



B.E/B.TECH DEGREE EXAMINATIONS: NOV/DEC 2024

(Regulation 2018)

Third Semester

AE/AUE/CE/MECH/MCE/EEE

U17MAT3101: Partial Differential Equations and Transforms

COURSE OUTCOMES

- CO1:** Form partial differential equations and solve certain types of partial differential equations.
CO2: Know how to find the Fourier Series and half range Fourier series of a function.
CO3: To know how to solve one dimensional wave equation, one dimensional heat equation in steady state using Fourier Series.
CO4: Apply Fourier series to solve the steady state equation of two dimensional heat equation in Cartesian coordinates.
CO5: Apply the Fourier Transform, Fourier Sine and Cosine transform to certain functions and use Parseval's identity to evaluate integrals.
CO6: Evaluate Z-transform for certain functions. Estimate Inverse Z-transform of certain functions and to solve difference equations using them.

Time: Three Hours

Maximum Marks: 100

Answer all the Questions:-

PART A (10 x 2 = 20 Marks)

(Answer not more than 40 words)

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|-----|--|-----|-------------------|
| 1. | Form the PDE by eliminating the arbitrary constants from $z = ax^n + by^n$ | CO1 | [K ₂] |
| 2. | Find the complete solution of $z = p^2 + q^2$ | CO1 | [K ₂] |
| 3. | State Dirichlet's Conditions on Fourier Series. | CO2 | [K ₁] |
| 4. | Define the root mean square value of the function. | CO2 | [K ₃] |
| 5. | Write the three possible solutions of one-dimensional wave equation | CO3 | [K ₁] |
| 6. | Write two-dimensional heat equation. | CO4 | [K ₁] |
| 7. | Write the Fourier transform pair. | CO5 | [K ₁] |
| 8. | State Convolution theorem on Fourier Transforms. | CO5 | [K ₁] |
| 9. | Find the Z-transform of 'n'. | CO6 | [K ₃] |
| 10. | State Initial and Final Value theorem on Z-Transforms. | CO6 | [K ₁] |

Answer any FIVE Questions:-
PART B (5 x 16 = 80 Marks)
(Answer not more than 400 words)

11. a) Solve $(y - z)p + (z - x)q = x - y$. (8) CO1 [K₃]
 b) Solve $z = px + qy + \sqrt{16 + p^2 + q^2}$. (8) CO1 [K₃]

12. a) Find the Fourier series for $f(x) = x^2$ in $(0, \pi)$ (8) CO2 [K₃]
 b) Compute the two harmonics of the Fourier series of $f(x)$ given by the following table (8) CO2 [K₄]

x	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π
$f(x)$	1.21	1.05	0.98	0.62	0.87	1.1	1.21

13. a) A tightly stretched string with fixed ends points $x = 0$ & $x = l$ is initially at rest in its equilibrium position. If it is set vibrating giving each points a velocity $kx(l - x)$. Find the displacement. (16) CO3 [K₃]

14. a) A square plate is bounded by the lines $x=0, y=0, x=l$ and $y=l$. Its faces are insulated. The temperature along the upper horizontal edge is given by $u(x, l) = x(l - x), 0 < x < l$ while the other three edges are kept at 0 degree centigrade. Write the boundary conditions. (16) CO4 [K₃]

15. a) Find the Fourier transform of $f(x) = \begin{cases} 1 - x^2, & |x| \leq 1 \\ 0, & |x| > 1 \end{cases}$. (8) CO5 [K₃]

- b) Evaluate $\int_0^{\infty} \frac{1}{(x^2 + a^2)^2} dx$ using Parseval's identity. (8) CO5 [K₃]

16. a) Find $z^{-1} \left(\frac{10z}{z^2 - 3z + 2} \right)$ using partial fraction method. (8) CO6 [K₃]

- b) Find $z \left(\frac{1}{n(n+1)} \right)$ (8) CO6 [K₃]
