10 x 1 = 10 Marks)

1. Match List I with List II and select the correct the answers with following codes. CO3 [K_1]

List I	List II
A. $R_{xx}(0)$	(i). Auto correlation is maximum at $\tau = 0$
B. $R_{xx}(\tau)$	(ii). Random process is stationary
C. $R_{xx}(0) \geq \lvert R_{xx}(\tau)\rvert$	(iii). $E(X^2(t))$
D. $\lim_{\tau\to\infty} R_{xx}(\tau) = \overline{X}^2$	(iv). $R_{xx}(-\tau)$

	A	B	C	D
a)	ii	i	iii	iv
b)	iii	iv	i	ii
c)	ii	iv	iii	i
d)	iii	i	ii	iv

2. When X is a random variable and a and b are constants, then $Var(aX) =$ CO1 [K_2]

a) $Var(X)$ b) $aVar(X)$
c) $a^2Var(X)$ d) $a^2 + Var(X)$

3. The following are statements on correlation analysis. Which of the statements are true? CO2 [K_1]

1) When the rate of increase of one variable increases the rate of the other variable, then the correlation is said to be a positive correlation.
2) When the rate of decrease of one variable decreases the rate of the other variable, then the correlation is said to be a negative correlation.
3) The range of correlation coefficient if [-1,1]
4) The range of correlation coefficient if (-1,1)

a) 1,3 b) 1,2,3
c) 1,4 d) 2,3

4. The binomial distribution whose mean is 3 and variance 2 is given by CO2 [K1]

a) $9C_r\left(\frac{2}{3}\right)^r\left(\frac{1}{3}\right)^{9-r}$

b) $3C_r\left(\frac{2}{3}\right)^r\left(\frac{1}{3}\right)^{3-r}$

c) $2C_r\left(\frac{2}{3}\right)^r\left(\frac{1}{3}\right)^{2-r}$

d) $9C_r\left(\frac{1}{3}\right)^r\left(\frac{2}{3}\right)^{9-r}$

5. Assertion(A): The Markov chain with the tpm $P=\begin{pmatrix}0 & 0 & 1\\ 1 & 0 & 0\\ 1/2 & 1/2 & 0\end{pmatrix}$ is periodic. CO3 [K2]

Reason(R):The Markov chain is periodic if the d_i = GCD(m: $p_{ii}^{(m)} > 0$) is equal to 1.

a) Both A and R are Individually true and R is the correct explanation of A

b) Both A and R are Individually true but R is not the correct explanation of A

c) A is true but R is false

d) A is false but R is true

6. What is the probability that the waiting of the customer in the system exceeds t in (M/M/1):(∞/FIFO) queueing model model? CO4 [K1]

a) $P(W_s > t) = e^{(\mu - \lambda)t}$

b) $P(W_s > t) = e^{-(\mu - \lambda)t}$

c) $P(W_s > t) = e^{(\mu + \lambda)t}$

d) $P(W_s > t) = e^{-(\mu + \lambda)t}$

7. Find the correct sequence of steps involved in the computation of posterior probability CO1 [K2]

(1) Find $P(Ei/A) = \frac{P(E_i)P(A/E_i)}{\sum_{i=1}^{n} P(E_i)P(A/E_i)}$

(2) Find $P(E_i)$

(3) Find $\sum_{i=1}^{n} P(E_i)P(A/E_i)$

(4) Find $P(A/Ei)$

The correct sequence of the procedure is

a) 3 – 2 – 4 – 1

b) 4 – 3 – 2 – 1

c) 2 – 3 – 1 - 4

d) 2 – 4 – 3 – 1

8. In $(M/M/1:K/FIFO)$ model, the waiting time in the queue is CO4 [K1]

a) $\frac{\lambda'}{L_S}$

b) $\frac{Ls}{\lambda'}$

c) $\frac{Lq}{\lambda'}$

d) $\frac{\lambda'}{Lq}$

9. Examine the two statements carefully and select the answer using the codes given below CO5 [K2]

Assertion(A) : In a queueing model if the average arrival time of the customers is 20 minutes and service time is 10minutes then the average number of customers in the system is approximately 1**.**

Reason (R) : In (M/G/1) queueing model $L_s = \frac{\lambda^2\sigma^2 + \rho^2}{2(1-\rho)} + \rho$

a) Both A and R are Individually true and R is the correct explanation of A

b) Both A and R are Individually true but R is not the correct explanation of A

c) A is true but R is false

d) A is false but R is true

10. Choose the correct answer. In (M/M/c):(k / FIFO) queuing model — CO5 [K4]

1) Effective arrival rate is $\lambda' = \lambda(1 - P_0)$

2) $L_s = L_q + \frac{\lambda'}{\lambda}$

3) $W_s = \frac{L_s}{\lambda'}$

4) $W_q = \frac{L_q}{\lambda'}$

a) 1,2 b) 1,4

c) 2,3 d) 3,4

PART B (10 x 2 = 20 Marks)
(Answer not more than 40 words)

11. If A and B are independent events with $P(A)=\frac{1}{2}$ & $P(B)=\frac{1}{3}$, find $P(\bar{A}\cap\bar{B})$. CO1 [K2]

