

B.E/B.TECH DEGREE EXAMINATIONS: NOV/ DEC 2024

(Regulation 2024)

First Semester

COMMON TO AERO / AUTO / CIVIL / MECH / MCE

24MAI111: Linear Algebra and Calculus

COURSE OUTCOMES

- CO1:** Apply eigenvalues for matrix diagonalization and transformations, and analyze results using computational tools.
- CO2:** Apply differentiation for solving optimization problems and enhance solutions through computational tools.
- CO3:** Apply partial differentiation for constrained optimization and evaluate with numerical techniques.
- CO4:** Apply integral calculus and computational tools to solve engineering problems.
- CO5:** Apply double integrals and computational tools for solving engineering problems.
- CO6:** Apply triple integrals techniques and computational tools to solve complex problems.

Time: Three Hours**Maximum Marks: 100****PART A (4 * 20 = 80 Marks)****Answer all the Questions**

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|----|----|---|---|-----|-------------------|
| 1. | a) | Define a real symmetric matrix and its importance in diagonalization. | 2 | CO1 | [K ₁] |
| | b) | State any two properties of eigenvalues. | 2 | CO1 | [K ₁] |
| | c) | Scenario: In a mechanical engineering application, a quadratic form $Q = 5x^2 + 8xy + 5y^2$ is used to analyze the stress on a material. Diagonalize this quadratic form to determine its principal axes. | 8 | CO1 | [K ₃] |
| | d) | Find the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$. | 6 | CO1 | [K ₃] |
| | e) | What role do eigenvalues play in an orthogonal transformation? Give an example. | 2 | CO1 | [K ₂] |
| 2. | a) | What is the total derivative? Provide an example. | 2 | CO3 | [K ₂] |
| | b) | State the significance of Jacobians in transformations. | 2 | CO3 | [K ₁] |
| | c) | Use Taylor's series to approximate $f(x, y) = \log(1 + x + y)$ near (0,0) up to second order derivatives. | 6 | CO3 | [K ₃] |
| | d) | Find the stationary points of $f(x, y) = x^2 + y^2 - 4x - 6y$ and classify them using Lagrange's multiplier method with the constraint $x + y = 4$. | 6 | CO3 | [K ₃] |
| | e) | Find the Jacobian of the transformation $u = x^2 + y, v = x - y^2$. | 4 | CO3 | [K ₃] |
| 3. | a) | Define integration by substitution and give an example | 2 | CO4 | [K ₁] |

- b) Evaluate $\int \frac{x^2}{(x^2+1)} dx$ using substitution. 2 CO4 [K₃]
- c) Solve $\int_0^\pi x^2 \cos x dx$ using integration by parts. 6 CO4 [K₃]
- d) Evaluate $\int \frac{dx}{(x+4)(x+3)(x+1)}$ using partial fraction method. 6 CO4 [K₃]
4. a) Define double integration and its application in finding area. 2 CO5 [K₃]
- b) Solve the double integral $\int_0^2 \int_0^x (x+y) dy dx$. 2 CO5 [K₃]
- c) Evaluate the double integral $\int_0^1 \int_0^{1-x} (x+y) dy dx$. 8 CO5 [K₃]
- d) Find the volume of ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ 8 CO6 [K₃]

PART B (1 x 20 = 20 Marks)
Answer any ONE Question

5. a) State Rolle's theorem. 2 CO2 [K₁]
- b) Verify Rolle's theorem for $f(x) = x^2 - 4x + 4$ in the interval [2,4]. 2 CO2 [K₃]
- c) Find the local maxima and minima of $f(x) = x^3 - 6x^2 + 9x + 2$. 6 CO2 [K₃]
- d) Using differentiation, determine the points of inflection for
 $f(x) = x^4 - 4x^3 + 6x^2$. 6 CO2 [K₃]
- e) Prove that the derivative of $f(x) = \sin(x) \cdot e^x$ is $f'(x) = e^x(\sin x + \cos x)$ 4 CO2 [K₁]

OR

6. a) Define Mean Value Theorem and state its significance. 2 CO2 [K₁]
- b) Verify the Mean Value Theorem for $f(x) = x^2 + 3x$ on the interval [0,2]. 2 CO2 [K₂]
- c) Find the maximum and minimum values of $f(x) = 2x^3 - 9x^2 - 24x - 20$ 8 CO2 [K₃]
- d) Give the significance of optimization problems using a real-world example. 2 CO2 [K₃]
- e) Verify Rolle's theorem for $f(x) = 2x^3 - 5x^2 - 4x + 3$ in the interval $[\frac{1}{2}, 3]$. 6 CO2 [K₃]

CO distribution summary:

	CO1	CO2	CO3	CO4	CO5	CO6
Marks (%)	20	20	20	20	10	10