12. A box contains 4 bad and 6 good record players. 2 are drawn from the box at a time. One of them is tested and found to be good. What is the probability that other one is also good? CO1 [K3]

13. Given the probability density function $f(x)=\frac{k}{1+x^2}, -\infty < x < \infty$, find k CO2 [K1]

14. Regression equations are $8x-10y+66=0$, $40x-18y = 214$. What are the mean values of X and Y? CO2 [K2]

15. Find the mean and Variance of a stationary random process whose auto correlation function is $R_{XX}(\tau) = 18 + \frac{2}{6+\tau^2}$. CO3 [K2]

16. Define auto correlation function. CO3 [K1]

17. Patients arrive randomly and independently at a doctors consulting room from 8.00A.M. at an average rate of 1 every 5 minutes. The waiting room can hold 12 persons. What is probability that the room will be full when the doctor arrives at 9 A.M.? CO4 [K3]

18. State and explain the Kendall's notation for queueing models. CO4 [K1]

19. State: Little's formulae. CO5 [K1]

20. State Pollaczek Khinchine formula CO5 [K1]

Answer any FIVE Questions:-
PART C (5 x 14 = 70 Marks)
(Answer not more than 300 words)

Q.No. 21 is Compulsory

21. a). The contents of urns I, II & III are as follows 1 white, 2 black & 3 red balls. 2 white, 1 black & 1 red and 4 white, 5 black and 3 red balls respectively. One urn is chosen at random & two balls are drawn from it. They happen to be white & red. What is the (7) CO1 [K3]

probability that they come from urns I, II or III?

b). The local authorities in a city installed 2,000 electric lamps in streets. If the lamps have an average life of 1000 burning hours with S.D of 200 hours. (7) CO2 [K_3]

(i) What number of lamps might be expected to fail in the first 700 burning hours?

(ii) After what period of burning hours would you expect that 10% of lamps Would be still burning? Assume that lives of the lamps are normally distributed.

22. a).A problem is given to 3 students A, B, C whose chances of solving it are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ respectively. What is the probability that (7) CO1 [K_3]

i) The problem is solved.

ii) Exactly one of them solves the problem.

b) Calculate the correlation coefficient for the following heights in inches of fathers (x) and their sons (y). (7) CO2 [K_1]

X	65	66	67	67	68	69	70	72
Y	67	68	65	68	72	72	69	71

23. a). Show that the random process $X(t) = A\sin(\omega t + \theta)$ is WSS where A and ω are constants and θ is uniformly distributed in $(0,2\pi)$. (7) CO3 [K_2]

b). A raining process is considered as a two-state Markov chain. If it rains, it is considered to be in state 0 and if it does not rain, the chain is in state 1. The transition probability of the Markov chain is defined as $P=\begin{bmatrix}0.6 & 0.4\\ 0.2 & 0.8\end{bmatrix}$ (7) CO3 [K_2]

i) Find he probability that it will rain after three days from today

ii) Find also the unconditional probability that it will rain after three days with the initial probabilities of state 0 and state 1 as 0.4 and 0.6 respectively.

24. Customers arrive at a one-man barber shop according to a Poisson process with a mean inter-arrival time of 20 minutes. Customers spend an average of 15 minutes in the barber's chair. If an hour is used as the unit of time, then CO4 [K_4]

i) What is the probability that a customer need not wait for a hair cut?
ii) What is the expected number of customers in the barber shop and in the queue?
iii) How much time can a customer expect to spend in the barbershop?
iv) Find the average time that a customer spends in the queue.
v) What is the probability that there will be 6 or more customers waiting for service?

25. An automatic car wash facility operates with only one bay. Cars arrive according to a distribution with a mean of 4 cars / hr. and may wait in the facility's parking lot if the CO5 [K_3]

bay is busy. Find $L_s, L_q, W_s, W_q,$ if the service time.

i) is constant and equal to 10 minutes.
ii) follows uniform distribution between 8 and 12 minutes.
iii) follows normal distribution with mean 12 minutes and S.D. 3 minutes.
iv) follows a discrete distribution with values 4, 8 and 15 minutes with corresponding probabilities 0.2, 0.6 and 0.2.

26. a). The following table infers the number of births in a particular city during a particular month. Fit a Poisson distribution for the data given. Also calculate the expected frequencies against the given frequencies. (7) CO2 [K_1]

No. of Births	0	1	2	3	4
Frequency	122	60	15	2	1

(7) CO3 [K_3]

b). A housewife buys 3 kinds of cereals A, B and C. She never buys the same cereal in successive weeks. If she buys cereal A, the next week she buys B. However, if she buys B or C, the next week she is 3 times as likely to buy A as the other cereal. In the long run, how often does she buy each of the three cereals?

27. At a railway station, only one train is handled at a time. The railway yard is sufficient only for 2 trains to wait while the other is given signal to leave the station. Trains arrive at the station at average rate of 6 / hour and the railway station can handle them at the rate of 12 / hour. Assuming Poisson arrivals and exponential service distribution, find the probability for the number of trains in system. Also find the average waiting time of a new train coming to the yard. CO5 [K_3]

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